Model Comparison for the Density Structure Across Solar Coronal Loops

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1. Motivation

AIM: To infer the properties of the spatial distribution of the mass density across solar coronal loops.

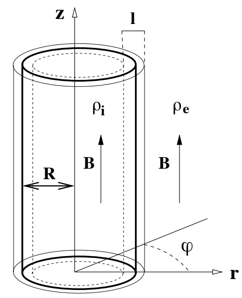
WHY: They govern time and spatial scales for wave damping and energy dissipation in MHD wave based heating mechanisms.

HYPOTHESIS: Observed transverse oscillations show evidence for in situ damping. If interpreted as MHD kink waves, resonant damping offers a plausible explanation. In that case, damping vs. dissipation scales are governed by the cross-field density structuring.

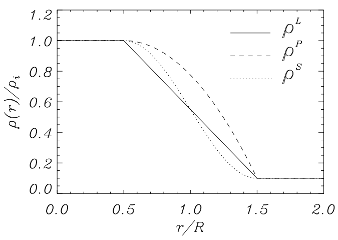
THIS WORK: We present a method to combine wave observations and theory to determine the cross-field density structuring of solar atmospheric waveguides.

2. Cross-Field Density Models

PHYSICAL MODEL: Classic one-dimensional waveguide with cross-field density variation over a length-scale l.



Three alternative density models



Relevant parameters

$\zeta = \rho_i/\rho_e$ density contrast

l/R transverse scale

ρ^L LINEAR PROFILE

ρ^P PARABOLIC PROFILE

ρ^S SINUSOIDAL PROFILE

3. Bayesian Data Analysis

BAYES' THEOREM:

$$p(\boldsymbol{\theta}|d) = \frac{p(d|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(d)}$$

$p(\boldsymbol{\theta}|d)$: posterior; $p(d|\boldsymbol{\theta})$: likelihood function; $p(\boldsymbol{\theta})$: prior; $p(d)$: evidence

State of knowledge on model parameters $\boldsymbol{\theta}$ is a combination of what is known a priori independently of the data, $p(\boldsymbol{\theta})$, and the likelihood of obtaining a data realization actually observed as a function of the parameter vector, $p(d|\boldsymbol{\theta})$.

3 LEVELS OF BAYESIAN INFERENCE:

PARAMETER INFERENCE: infer unknown parameters from marginalisation of the full posterior.
MODEL COMPARISON: compare performance of alternative models in explaining data by taking posterior ratios.
MODEL AVERAGING: compute averaged posterior weighted with evidence for each model, in view of data.

4. Parameter Inference

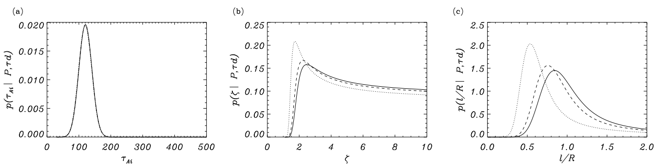
Bayesian inversion of cross-field density structuring.

Observations: loop oscillation period and damping times.
Theory: resonantly damped MHD kink modes.

Alfvén travel time τ_{Ai}, density contrast ζ, transverse scale l/R

Inference with Alternative Density Models

MARGINAL POSTERIOR DENSITIES



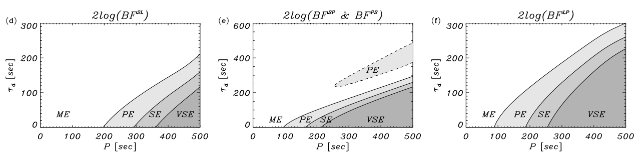
Alfvén travel time inference independent of density model.
Density contrast inference weakly depends on density model.
Transverse length-scale affected by employed density model.

5. Model Comparison

Bayesian model comparison between density models. Bayes factors as a function of period and damping time. Assign levels of evidence to Bayes factor magnitude.

Comparison between Alternative Density Models

BAYES FACTOR DISTRIBUTION



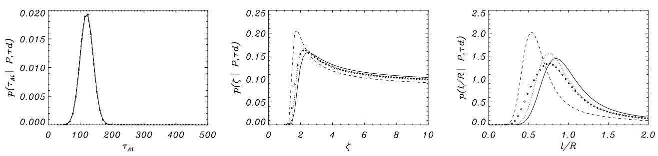
Regions with moderate evidence (ME); positive evidence (PE); strong evidence (SE); and very strong evidence (VSE) for one model against the alternative.

6. Model Averaging

Bayesian model averaging. Take into account the evidence for each model for constructing an averaged posterior.

Computation of Averaged Posterior

POSTERIORS WEIGHTED WITH EACH MODEL EVIDENCE



Line styles show the inversion using the three alternative models. Symbols represent averaged posteriors, weighted with the evidence for each of the considered models.

Conclusions

- ⋆ Inference of the unknown parameters depends on the density model used.
- ⋆ Model comparison enables us to assess which density model better explains observed period and damping times.
- ⋆ Model averaging offers the most general inference, using all the available information in a consistent way: prior information, data with uncertainty, model predictions, and model evidence