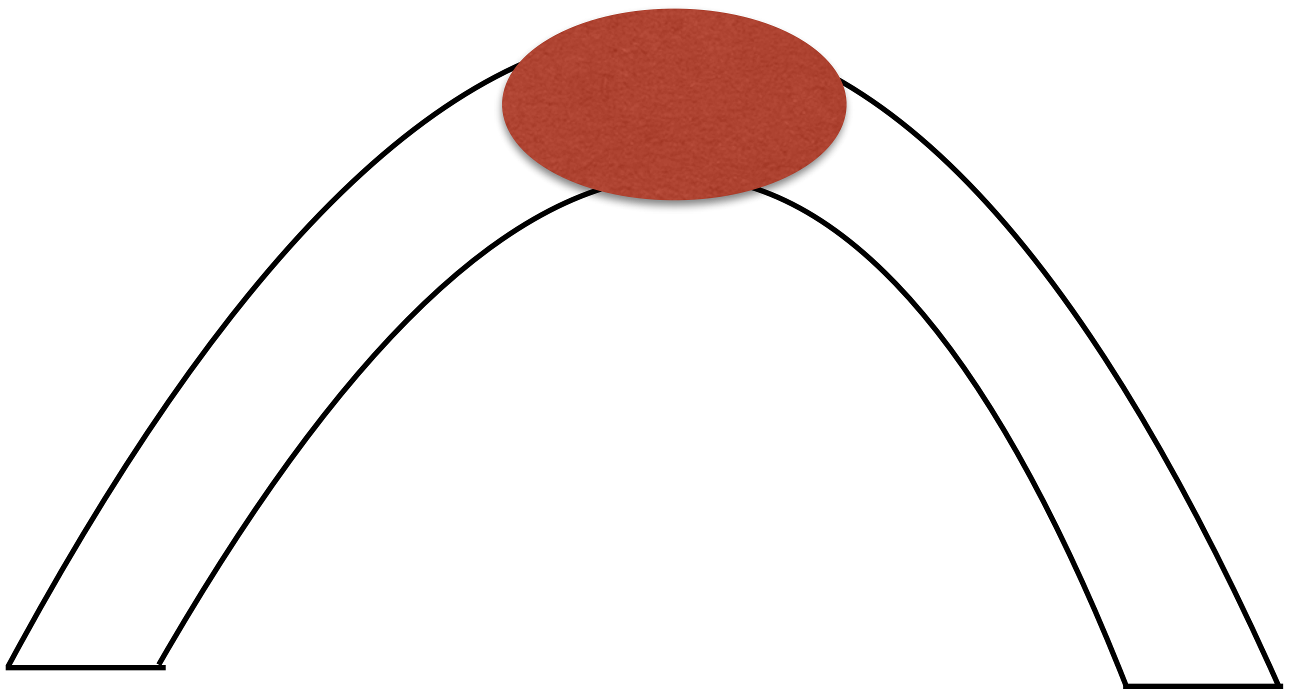
Beyond Hydrodynamics or MHD Modelling

Tony Arber
University of Warwick

Coronal Loops Workshop VII, Cambridge 21 July 2015

THE UNIVERSITY OF WARWICK

What do I want my next big coronal code to have?



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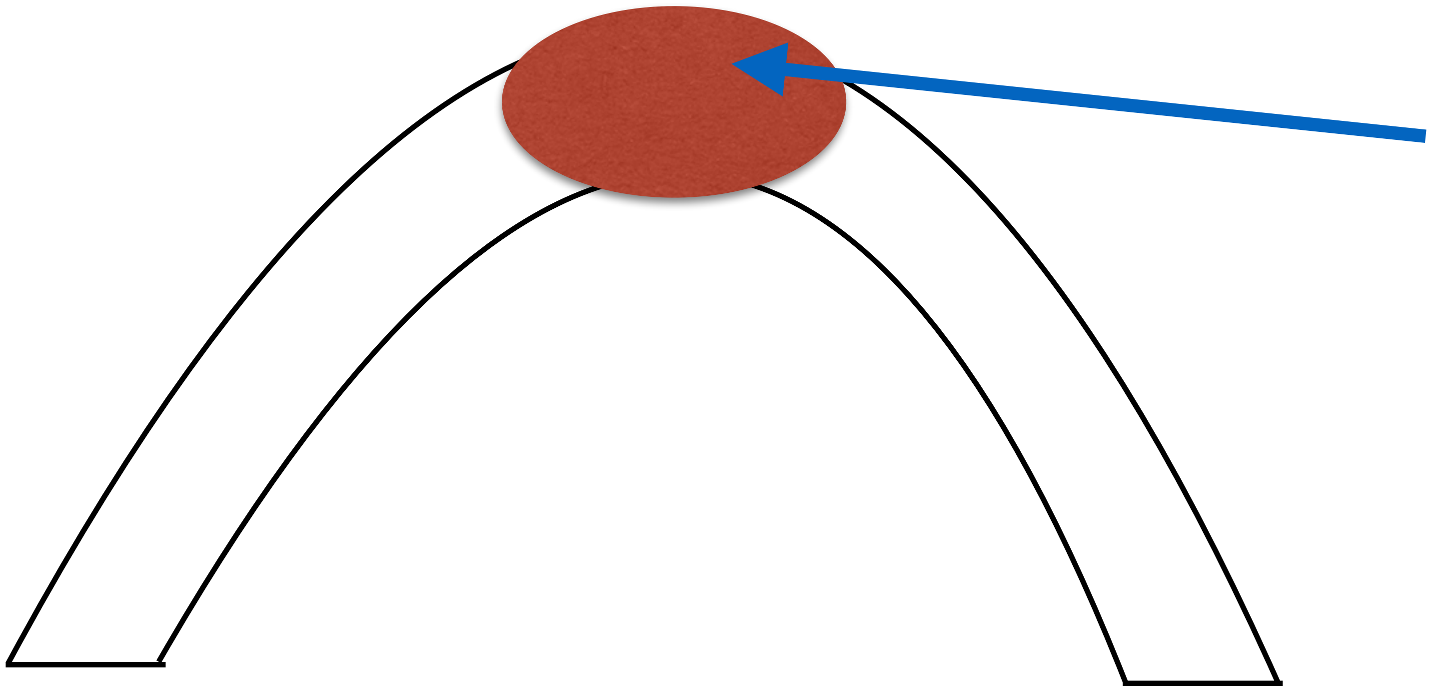

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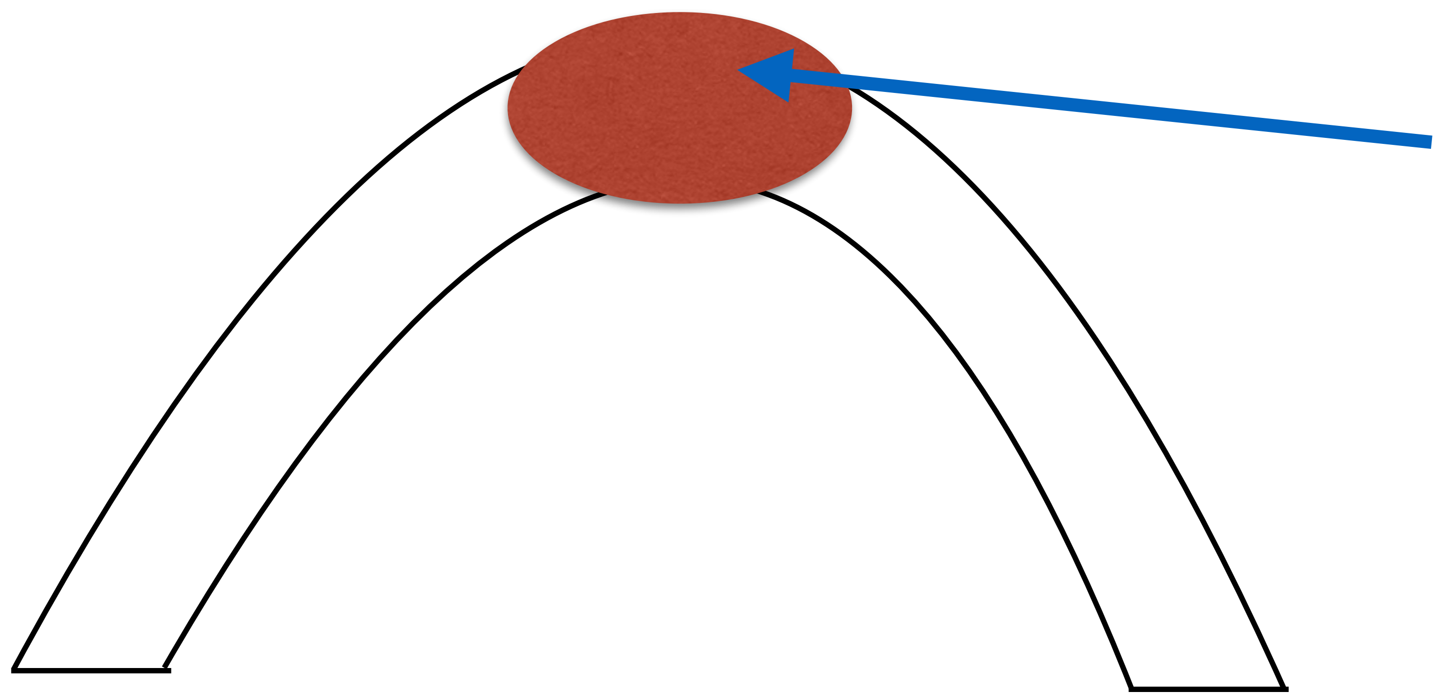

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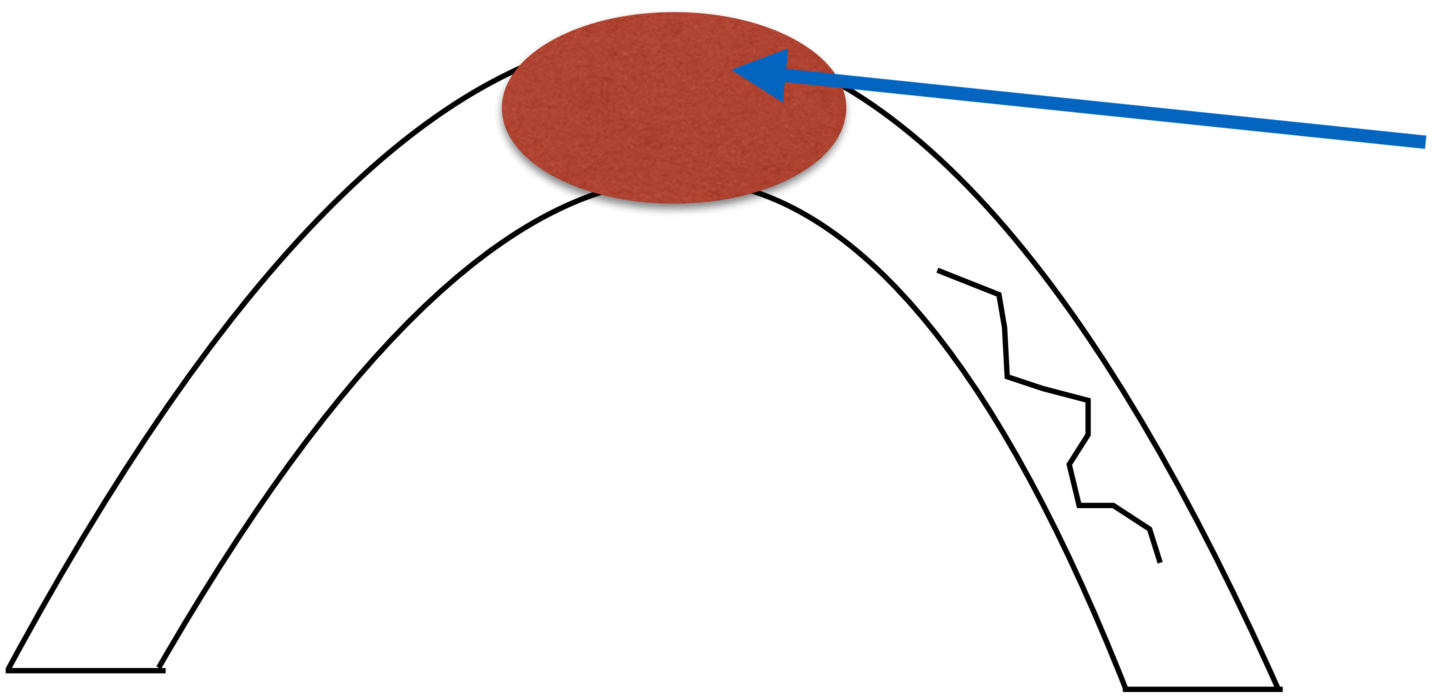

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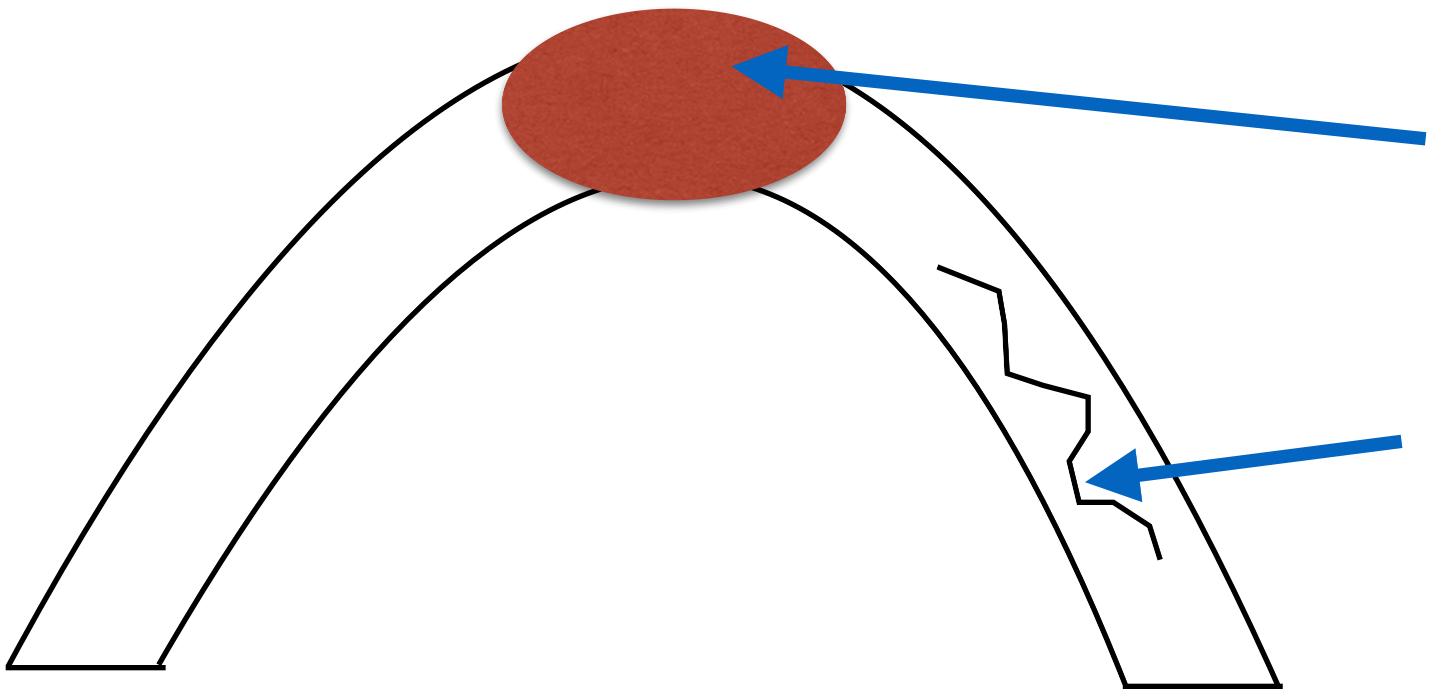

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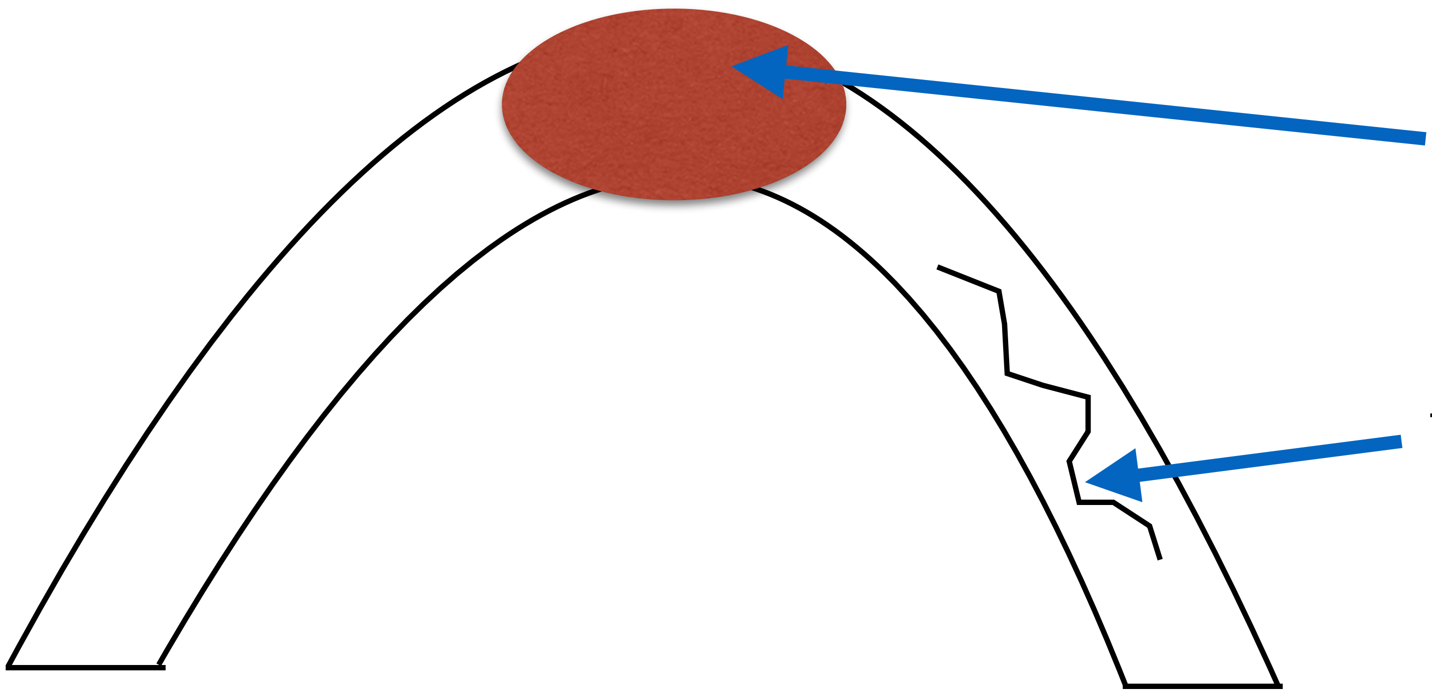

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- Return current
- Non-local thermal conduction
- Model for wave-particle interaction
- Quasi-linear instability saturation

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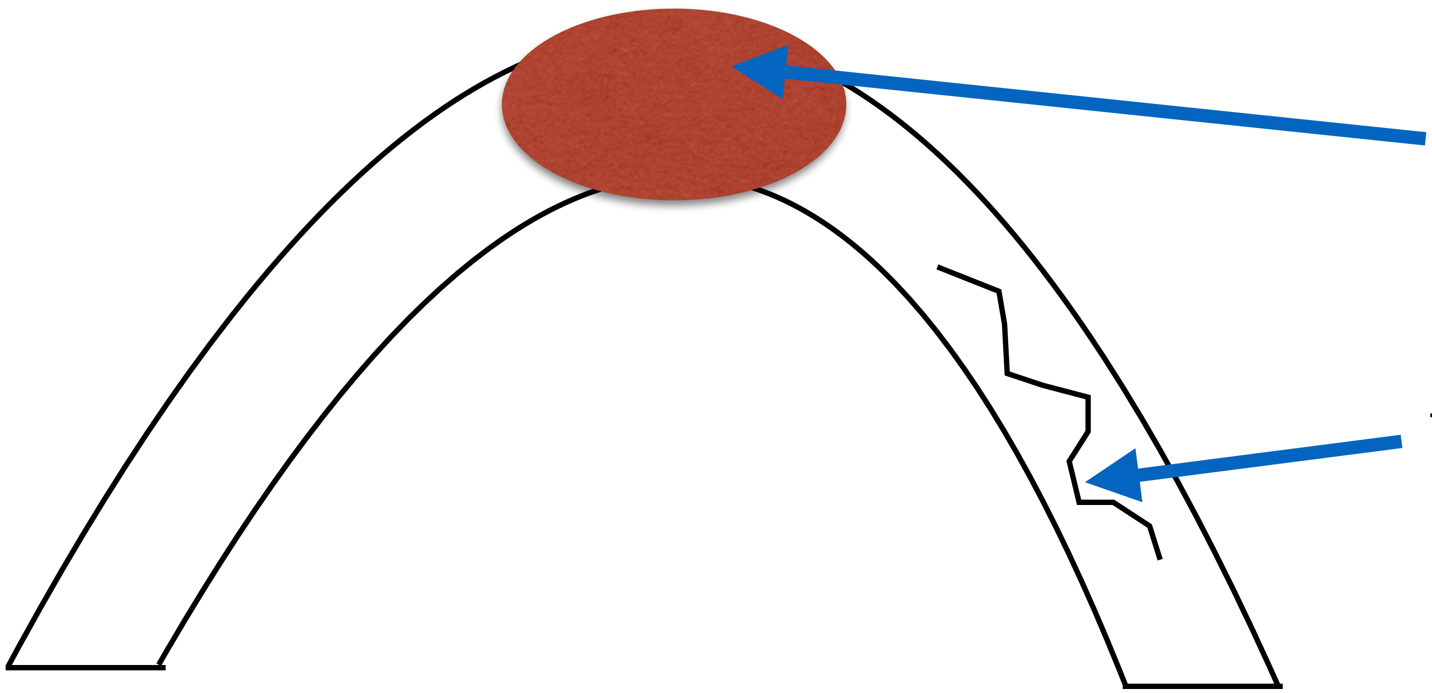

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3D model to couple to real MHD large scale driver
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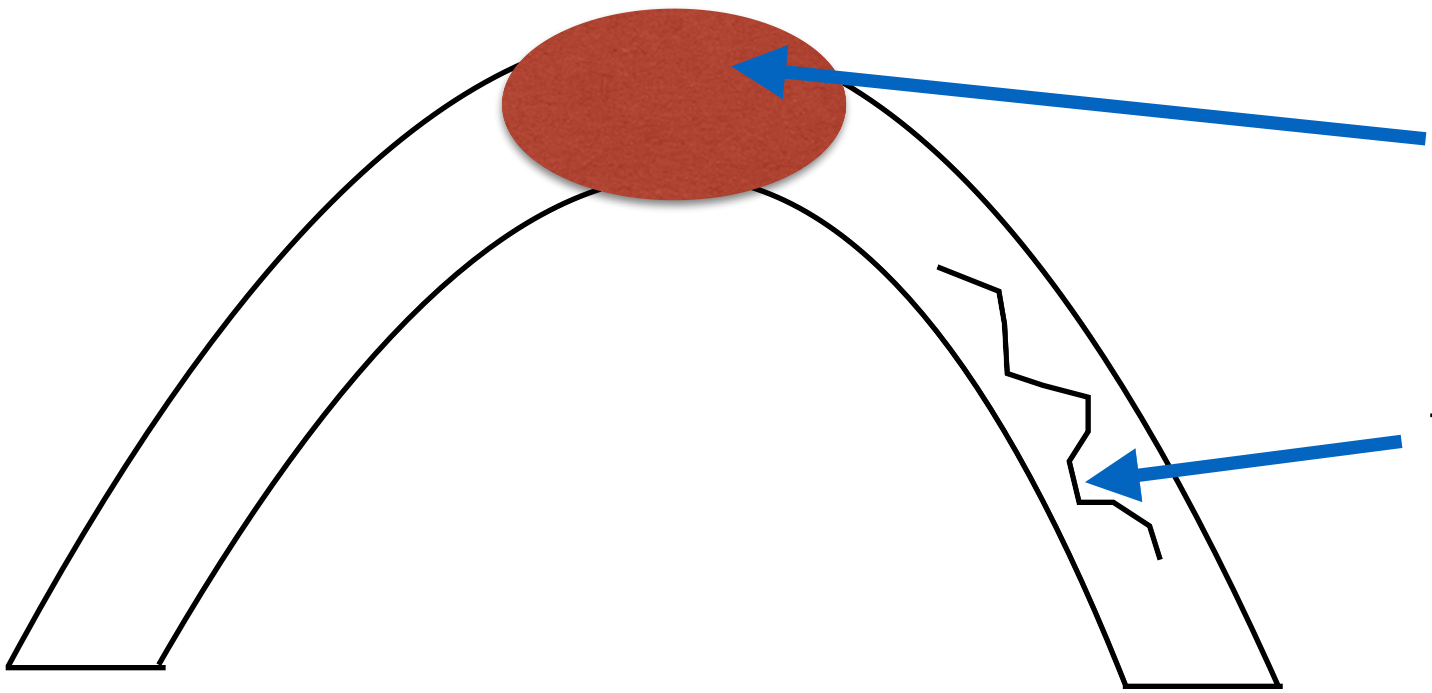

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Not included in this talk:
- Chromosphere
- Radiation transport except optically thin losses in TR

Overview

Where more than MHD do we want?

- Accuracy of MHD modelling for coronal loops
- Guiding centre vs. tracer particles
- Return current
- Thermal conduction
- Conduction fronts & double layers?

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Options for modelling

- Thermal flux limiters or SNB
- Return current
- Treating test particles self-consistently
- Stable double layer in MHD suppresses heat flow?

MHD Validity in Coronal Loops

Mean-free-path at T~1 MK and number density 10^9 cm^{-3} is $\lambda_{mfp} \simeq 10\mathrm{km}$

In a 10 G magnetic field ion Larmor radius $a_i \simeq 1\mathrm{m}$

Electron Larmor radius $a_e \simeq 1\mathrm{mm}$

Debye length $\lambda_D \simeq 1\mathrm{mm}$

Collisionless skin depth $\lambda_i = \dfrac{c}{\omega_{pi}} \simeq 10\mathrm{m}$

For coronal loops *L*~10-100 Mm and any *a*>>1 m MHD valid
…so what's the problem?

Fast particles - breaks Maxwellian approximation, non-local transport
Rapid localised heating - non-local transport
Particle acceleration - wave-particle interactions
Reconnection - scale-collapse down to λ_i
…

Study particle acceleration by test particles

Common to assess particle acceleration by using test particles

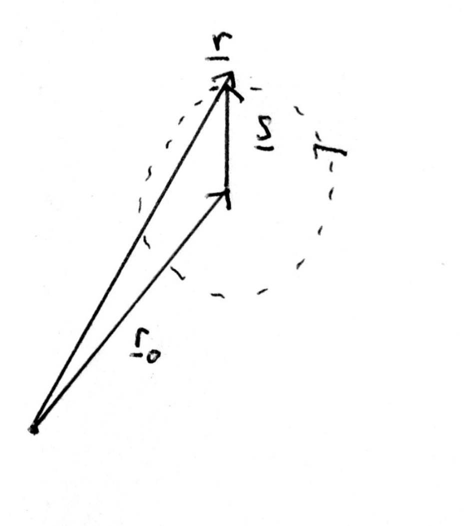
$$\frac{dv}{dt} = \frac{q}{m}(E + v \times B)$$

With E and B given from MHD simulations

Alternative approach uses guiding centre motion of particles in E and B from MHD

MHD usually derived from Boltzmann equation, take moments, long-length, slow approximation etc.

Can derive MHD from particle orbit theory

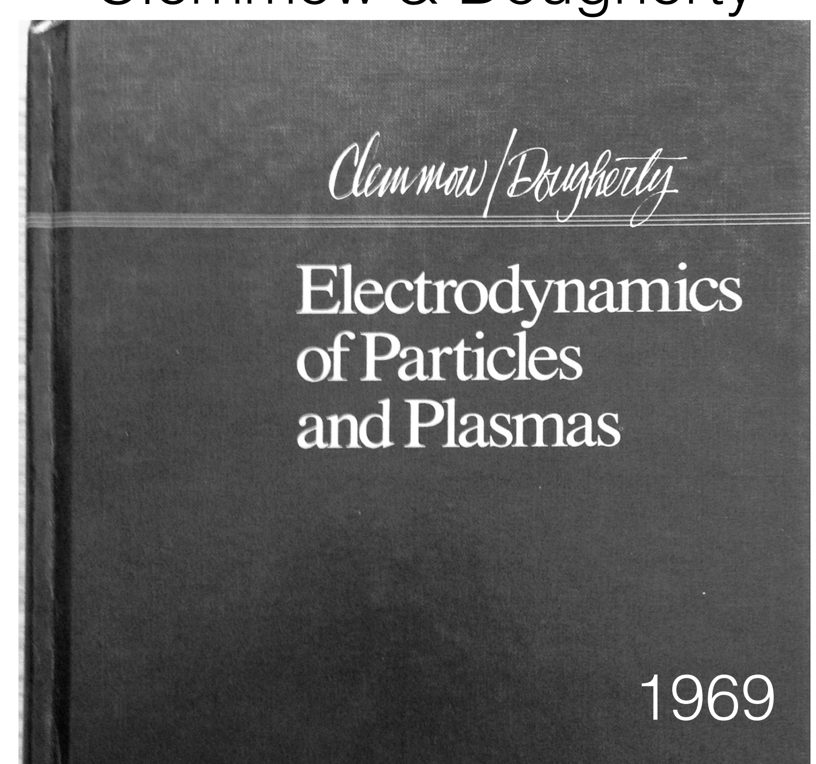


$$\mathbf{r} = \mathbf{r_0} + \mathbf{s} + \mathbf{s}*$$

$$\mathbf{s} = \sum_{n=1}^{\infty} \epsilon^n e^{in\theta} \mathbf{a}_n$$

$$\theta = \int^t \Omega(\mathbf{r_0}) dt$$

Clemmow & Dougherty



Guiding centre drift

With M as magnetic moment Clemmow & Dougherty show the perpendicular drift to be

$$\mathbf{u}_\perp = \mathbf{u}_E + \frac{\varepsilon\mathbf{B}}{B^2} \times \left\{\frac{M}{m}\,\mathrm{grad}\,B + \frac{u_\parallel^2}{B^2}(\mathbf{B}\cdot\mathrm{grad})\mathbf{B} + \frac{u_\parallel}{B}[(\mathbf{B}\cdot\mathrm{grad})\mathbf{u}_E + (\mathbf{u}_E\cdot\mathrm{grad})\mathbf{B}] + (\mathbf{u}_E\cdot\mathrm{grad})\mathbf{u}_E + u_\parallel\frac{\partial}{\partial t}\left(\frac{\mathbf{B}}{B}\right) + \frac{\partial\mathbf{u}_E}{\partial t}\right\}.$$

and the parallel moving given by

$$\dot{u}_\parallel = -\frac{M}{m}(\mathrm{grad}\,B)_\parallel + \frac{1}{\varepsilon}E_\parallel + \frac{\mathbf{u}_E}{B}\cdot\left\{\frac{\partial\mathbf{B}}{\partial t} + \left[\left(\frac{u_\parallel}{B}\mathbf{B} + \mathbf{u}_E\right)\cdot\mathrm{grad}\right]\mathbf{B}\right\}.$$

Average these drifts over a fluid element

$$\mathbf{j}_\perp = Ne\mathbf{u}_E + \frac{Nm}{B}\hat{\mathbf{B}} \times \left\{\frac{p_\perp}{NmB}\nabla B + \frac{\overline{(v_\parallel^2)}}{B^2}(\mathbf{B}\cdot\nabla)\mathbf{B} + \frac{\bar{v}_\parallel}{B}\left[\frac{\partial\mathbf{B}}{\partial t} + (\mathbf{B}\cdot\nabla)\mathbf{u}_E + (\mathbf{u}_E\cdot\nabla)\mathbf{B}\right] + (\mathbf{u}_E\cdot\nabla)\mathbf{u}_E + \frac{\partial\mathbf{u}_E}{\partial t}\right\} + \hat{\mathbf{B}} \times \left\{\nabla\left(\frac{p_\perp}{B}\right) - \frac{p_\perp}{B^3}(\mathbf{B}\cdot\nabla)\mathbf{B}\right\}.$$

MHD equations

Starting from fluid equation of motion and expanding to lowest order

$$\mathbf{u} = \mathbf{u}_E + v_{||}\mathbf{B}$$

Gives exactly the answer for the drift currents from guiding centre theory
Formally this gives a double adiabatic model but if we assume isotropy…

The sum of guiding centre motion plus the diamagnetic drifts is exactly equivalent to MHD

If test particles differ from the current distribution in MHD then this highlights a breakdown of MHD model due to FLR, wave-particle …

If this current is significant, i.e. worth studying!, then the MHD current must be changed to maintain quasi-neutrality

Return Currents - 1

The largest current that a charged beam can carry is given by the Alfven limited current. Above this the self generated B-field gyrates electrons within beam radius.

$$I \simeq \frac{\gamma m_e v}{e\mu_0}$$

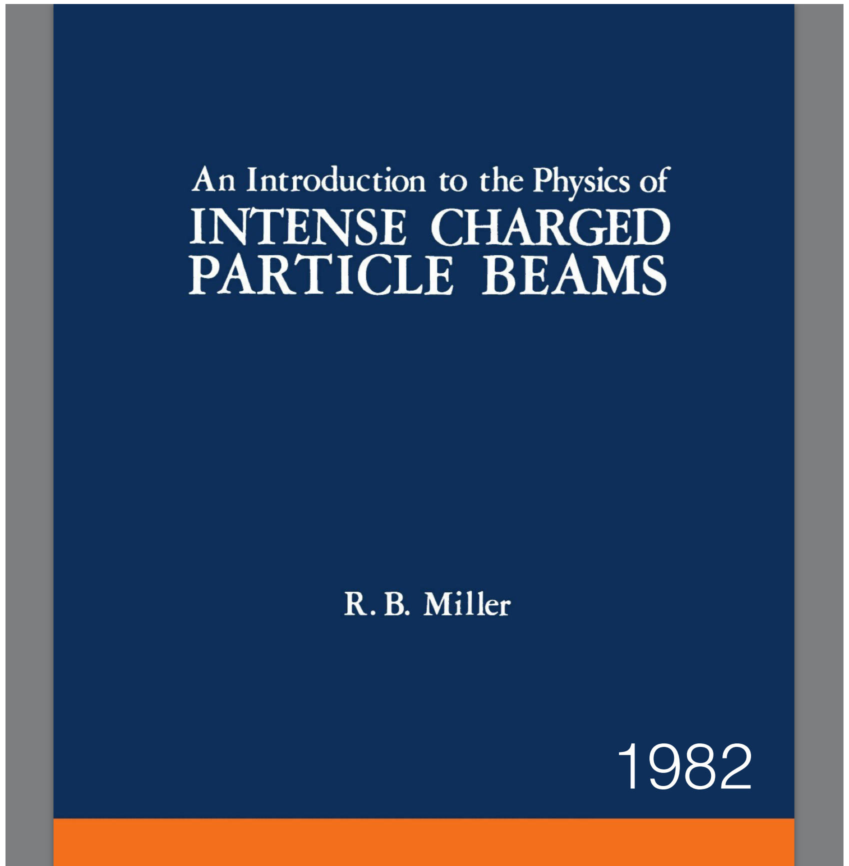
In practices beams in plasmas can exceed this due to a return current in the background. Return current either inductive or electrostatic.

On MHD length/time scales for coronal plasmas the whole plasma is quasi-neutral

$$\nabla . \mathbf{j} = 0$$

Fast current, not from MHD, and background MHD current, must then satisfy

$$\mathbf{j_f} + \mathbf{j_b} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$$



Return Currents - 2

$$\mathbf{j_b} = -\mathbf{j_f} + \frac{1}{\mu_0}\nabla \times \mathbf{B}$$

Note that now there is no requirement that $\nabla.\mathbf{j_b} = 0$

Consequently the Lorentz force on the background plasma is not

$$\mathbf{j}_{Total} \times \mathbf{B} = (\mathbf{j_b} + \mathbf{j_f}) \times \mathbf{B}$$

But…

$$\mathbf{j_b} \times \mathbf{B} = (-\mathbf{j_f} + \frac{1}{\mu_0}\nabla \times \mathbf{B}) \times \mathbf{B}$$

Fast electrons are collisionless so the electric field is given by

$$\mathbf{E} = \eta_b \mathbf{j_b} - \mathbf{v} \times \mathbf{B} = \eta_b(-\mathbf{j_f} + \frac{1}{\mu_0}\nabla \times \mathbf{B}) - \mathbf{v} \times \mathbf{B}$$

The background Spitzer resistivity may need to be modified due to beam instabilities.

1982

Cosmic Ray Modelling

Interaction of cosmic rays with MHD fluid.
Cosmic rays come from 'somewhere else' and enter region treated by MHD.

$$\frac{d\mathbf{v}}{dt} = (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Find fast current density from cosmic rays

$$\rho \frac{d\mathbf{u}}{dt} = (-\mathbf{j_f} + \nabla \times \mathbf{B}) \times \mathbf{B} - \nabla P$$

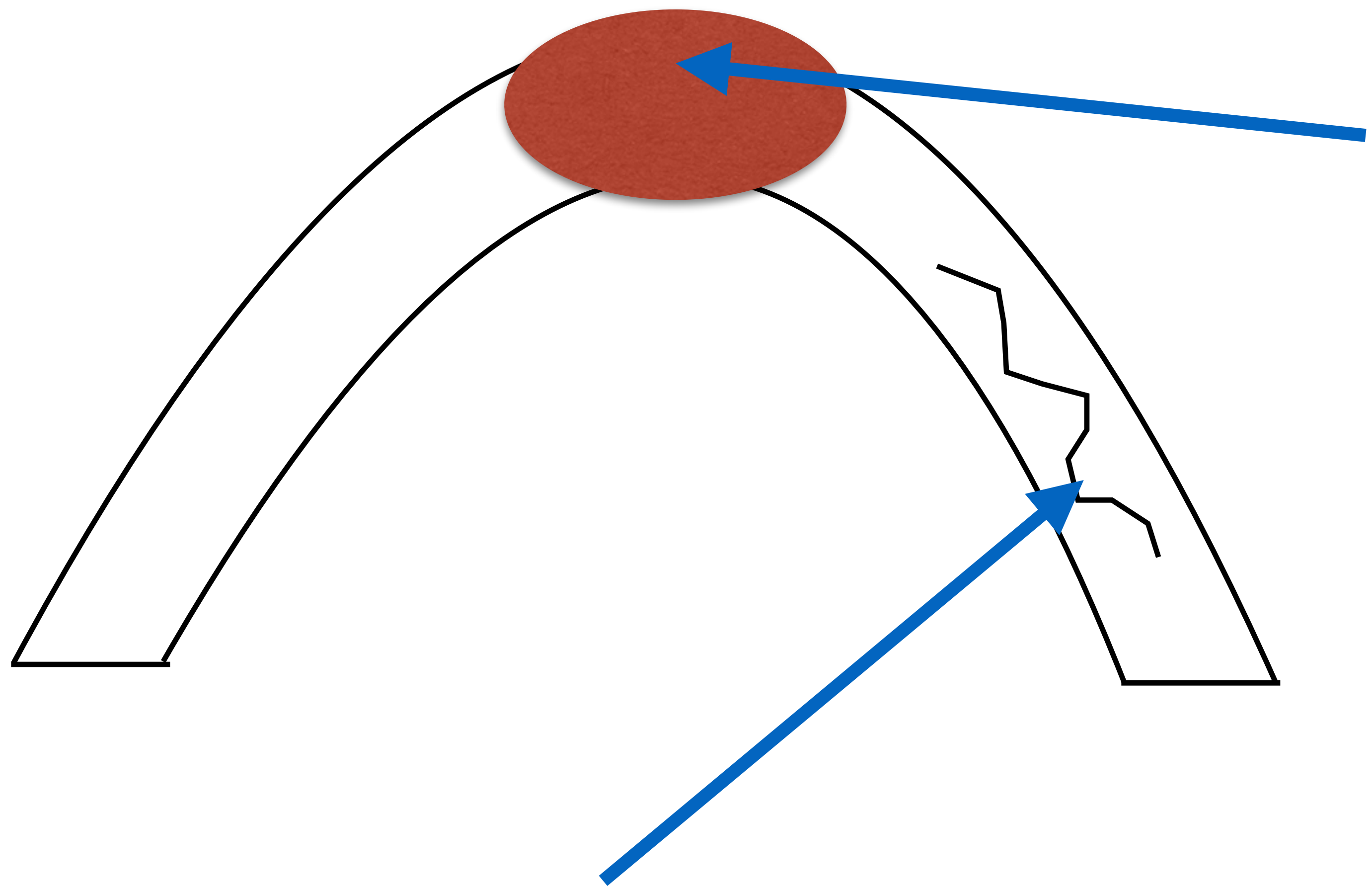
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = \eta_b \mathbf{j_b} - \mathbf{v} \times \mathbf{B} = \eta_b(-\mathbf{j_f} + \frac{1}{\mu_0} \nabla \times \mathbf{B}) - \mathbf{v} \times \mathbf{B}$$

This assumes the number density of cosmic rays is small but their current is large

Could a similar model be used to include self-consistent fast particles from flares in global MHD simulations of coronal loop?

Towards an MHD loop model with fast electrons



Localised source of fast electrons from models of reconnection, islands, turbulence etc. Consistent with global energy budget and split between thermal/non-thermal

$$f_{source}(v)$$

Sample with particles as in PIC codes

Models of beam instabilities, quasi-linear relaxation, wave-particle heating condensed into fast particle generalised force

$$\mathbf{F}_{kinetic}(v)$$

May need local source of $f_{source}(v)$ along loop

MHD/fast electron model

Sample fast electrons to match $f_{source}(v)$

$$\frac{d\mathbf{v}}{dt} = (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \;+\; \mathbf{F}_{kinetic}(v)$$

Find fast current density from $f_{source}(v)$ recalculated from PIC particles

$$\rho\frac{d\mathbf{u}}{dt} = (-\mathbf{j_f} + \nabla \times \mathbf{B}) \times \mathbf{B} - \nabla P$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = \eta_b \mathbf{j_b} - \mathbf{v} \times \mathbf{B} = \eta_b(-\mathbf{j_f} + \frac{1}{\mu_0}\nabla \times \mathbf{B}) - \mathbf{v} \times \mathbf{B}$$

$$\frac{\partial \rho}{\partial t} = -\nabla.(\rho\mathbf{u})$$

This assumes the number density of fast electrons is small but their current is large. Could be an issue for coronal loops.

Thermal Conduction

3D MHD models of dynamic coronal loops increasingly common with thermal conduction.

Classical, near equilibrium, thermal conduction along a field line given by

$$\rho \frac{\partial \epsilon}{\partial t} = \nabla . \left(\kappa \mathbf{b} . \nabla T \mathbf{b} \right)$$

Where the Spitzer-Harm thermal conductivity is

$$\kappa = \frac{1.84 \times 10^{-10}}{\ln \Lambda} T(K)^{5/2}$$

There is a curious coronal physics approximation to always take the Coulomb log as 18.4

$$\kappa = 10^{-11} T(K)^{5/2}$$

This assumes mean free path << scale of interest

This assumption usually broken for nano-flares and the transition region

VFP solution for thermal flux

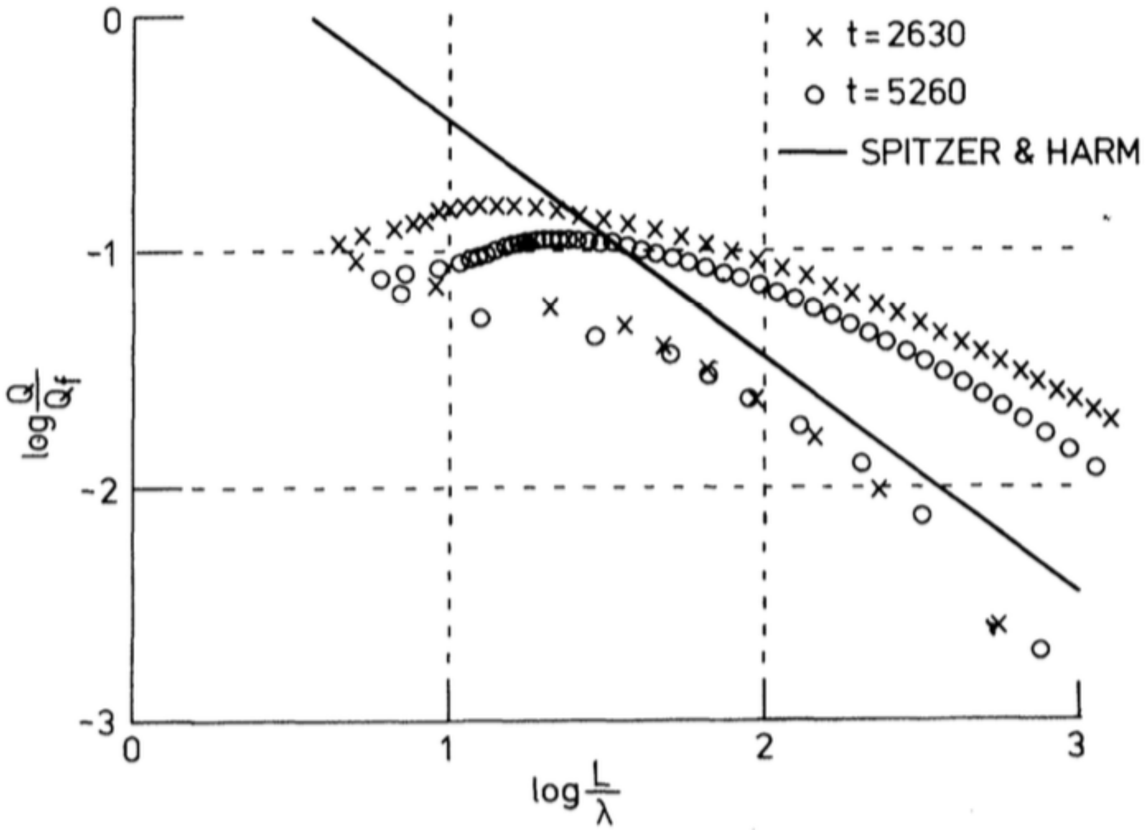
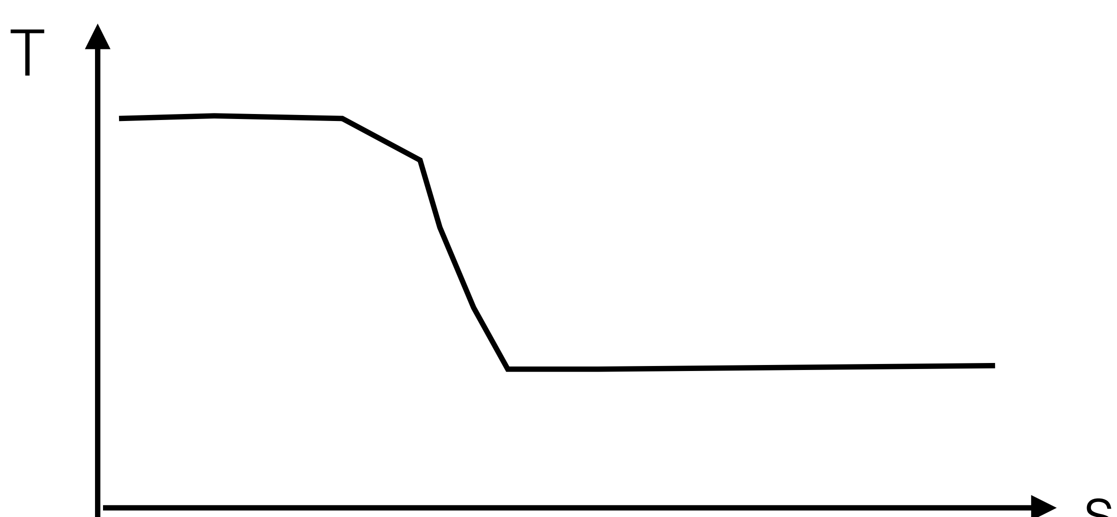


FIG. 2. Plot of heat flow Q against inverse temperature gradient. $Q_f = nkT(kT/m)^{1/2}$, λ is the mean free path of an electron with energy $\frac{3}{2}kT$, and $L = T(dT/dx)^{-1}$. All quantities are defined locally. The solid line gives the Spitzer-Harm conductivity for $Z = 4$.



A. R. Bell, R. G. Evans & D. J. Nicholas, PRL 1981

Heat flux $q_T = -\kappa \nabla T$

$$q_T = -3.2 Q_f \frac{\lambda_{mfp}}{L}$$

Where free-streaming flux is

$$Q_f = n k_B T \left(\frac{k_b T}{m_e} \right)^{1/2}$$

L is the temperature gradient scale

$$L = \frac{T}{\nabla T}$$

Figure shows full VFP solution for an initial temperature profile

Full VFP not practical in 3D on loop scale

Thermal flux limiters

Two key features of non-local transport

- Suppression of thermal conductivity below Spitzer-Harm
- Non-local source of remote fast electrons

First can be roughly approximated by insisting th heat flux cannot exceed the free streaming limit

$$\frac{1}{q} = \frac{1}{q_{Spitzer}} + \frac{1}{q_{free}}$$

Where $q_{free} = \alpha Q_f$ and typically $\alpha \simeq 0.06$

Such a treatment is the absolute minimum for thermal conduction in any loop model

Free-streaming limit (alpha) made-up!

Misses non-local physics and generally not accurate

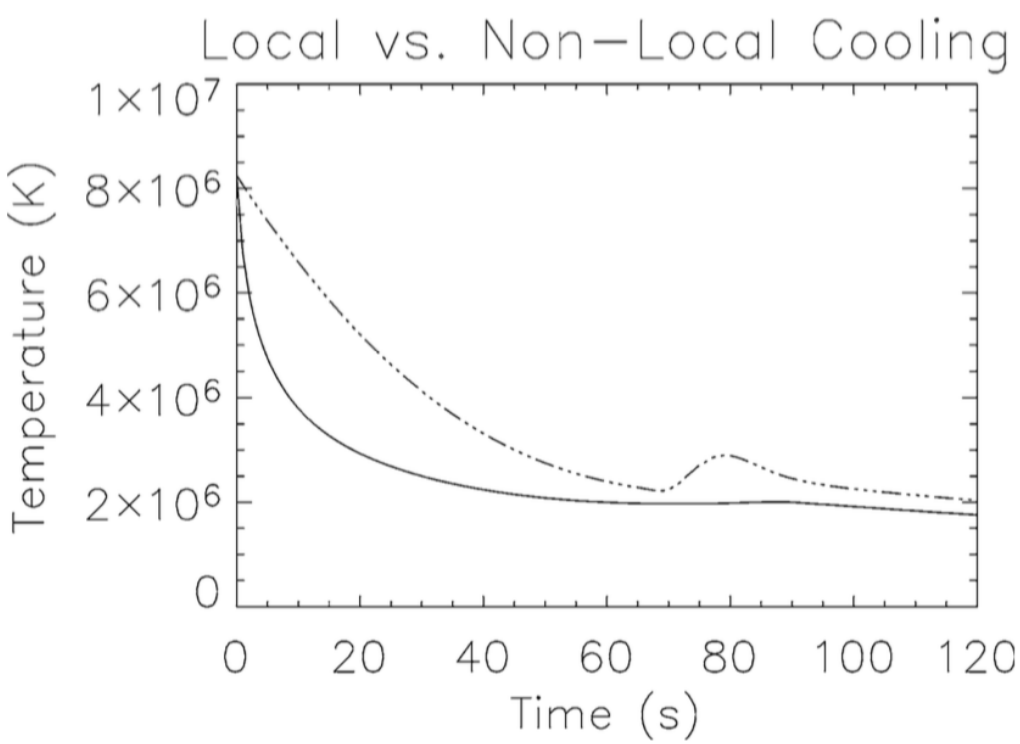
Non-local transport models

Luciani, Mora & Virmont (1982) proposed a simplified non-local transport model

$$q(s) = \int q_{Spitzer}(y) w(s, y) dy$$

$$w(s, y) = \frac{1}{\lambda_{NL}(y)} \exp\left(-\frac{1}{\lambda_{NL}(y) n(y)} \int_y^s n(z) dz\right)$$

$\lambda_{NL}(y)$ is the effective mean-free-path from point *y*



1D work on the LMV model showed slowing of cooling time.

LMV difficult to implement in 3D in parallel as required to follow field lines in integral. Also still not rigorously based on VFP

West, Bradshaw & Cargill, Solar Physics (2008)

SNB model

The standard non-local transport model used in laser-plasma codes is now from Schurtz, Nicolai & Busquets - Physics of Plasmas (2000).

Removes some of the *ad hoc* form of LMV and is easier to compute in multi-dimensions with B-field.

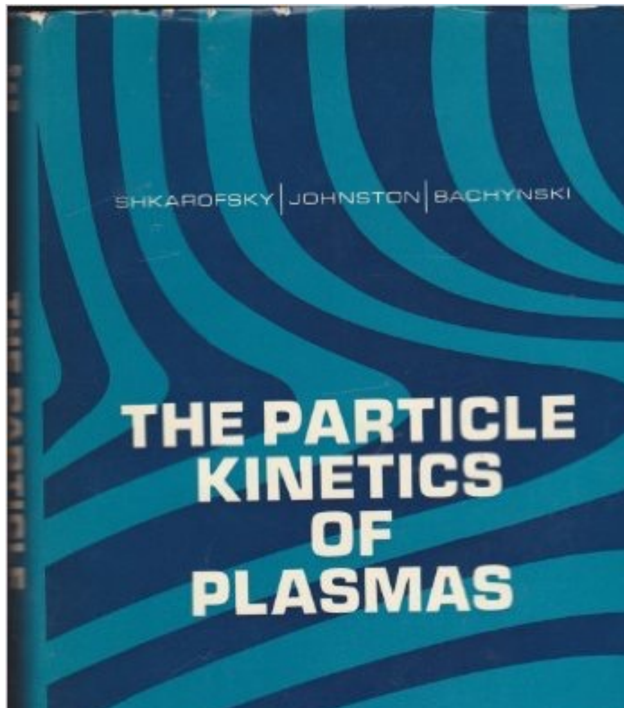
Start from Vlasov-Fokker-Planck

$$\partial_t f_e + \mathbf{v} \cdot \nabla f_e - \frac{e\mathbf{E}}{m_e} \cdot \nabla_{\mathbf{v}} f_e = C_{ee} + C_{ei}$$

Expand into isotropic and first order 'diffusive' approximation

$$f_e(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{r}, v, t) + \frac{\mathbf{v}}{v} \cdot \mathbf{f}_1(\mathbf{r}, v, t)$$

Shkarovsky (1966)



Assume istotropic part slowly varies giving fluid variables then

$$\frac{v}{3} \nabla \cdot \mathbf{f}_1 - \frac{e\mathbf{E}}{3m_e v^2} \cdot \nabla_v (v^2 \mathbf{f}_1) = \nu_{ee}(f_0 - f_0^m),$$

$$v \nabla f_0 - \frac{e\mathbf{E}}{m_e} \nabla_v f_0 = -\nu_{ei} \mathbf{f}_1,$$

Electric field small from zero current condition

ro

Separate into leading order Maxwellian and correction from Spitzer-Harm

$$f_0 = f_0^m + \Delta f_0, \;\; \mathbf{f}_1 = \mathbf{f}_1^m + \Delta \mathbf{f}_1$$

$$\mathbf{f}_1^m = -\frac{v}{\nu_{ei}}\left(\frac{m_e v^2}{2T_e} - 4\right) f_0^m \frac{\nabla T_e}{T_e}$$

Expand and simplify E-field effects into effective mean-free-path

$$\nu_{ee}\Delta f_0 + \frac{v}{3}\nabla \cdot \Delta \mathbf{f}_1 = -\frac{v}{3}\nabla \cdot \lambda_{ei}^{\mathrm{FP}} f_0^m \nabla T_e / T_e,$$

$$v \nabla \Delta f_0 + \nu_{ei} \Delta \mathbf{f}_1 = 0,$$

Finally gives heat flux

$$\begin{aligned}\mathbf{Q} &= \frac{2\pi m_e}{3}\int_0^\infty \mathbf{f}_1 v^5 dv \\ &= \frac{2\pi m_e}{3}\int_0^\infty \mathbf{f}_1^m v^5 dv - \frac{2\pi m_e}{3}\int_0^\infty \lambda_{ei}^E \nabla \Delta f_0 v^5 dv,\end{aligned}$$

Where

$$1/\lambda_{ei}^E = 1/\lambda_{ei}^{\mathrm{FP}} + |eE|/\varepsilon$$

Marochino et al., Physics of Plasmas (2013)

Parallel E-field also determined by SNB

A Beyond MHD Model for Coronal Loops

Fast electrons specified from detailed kinetic, or high resolution, reconnection.

Fast electrons also accelerated/slowed by wave-particle interactions

Sample fast electrons to match $f_{source}(v)$

$$\frac{d\mathbf{v}}{dt} = (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \ + \ \mathbf{F}_{kinetic}(v)$$

Find fast current density from $f_{source}(v)$ recalculated from PIC particles

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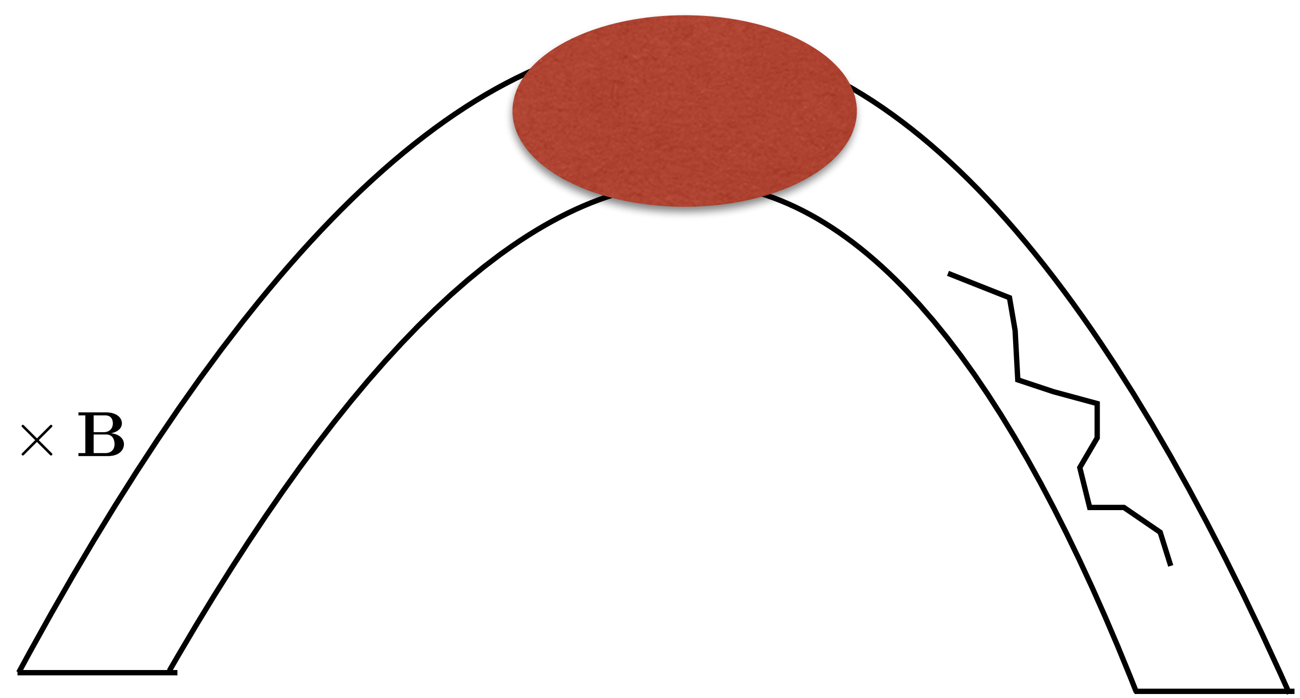


Non-local heat conduction handled by SNB model

Summary - 1

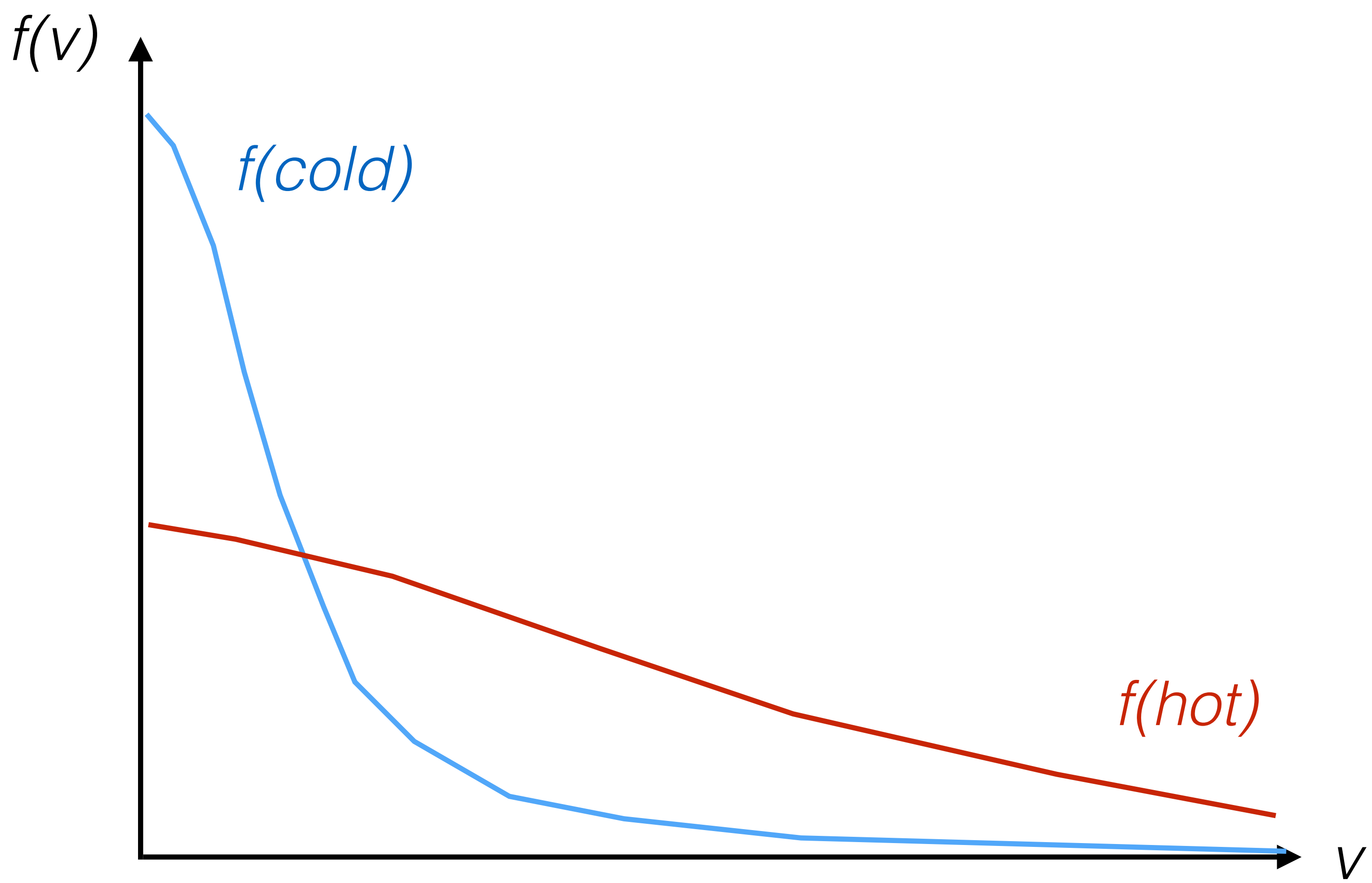
Simulations with fast electrons self-consistently coupled to MHD are possible

- First approximation to use SNB model for thermal conduction
  - Improvement over LMV
  - Already implemented in most national lab. codes
  - Has been extended to include B-field

- Highly non-Maxwellian sources of fast particles could be handled using techniques developed for cosmic rays
  - Return current in MHD driven by fast kinetic electrons
  - Need initial source of fast electrons
  - Need model for effective force on fast electrons due to wave-particle

Hot Electron Instabilities

Thermal hot electrons can propagate freely



$$k^2 = -\omega_{pe}^2 \int \frac{d_v g(v)}{\omega/k - v} dv$$

$$E_1(x,t) = E_1 \exp\left(ikx - i\omega_r t\right)\exp(\gamma t)$$

$$\gamma = \frac{\pi \omega_{pe}^3}{2k^2 n_0} \left(\frac{df}{dv}\right)_{v=\omega/k}$$

Time of flight effects can hollow distribution leading to instability

Relaxes to almost flat distribution through instability, wave-particle to quasi-linear relaxed state

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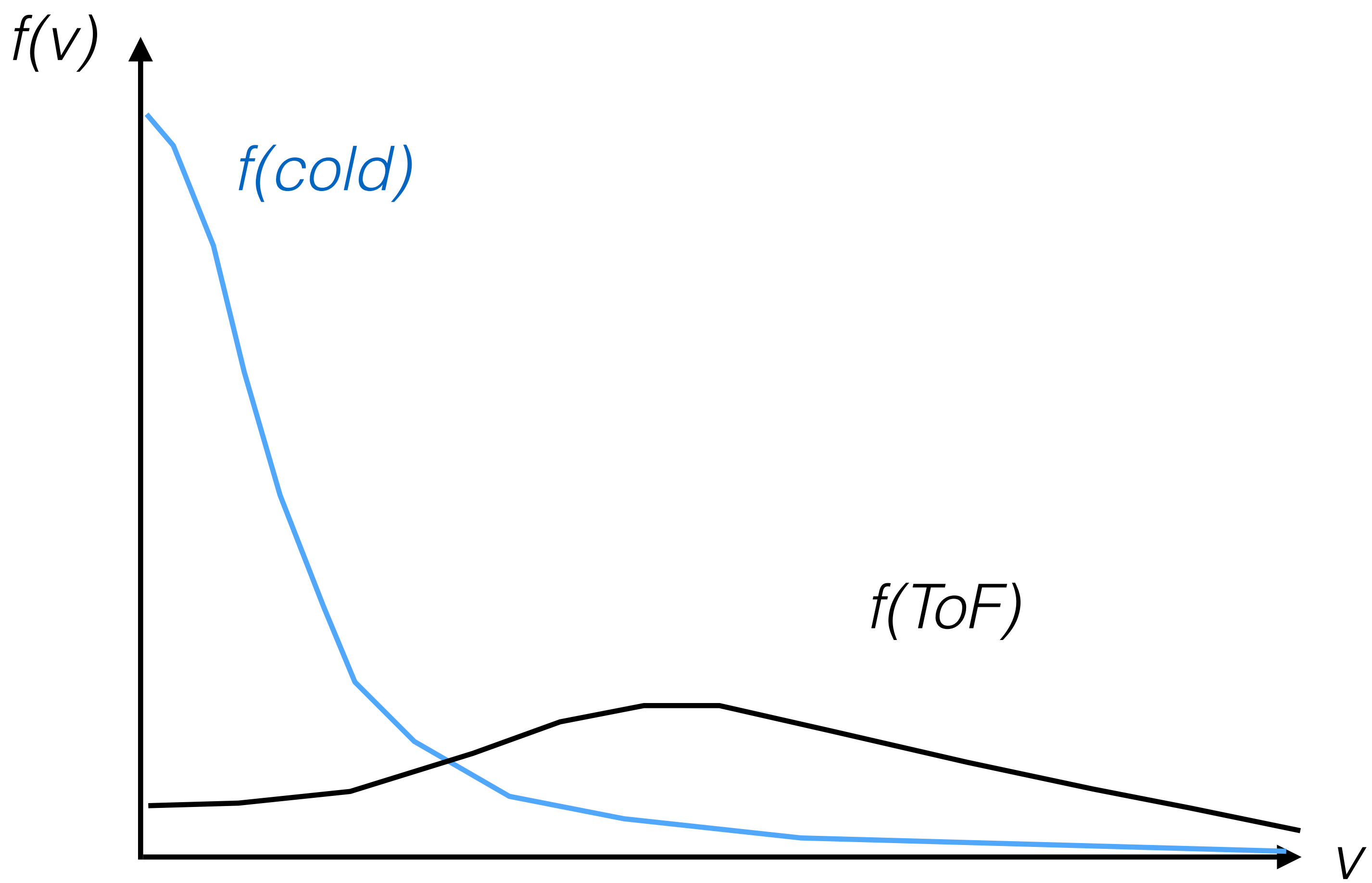

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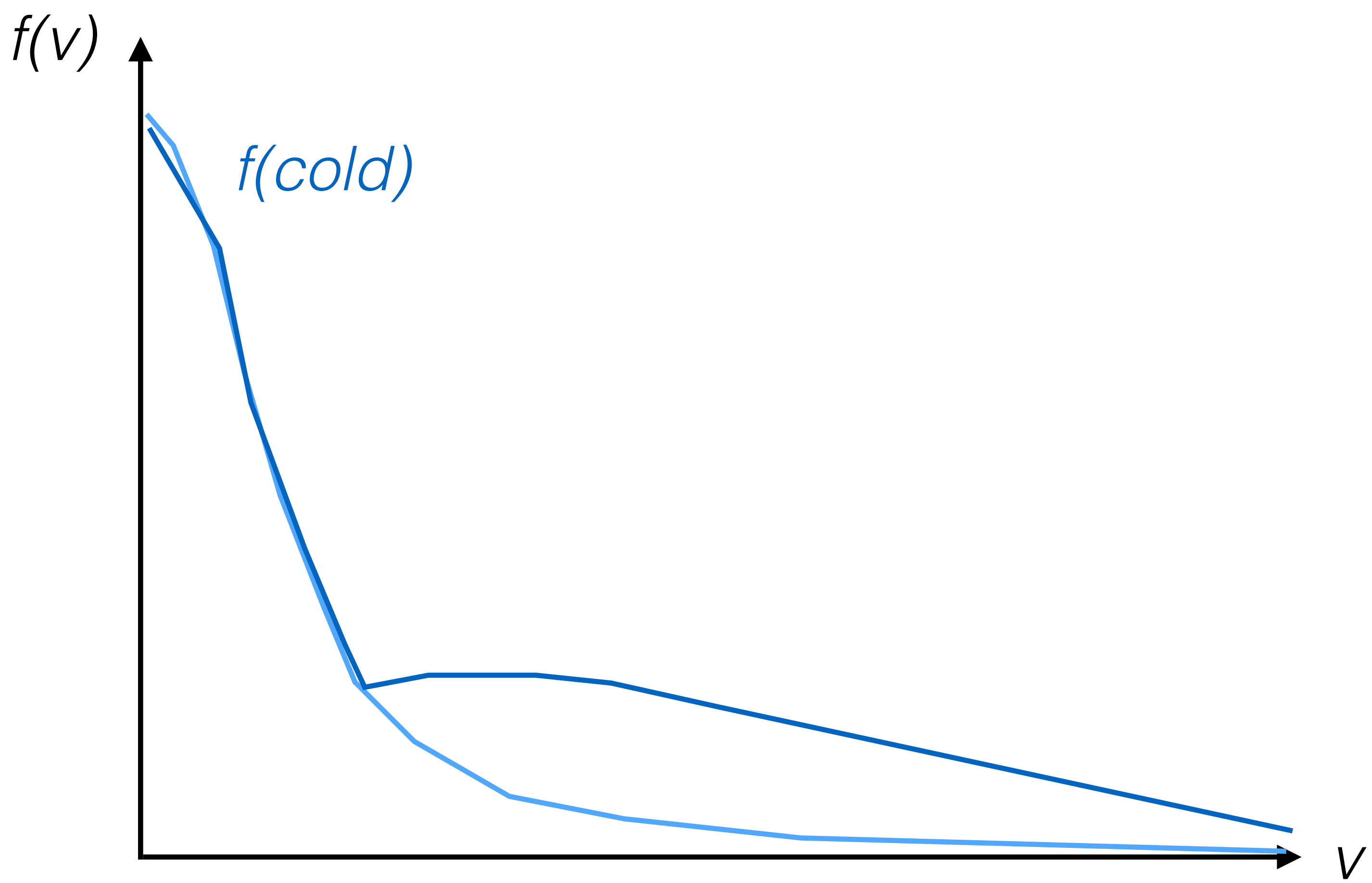

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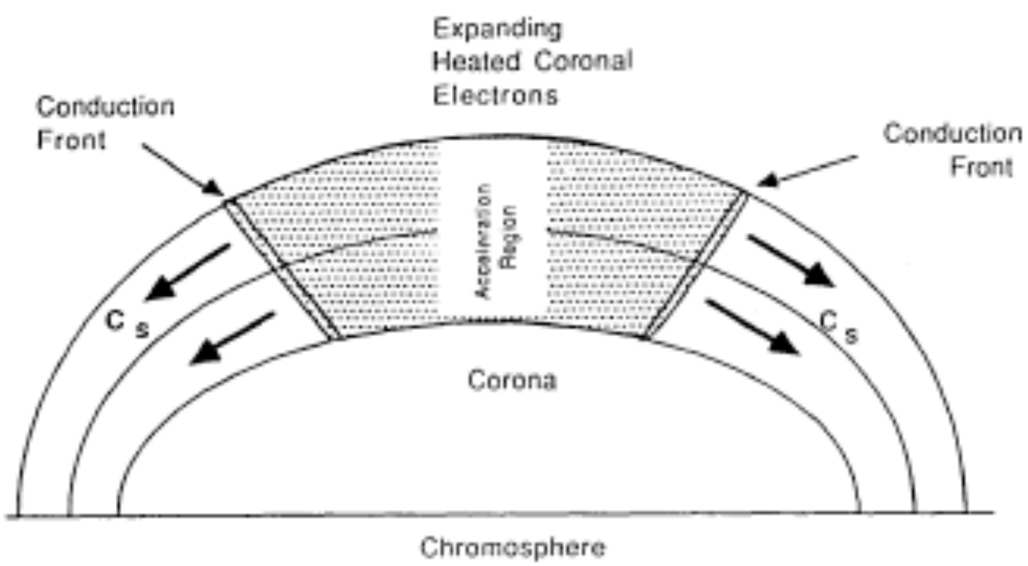
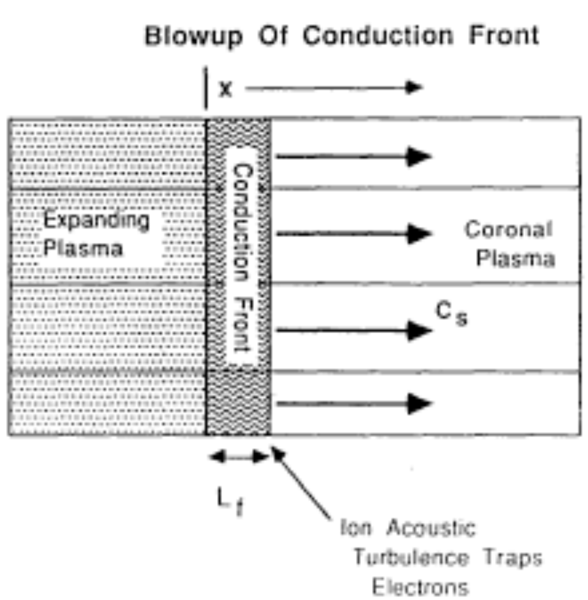
Current Free Double Layers

While most hot electron beams can propagate, maybe after quasi-linear relaxation, this may not be true for the return current.





Adapted from Batchelor (ApJ, 1985)

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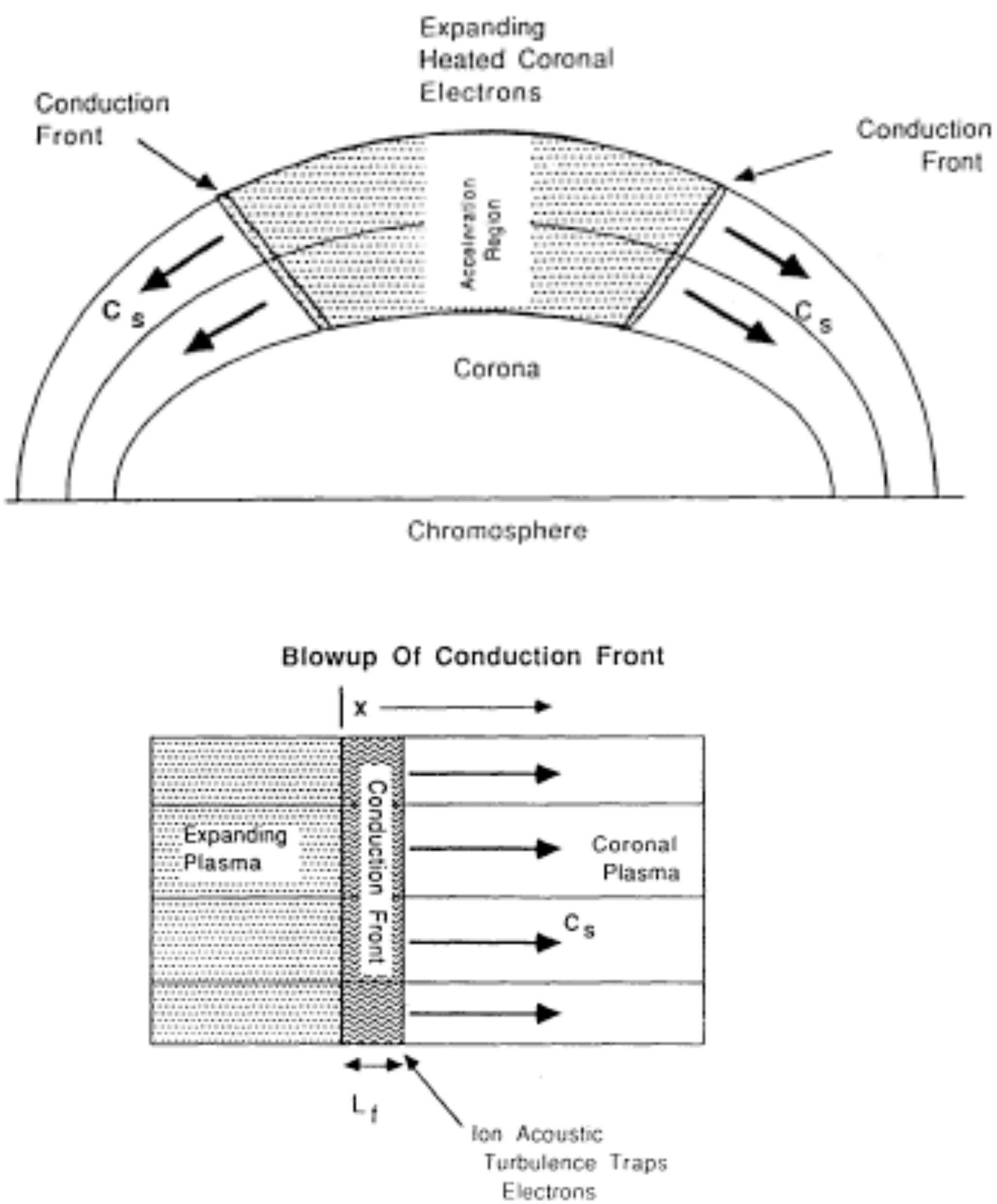
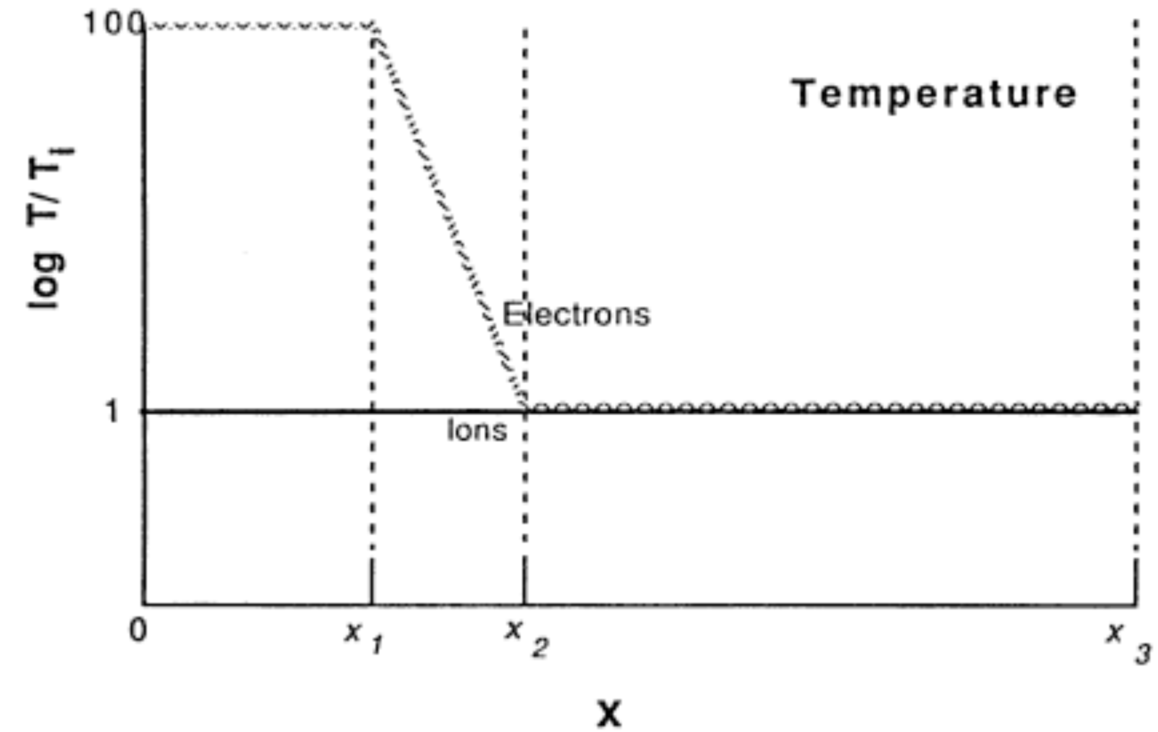


Initial Conditions



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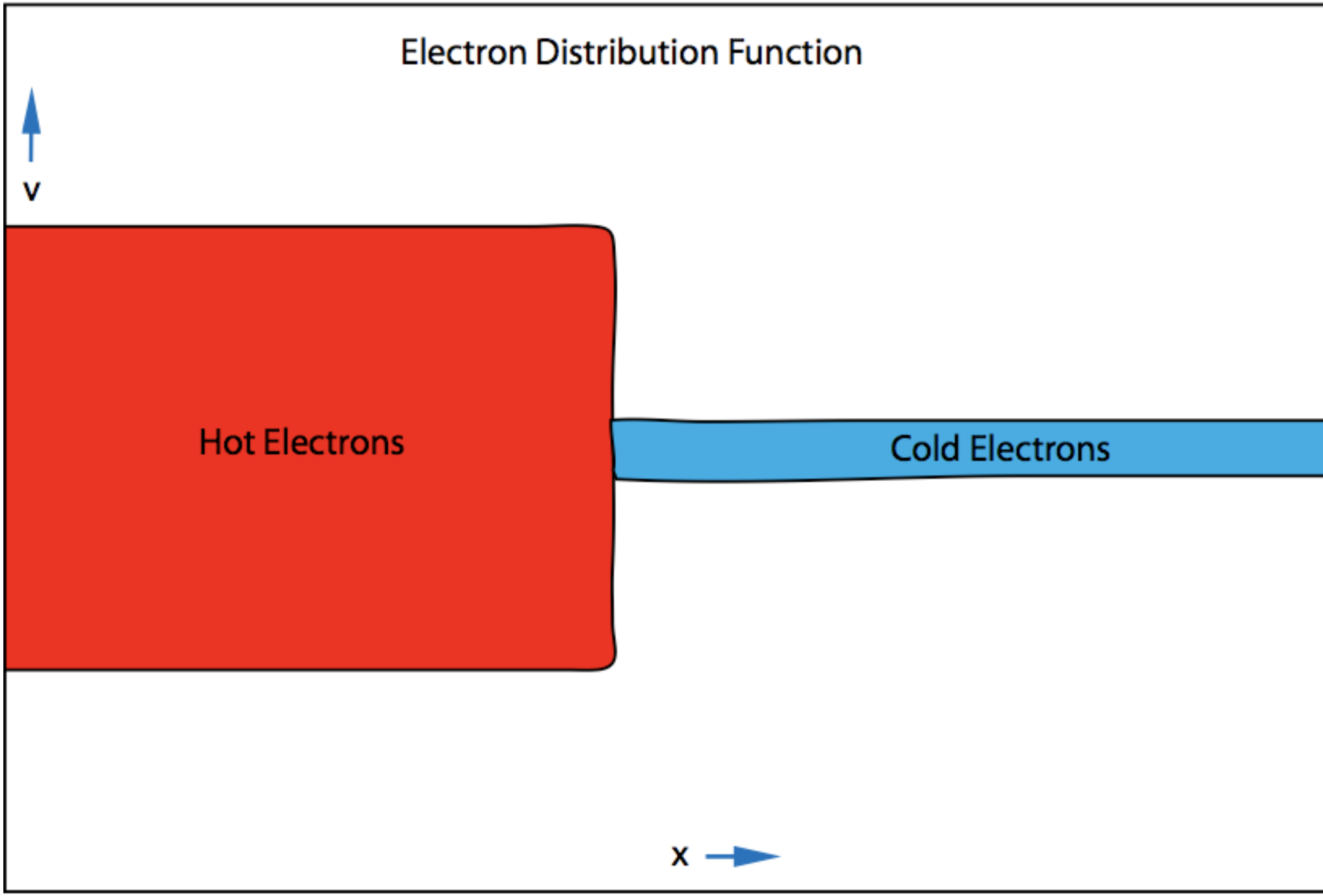
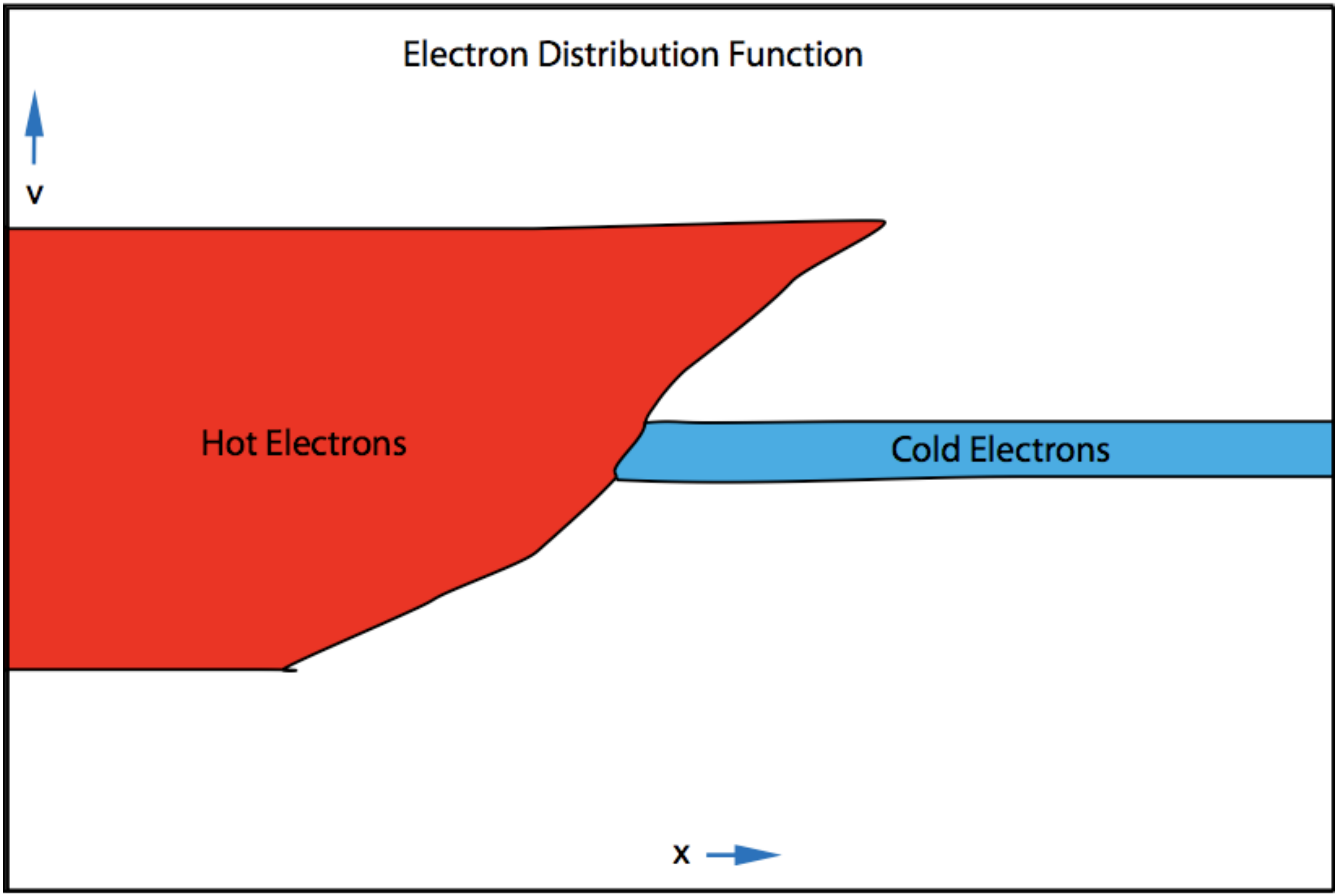
Electrostatic Return Current Model



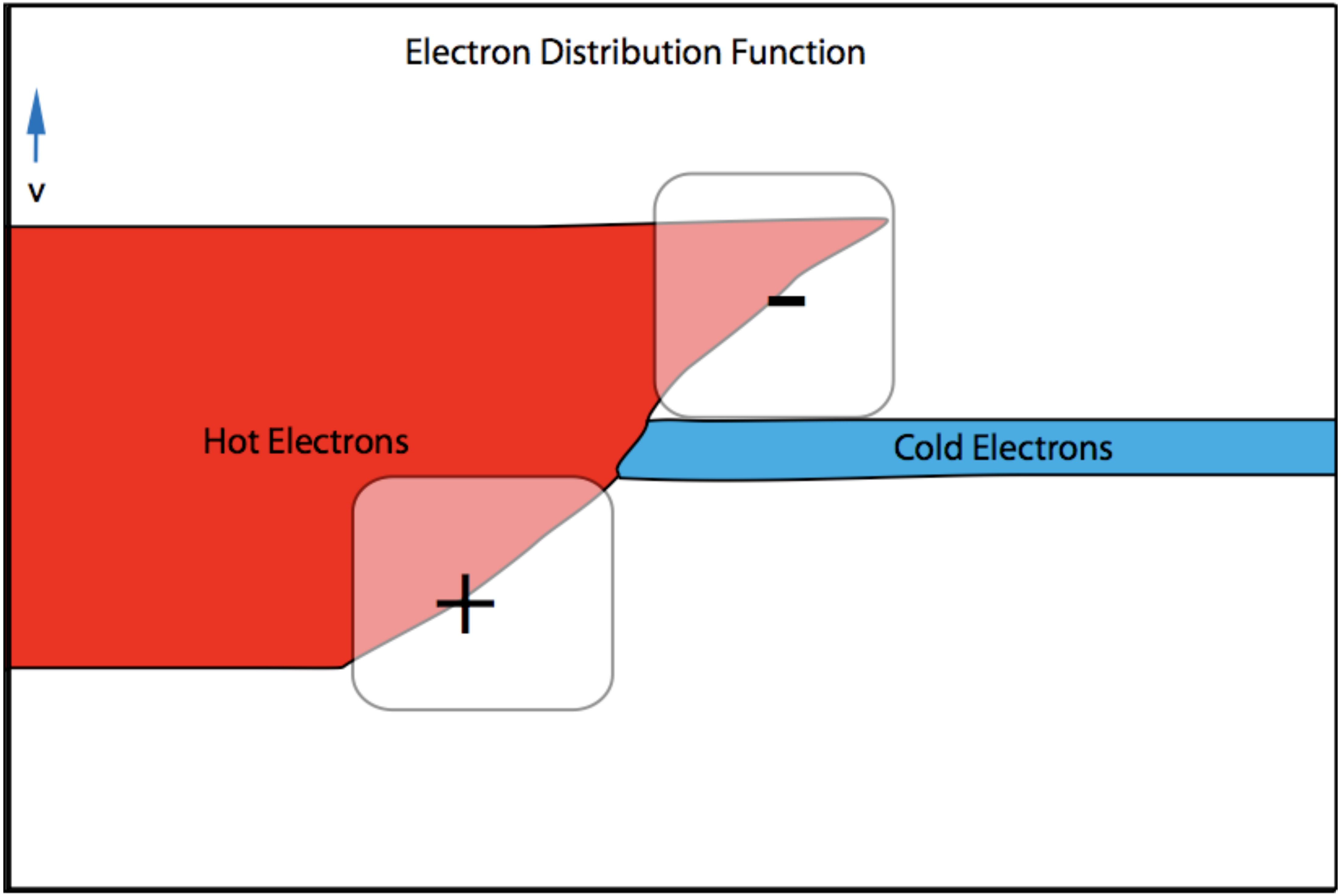
Batchelor (ApJ, 1985), Brown, Melrose & Spicer (ApJ, 1979),
Levin & Melnikov (Solar Physics, 1993), Bychenkov et al.

Electrostatic Return Current Model



Batchelor (ApJ, 1985), Brown, Melrose & Spicer (ApJ, 1979),
Levin & Melnikov (Solar Physics, 1993), Bychenkov et al.

Electrostatic Return Current Model



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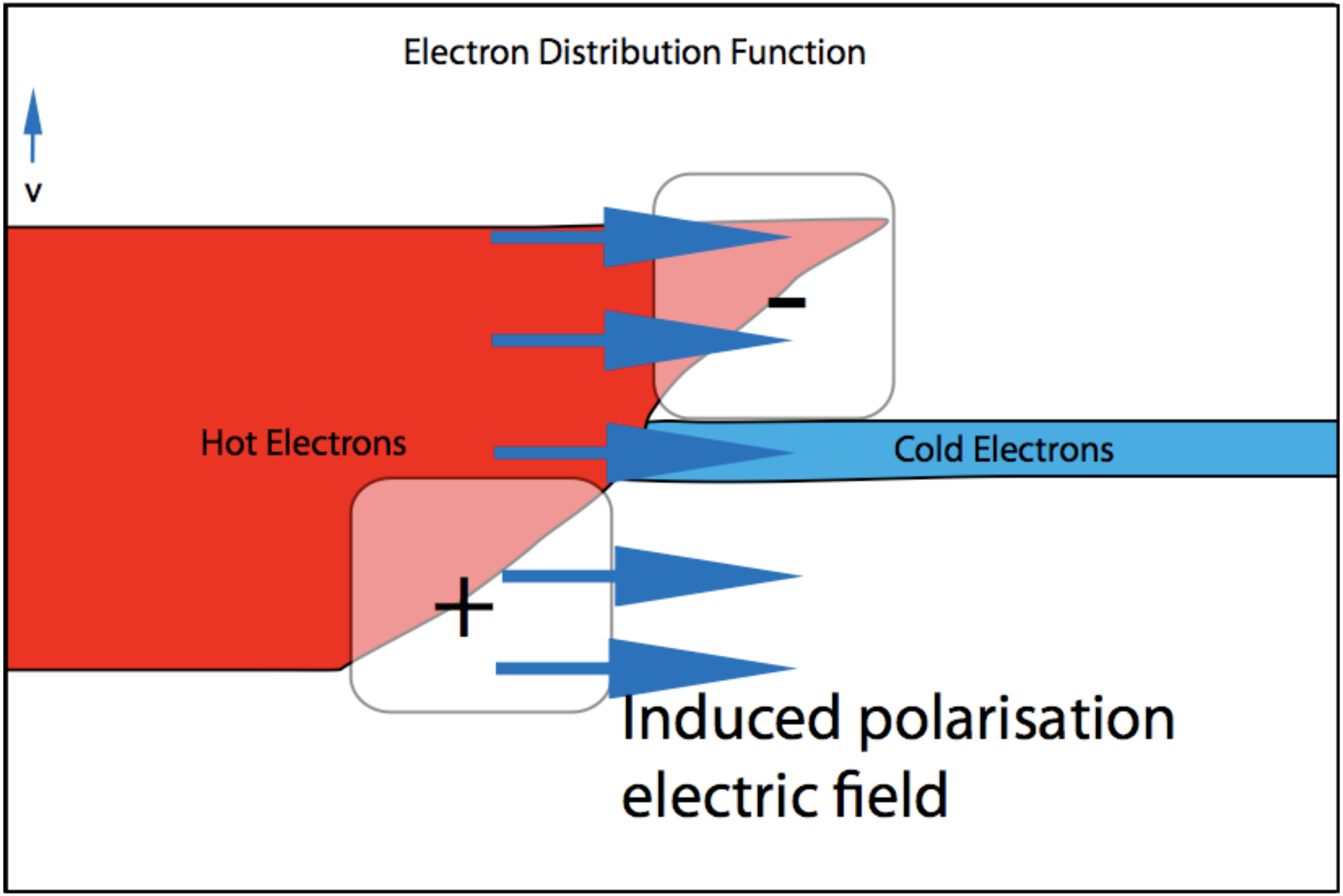

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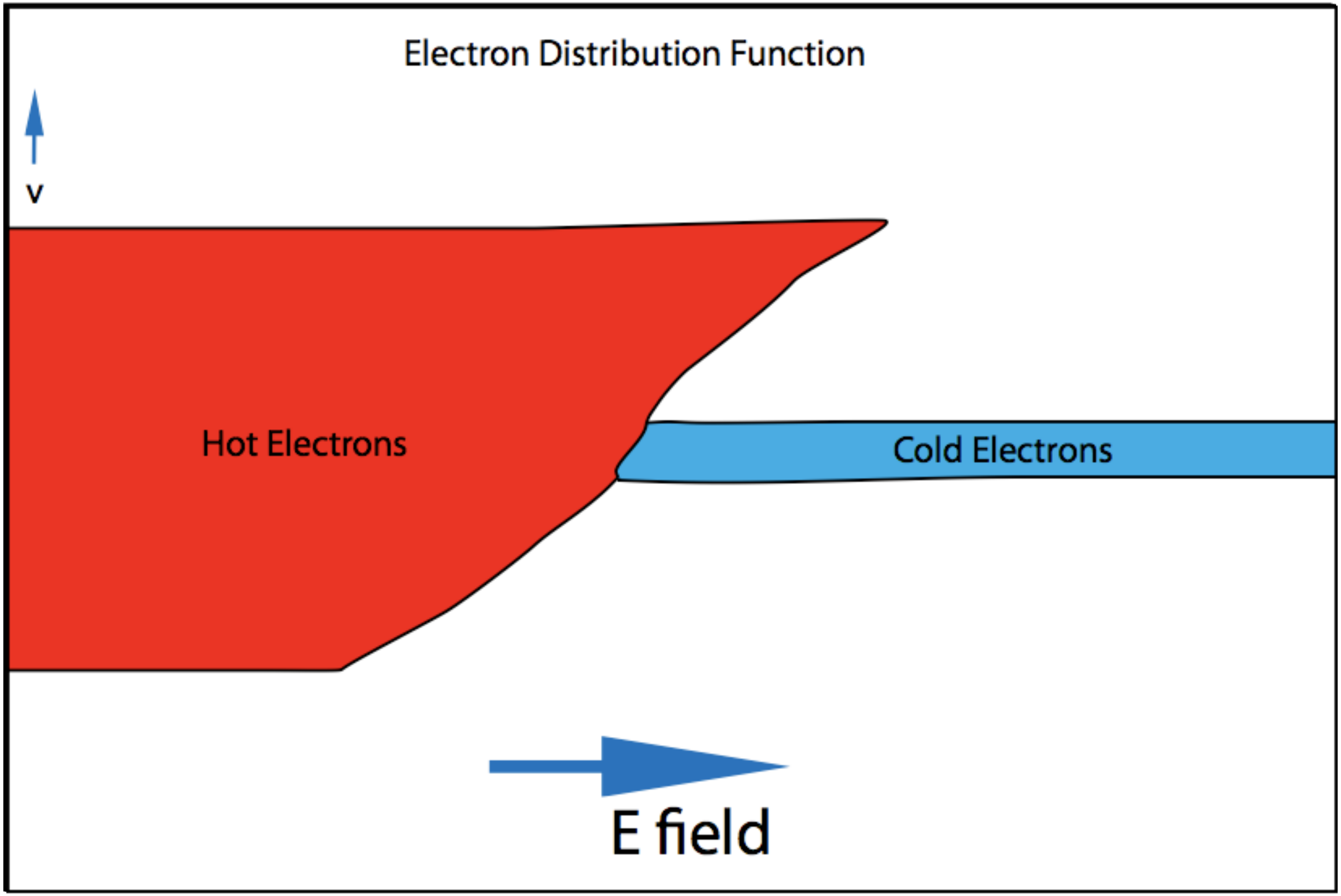

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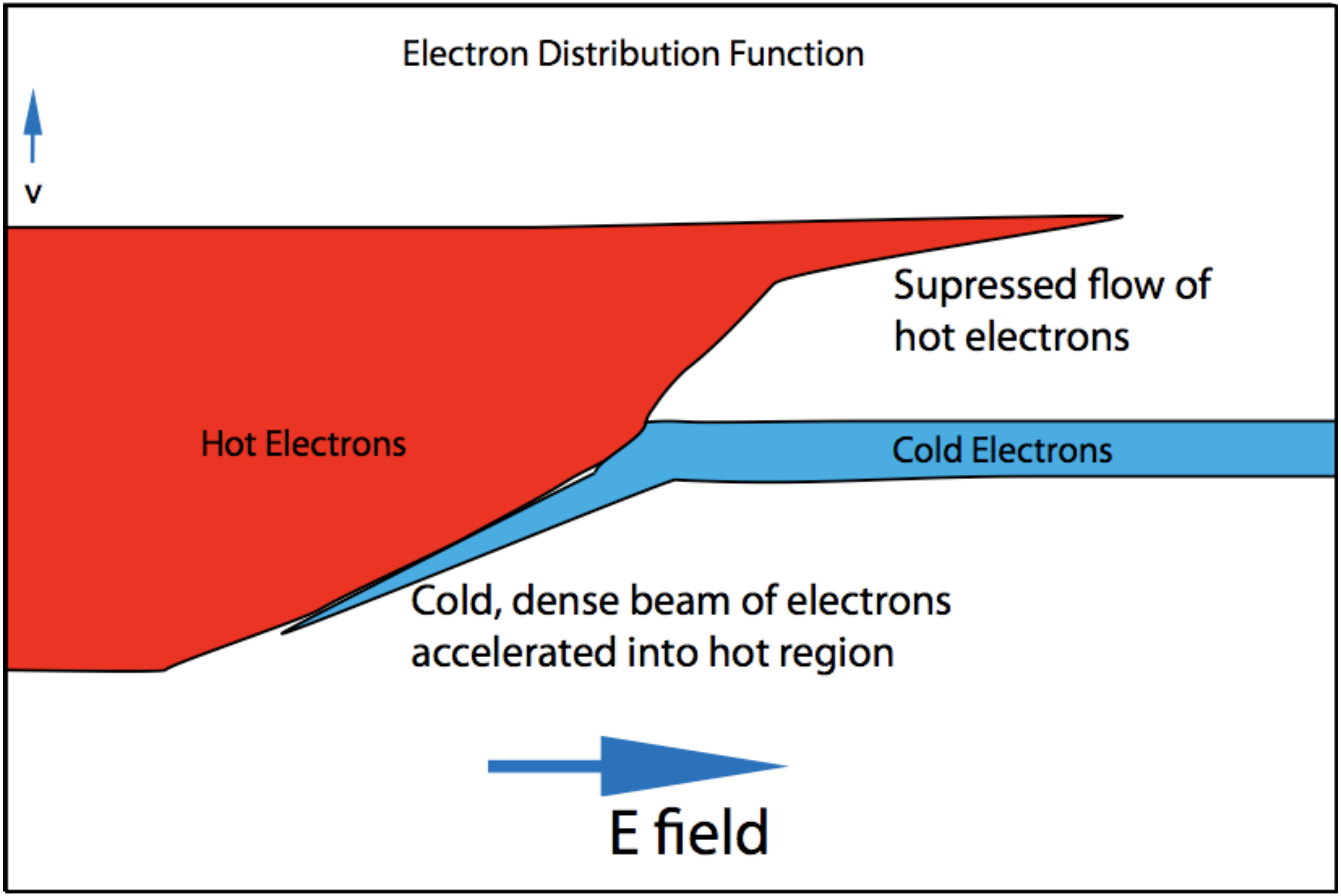


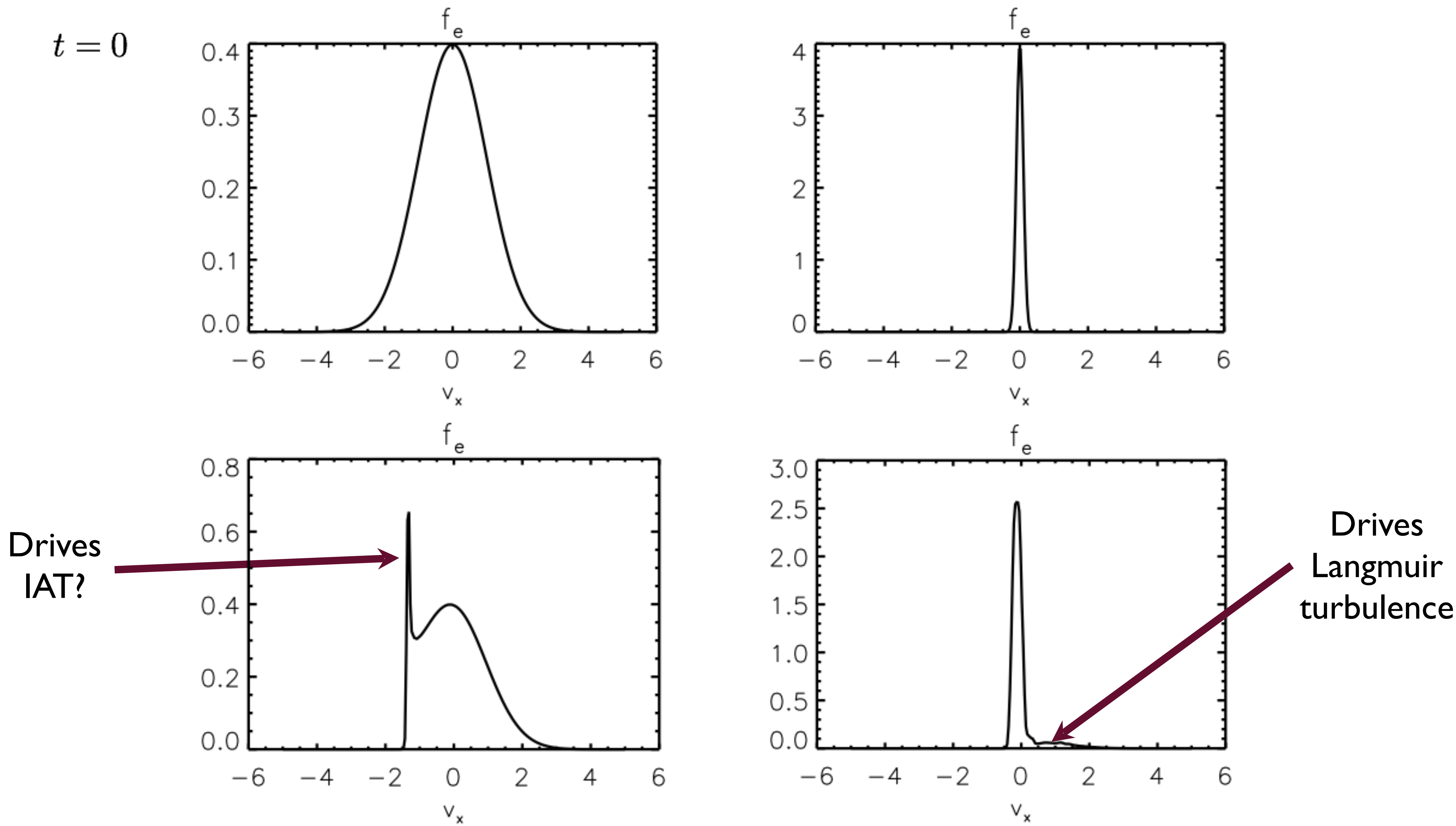
Batchelor (ApJ, 1985), Brown, Melrose & Spicer (ApJ, 1979),
Levin & Melnikov (Solar Physics, 1993), Bychenkov et al.

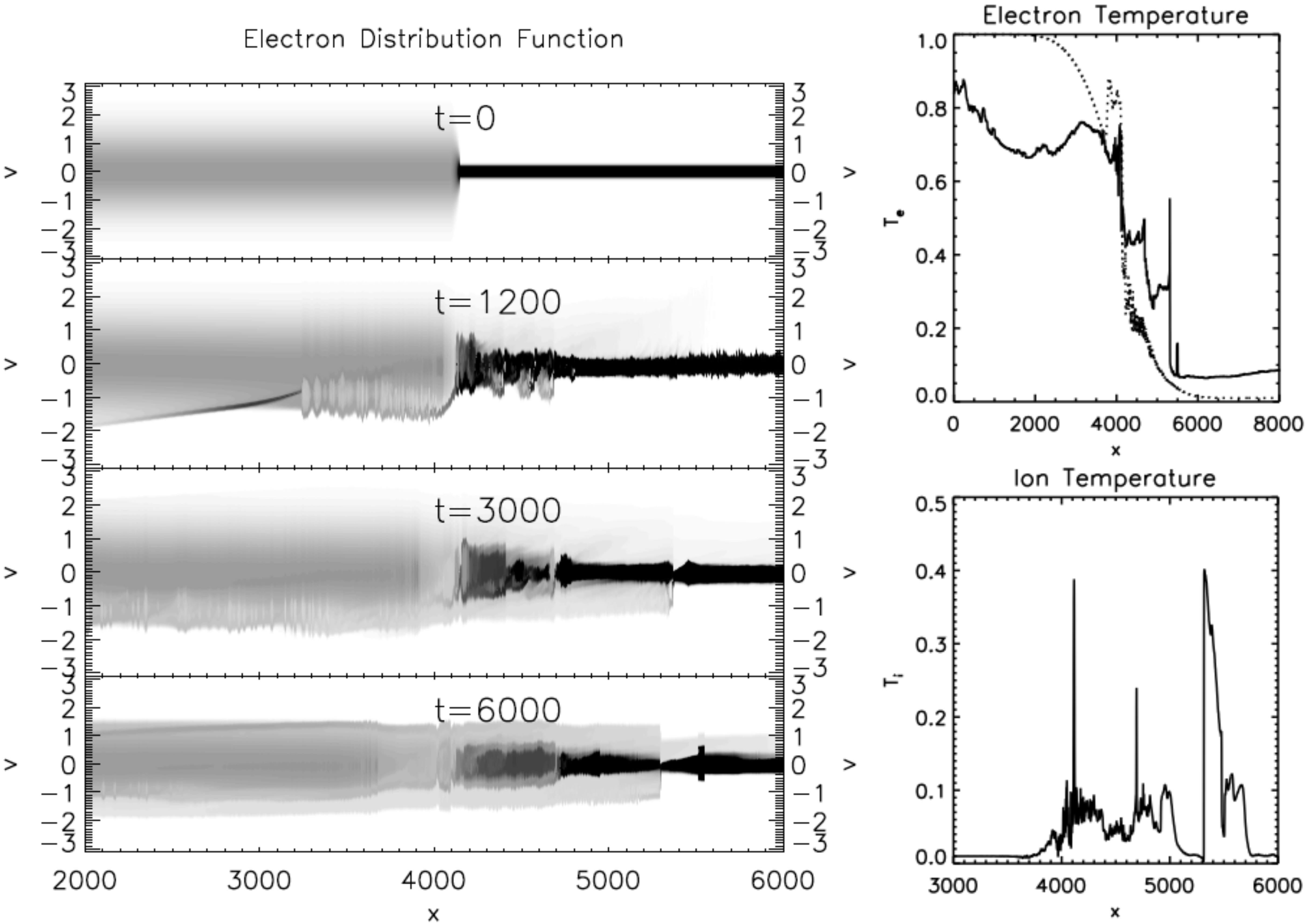
Distribution Functions

Hot electron region

Cold electron region

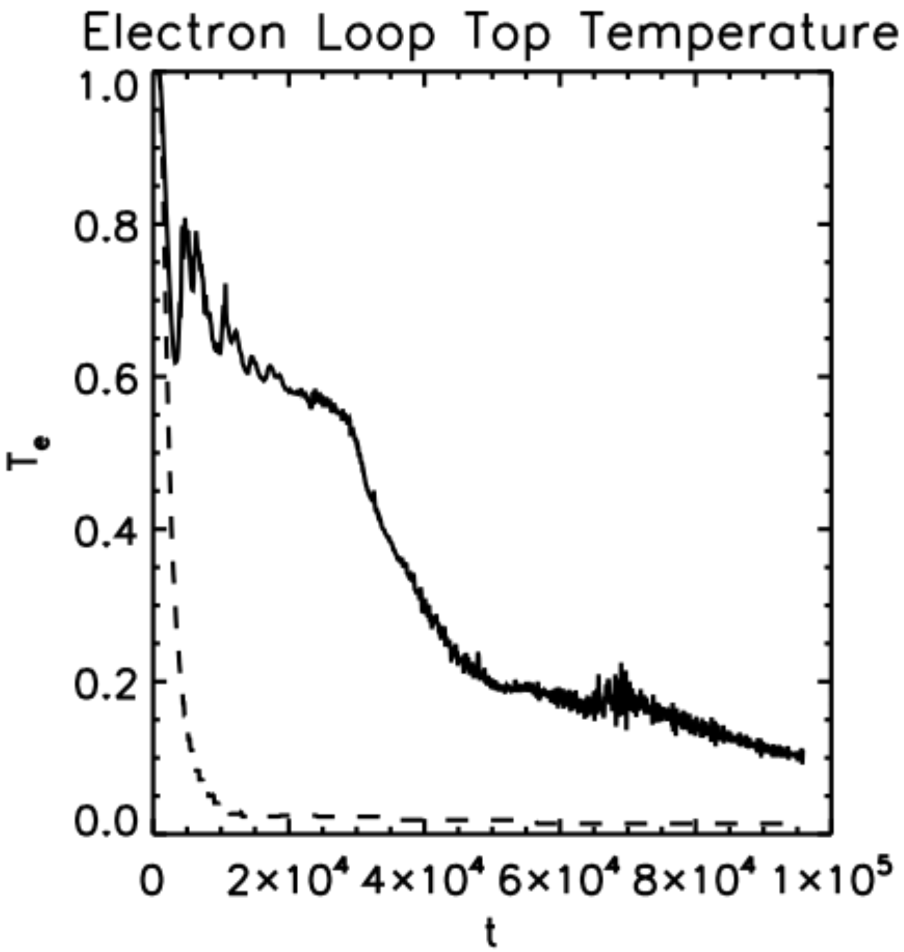
$t = 0$





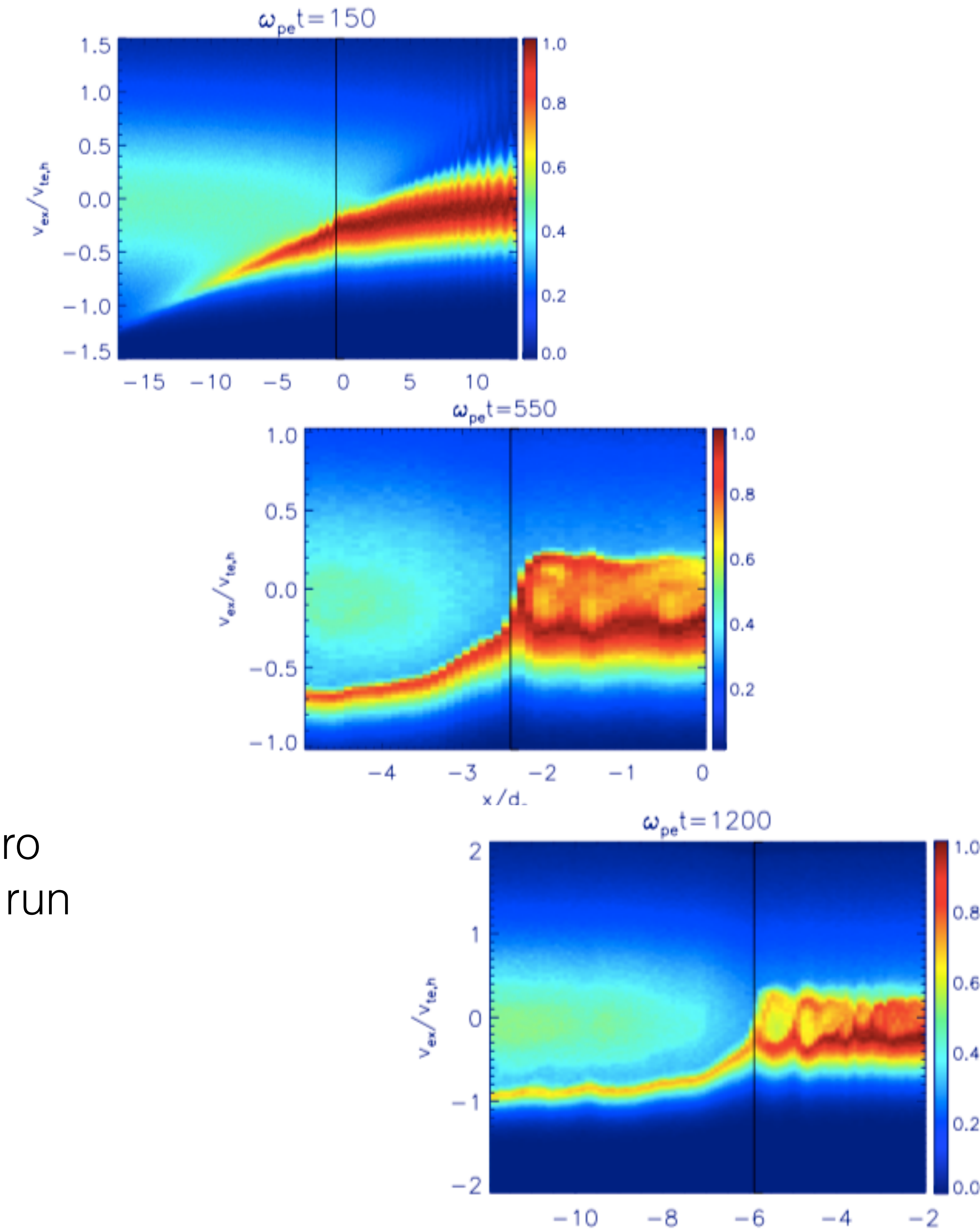
Arber & Melnikov, ApJ (2009)

Supressions of Electron Heat Flow



Arber & Melnikov, ApJ (2009)

Li, Drake & Swisdak, ApJ (2012)



Curiously if this was to be modelled on hydro time-scale best approximation would be to run with an adiabatic energy equation!

Summary - 2

If heating highly localised electron transport may be suppressed

Return current drives plasma-wave turbulence

Sets up a stable, current free double layer with large-scale E-field

Net current zero due to return current and accelerated ions