Three-dimensional geometry of coronal loops inferred by the Principal Component Analysis

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Introduction

Knowing the three dimensional (3D) structure of coronal loops is an important task in the wave-based diagnostics of the solar corona. The combination of different points of observation in space, e.g. those provided by the Solar Dynamics Observatory (SDO) and the Solar TErrestrial RElations Observatory (STEREO), allows us to infer information about the geometrical shape of coronal loops in 3D [1]. Here, we propose a new method to reconstruct the loop shape starting from stereoscopically determined 3D points, which sample the loop length, by the Principal Component Analysis (PCA). This method is shown to retrieve in an easy way the main parameters that define a coplanar loop, such as the minor and major axis, the loop plane, the azimuthal and inclination angle.

Principal Component Analysis

3D tie-points sampling the loop in a given coordinate system (e.g., HEEQ) are generally "interrelated". PCA allows us to transform this set of points into a new one of "uncorrelated" or independent variables, known as "principal components", which preserve the variation of the data distribution [2]. Let be $\boldsymbol{X}$ the matrix of our N measurements, $\boldsymbol{C}=(x_C,y_C,z_C)$ the loop centre, and $\boldsymbol{S}$ the covariance matrix:

$$\boldsymbol{X}=\begin{pmatrix} x_1-x_C & x_2-x_C & \dots & x_N-x_C \\ y_1-y_C & y_2-y_C & \dots & y_N-y_C \\ z_1-z_C & z_2-z_C & \dots & z_N-z_C \end{pmatrix} \rightarrow \boldsymbol{S}=\boldsymbol{X}\boldsymbol{X}^T=\begin{pmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 & \sigma_{xz}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 & \sigma_{yz}^2 \\ \sigma_{zx}^2 & \sigma_{zy}^2 & \sigma_{zz}^2 \end{pmatrix}$$

The main idea is to find a new reference frame, whose axes maximise the variances and minimise the covariances of $\boldsymbol{S}$. This is equivalent to finding the eigenvalues and eigenvectors that diagonalise $\boldsymbol{S}$ in a matrix $\boldsymbol{\Lambda}$. Let us obtain the eigenvalues and sort them in ascending order, as:

$$\lambda_n \leqslant \lambda_a \leqslant \lambda_b$$

The corresponding eigenvectors will be $\boldsymbol{e}_n$, $\boldsymbol{e}_a$, $\boldsymbol{e}_b$, whose components are relative to the original frame. Thus, the matrix:

$$\boldsymbol{E}=\begin{pmatrix} e_{n,x} & e_{n,y} & e_{n,z} \\ e_{a,x} & e_{a,y} & e_{a,z} \\ e_{b,x} & e_{b,y} & e_{b,z} \end{pmatrix}$$

defines the basis change $\boldsymbol{X}'=\boldsymbol{E}\boldsymbol{X}$ between $\boldsymbol{X}$ and $\boldsymbol{X}'$, the coordinates in the new reference frame of the loop, and diagonalises $\boldsymbol{S}$ in $\boldsymbol{\Lambda}=\boldsymbol{E}^T\boldsymbol{S}\boldsymbol{E}$. The eigenvectors indicate the axes of the new reference frame for the loop:

- $\boldsymbol{e}_n$, associated with the smallest eigenvalue, represents the vector normal to the loop plane;
- $\boldsymbol{e}_a$ is directed along the minor axis of the ellipse that fits the loop shape;
- $\boldsymbol{e}_b$, associated with the largest eigenvalue, is oriented along the major axis. The minor radius a an the major radius b of the ellipse are related to the eigenvalues λ_a, λ_b, and the number of points N:

$$a=\sqrt{\frac{2\lambda_a}{N}},\quad b=\sqrt{\frac{2\lambda_b}{N}}$$

The ellipse can be traced by using the parametric equations:

$$x'_{ell}=a\cos(t) \qquad y'_{ell}=b\sin(t)$$

with $t=[0,2\pi]$. The points $\{(x'_{ell}, y'_{ell})\}$ can be transformed into $\{(x_{ell}, y_{ell}, z_{ell})\}$ by $\boldsymbol{E}$. Those that satisfy the condition $r^2= x_{ell}^2 + y_{ell}^2 + z_{ell}^2 \geq 1.0\, R_\odot$, define the loop curve. The orientation of the loop in the space is quantified by the inclination θ and the azimuthal angle α of the plane:

$$\theta=\frac{\pi}{2}-\cos^{-1}(\boldsymbol{e}_r\cdot\boldsymbol{e}_n) \qquad \alpha=\frac{\pi}{2}-\cos^{-1}(\boldsymbol{e}_\varphi\cdot\boldsymbol{e}_n)$$

where $\boldsymbol{e}_r$ is the normal to the solar surface, $\boldsymbol{e}_\varphi$ is the longitudinal vector along the east-west solar direction. The loop length L is estimated numerically.

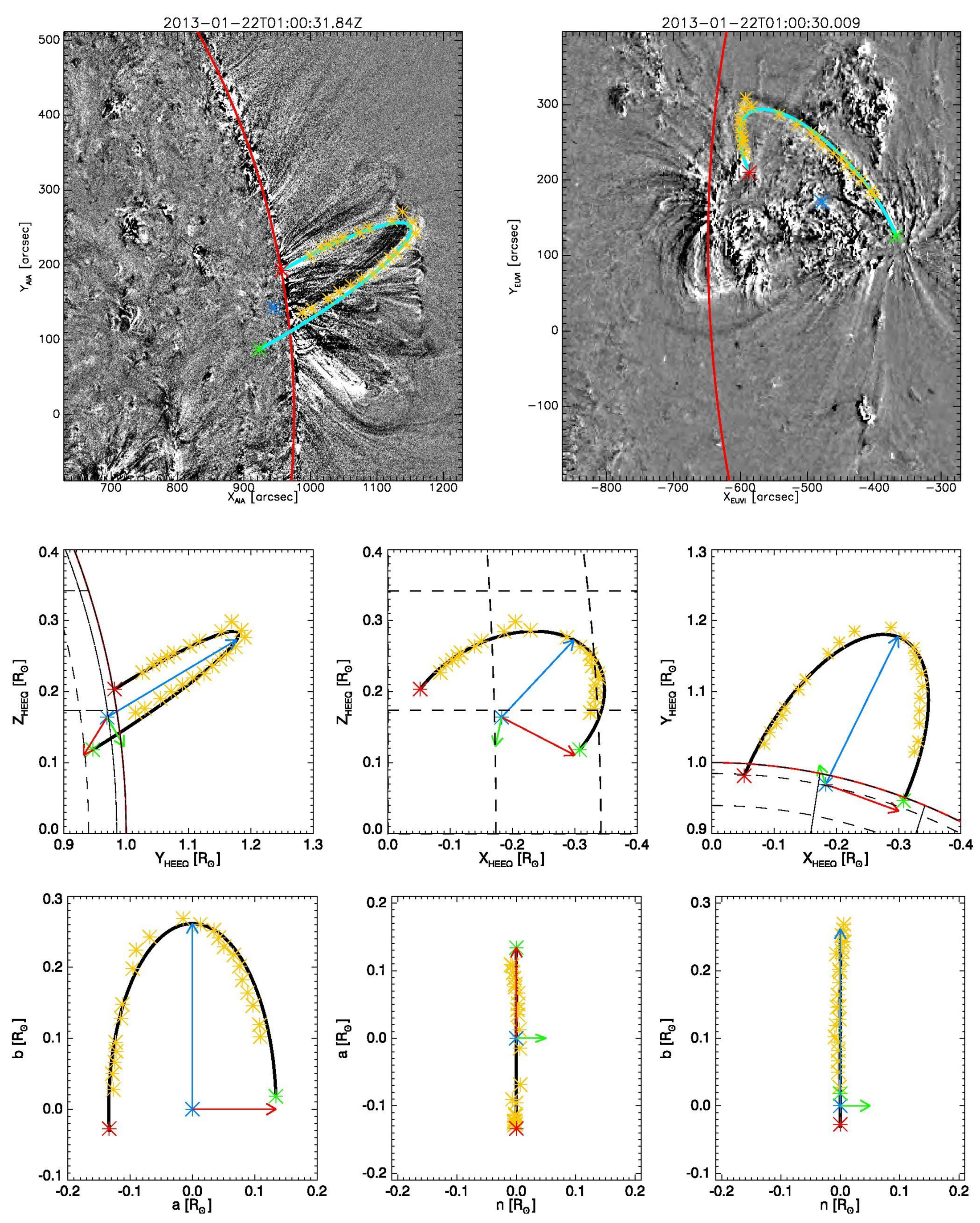
References

[1] Aschwanden, M.J.; Fletcher, L.; Schrijver, C.J.; Alexander, D. ApJ. 1999, 520, 880–894.

[2] Nisticò, G.; Verwichte, E.; Nakariakov, V. M. Entropy, 2013, 15, 10, 4520-4539.

[3] Sonnerup, B.U.O. & Scheible, M. ISRS, 1998, 1, 185-220.

3D loop reconstruction



Sample

N= 22 tie-points

Loop centre in heliographic coordinates:

$\boldsymbol{C}$=(100.67, 9.43) deg

PCA outputs

Loop axes:

a= 0.134 $R_\odot$
b= 0.262 $R_\odot$

eccentricity=0.86

Orientation in 3D space

θ=20.66 deg
α=-17.17 deg

Loop length

L = 457.96 Mm

Example of 3D reconstruction of a loop belonging to the active region (AR) NOAA11654 [2]. From SDO/AIA (top-left) the AR is partly off-limb and the footpoints are not visible. The same AR is seen well from STEREO-A (top-right). The yellow symbols are the 3D data points measured with the routine *scc_measure.pro*. The light-blue line is the fitted curved loop by PCA. The centre at the loop baseline is in blue and the footpoints are distinct in red and green. The middle plots show the reconstructed loop (black) against the data points (yellow) for the different orientations of the Heliocentric Earth EQuatorial (HEEQ) coordinate system. The coloured vectors represent the new set of three axes: in red and blue, the minor and major axis; in green, the normal to the loop plane. The bottom figures show the loop in this new reference frame for different orientations.

Discussion and conclusions

Understanding the shape of loops is important for a better estimate of the loop length, which is a crucial parameter in inferring the magnetic field by coronal seismology, or comparing the geometry with magnetic field models. The previous approach to 3D reconstruction is based on fitting stereoscopic tie-points, taken along the loops, and requires the determination of six free parameters (e.g., inclination, azimuth angle, baseline, etc). All this information is implicitly stored in the distribution of a sample of 3D measurements, and it can be retrieved easily by PCA. The advantage of this method is that it works for a reasonably small number of data points (10-20), without requiring any intermediate step, like interpolation. The only assumption made is that the loop are almost planar. A further step for a more appropriate modelling of the magnetic field would be fitting the loop by a dipole or a stretched dipole line and error analysis. Bootstrap method will further provide error estimates of the principal components [3], hence of the 3D reconstruction .