Statement of Research

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My interests within applied mathematics are broad, ranging from well-posedness problems arising in hydrodynamics, to Lie symmetry analysis and infinite dimensional Hamiltonian systems. However, I am primarily interested in nonlocal approaches to boundary value problems (BVPs) which employ a unified formalism developed by A.S. Fokas [1]. Using this nonlocal approach I have studied abstract BVPs [2], generalised the $\bar{\partial}$-approach to higher dimensions using Clifford analysis [3], provided a Hamiltonian structure for water waves with constant vorticity [4], derived well-posedness results for a classical problem in hydrodynamics [5] and classified stability criteria for a complex dynamical problem involving fluid loaded structures [6]. In other areas, I have derived results pertaining to the link between non-Lie symmetries and conservation laws [7] and given a non-existence proof for certain finite energy solutions to the classical water wave problem associated with a large class of non-trivial bottom surface topographies [8].

The nonlocal approach to boundary value problems involves the analysis of the so called *Global Relation*. The global relation is a family of integral equations coupling the boundary values of a given BVP. A salient feature of these integral equations is that they depend meromorphically on a parameter $k \in \mathbf{C}^n$, referred to as the spectral parameter. This parameter is analogous to the wave number associated with the Fourier transform. The analysis of the global relation *in spectral space* allows one to gain new insight into many difficult problems. The fact that the spectral dependence is meromorphic allows one to use techniques from complex analysis.

This new approach is highly applicable and incredibly versatile. It has been successful in providing explicit solutions for BVPs associated with linear and integrable nonlinear PDEs [1], deriving generalised Dirichlet-Neumann maps [4] and providing a framework to attack rigorous well-posedness issues [3, 5, 6]. In particular, it provides a map between the unknown boundary values and the known boundary data; the explicit nature of the global relation means that many problems can either be solved explicitly or can be simplified. For example, using classical results from functional analysis, it is possible to deduce regularity properties (see [5]) in a straightforward manner. For integrable nonlinear PDEs this approach allows one to extend the so-called inverse scattering method for initial-value problems to more complicated but also more physically relevant BVPs.

By combining the new unified approach of [1] with rigorous techniques it is now possible to investigate a large class of problems. These problems range from rigorous questions in the analysis of linear and nonlinear BVPs, to classification of stability problems in hydrodynamics. For example, currently I am revisiting the classical problem of classifying existence and regularity of solutions to elliptic BVPs. The nonlocal approach appears natural for this problem, due to the simple definition of the Sobolev spaces $H^s(\mathbf{R}^n)$ in spectral space. By analysing the global relation, and invoking some novel integral representations, we expect to classify existence and regularity results for a large class of boundary value problems, illustrating further the advantages of the nonlocal approach.

References

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