A Computational Method for Modelling Chromospheric Evaporation

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Abstract

We present the results of 1D field-aligned simulations that study the plasma response to variations in coronal heating. We treat thermal conduction by using Super Time Stepping methods (Meyer et al. 2012). The basic loop model consists of a hydrostatic equilibrium in thermal balance between conduction, optically thin radiation and heating. Then we include an additional release of energy, across the coronal part of the loop to increase the temperature. This drives a conduction front downwards into the transition region and facilitates chromospheric evaporation. We show that, using Super Time Stepping methods for thermal conduction, we can fully resolve the temperature and density profiles without prohibitive time-step restrictions.

Introduction to Super Time Stepping Methods (Meyer et al. 2012)

Thermal energy equation:

$$\rho\frac{\partial \epsilon}{\partial t} = -\rho(\mathbf{v}\cdot\nabla)\epsilon - P\nabla\cdot\mathbf{v} - \nabla\cdot\mathbf{q} - \rho^2\chi T^\alpha + H,$$

where $-\mathbf{q} = \kappa_\parallel(\mathbf{B}\cdot\nabla T)\frac{\mathbf{B}}{B^2} + \kappa_\perp(\mathbf{B}\times(\nabla T\times\mathbf{B}))/B^2$ is the heat flux vector.

Conduction term in the thermal energy equation is a parabolic operator.
Explicit stability condition : $\Delta t_{parab} \leq \frac{(\Delta x)^2}{2D}$.

Ideal MHD equations form a hyperbolic system.
CFL condition for advection : $\Delta t_{adv} \leq \frac{\Delta x}{\max(v)}$.

We have a mismatch in time-step restrictions because for small enough mesh sizes $\Delta t_{parab} << \Delta t_{adv}$. Therefore, explicit methods must sub-cycle conduction to catch up with the advection.

Super Time Stepping (STS) methods have been designed to relax the explicit time-step restriction while using only a limited number of stages. These methods use s strategically designed explicit Runge-Kutta stages that are choosen so that the overall time-step is stable up to $\approx s^2\Delta t_{parab}$.

- Select the number of stages so that $s^2\Delta t_{parab} = \Delta t_{adv}$.
- Update the conduction term to the hyperbolic time-step using only s-stages.
- $\therefore$ s fold gain in computational efficiency over explicit time-step sub-cycling.

Non-Linear Thermal Conduction Problem

To demonstrate the potential of Super Time Stepping methods we first consider the following non-linear thermal conduction problem,

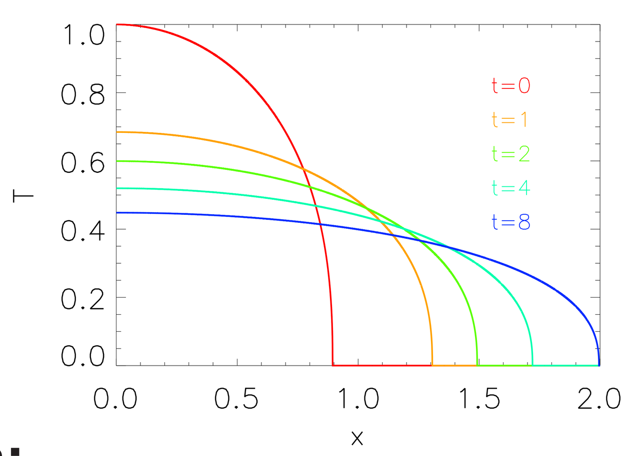
$$\begin{cases} \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(T^{5/2}\frac{\partial T}{\partial x}\right), & t\in(0,8],\ x\in[0,2], \\ T'(0,t) = 0, & t\in(0,8], \\ T(x,0) = \begin{cases} \left(1-\frac{5}{4}x^2\right)^{2/5}, & x\in[0,2/\sqrt{5}], \\ 0, & x\in(2/\sqrt{5},2]. \end{cases} \end{cases}$$

Following Mayer et al (1983) we obtain a self-similar solution for this problem,

$$T(x,t) = \begin{cases} \frac{1}{(1+\frac{9}{2}t)^{2/9}}\left(1-\frac{x^2}{\frac{4}{5}\left(1+\frac{9}{2}t\right)^{4/9}}\right)^{2/5}, & x\in\left[0,\frac{2}{\sqrt{5}}\left(1+\frac{9}{2}t\right)^{2/9}\right], \\ 0, & x\in\left(\frac{2}{\sqrt{5}}\left(1+\frac{9}{2}t\right)^{2/9},2\right]. \end{cases}$$

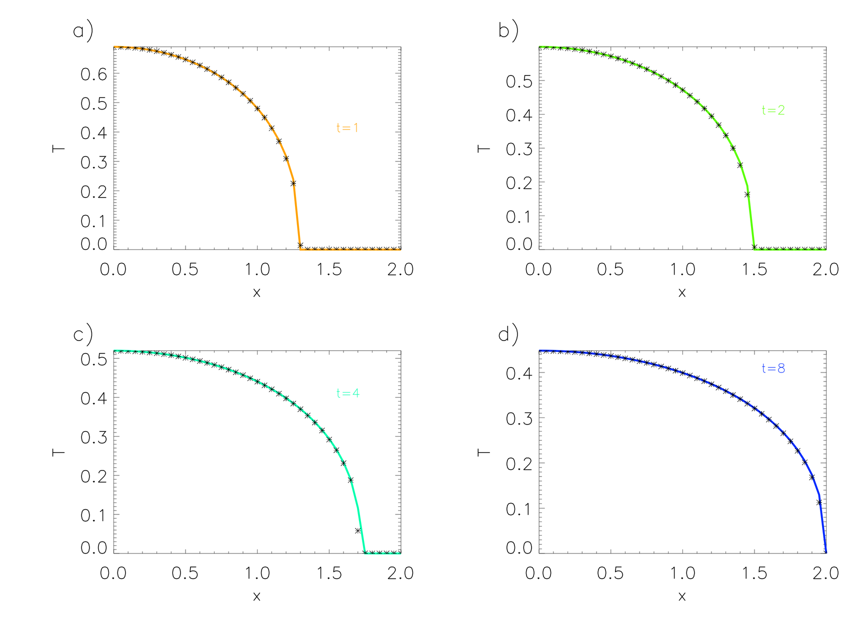
Shown on the RHS, solutions propagate along the x-axis following the travelling wavefront location,

$$x_f(t) = \frac{2}{\sqrt{5}}\left(1+\frac{9}{2}t\right)^{2/9}.$$

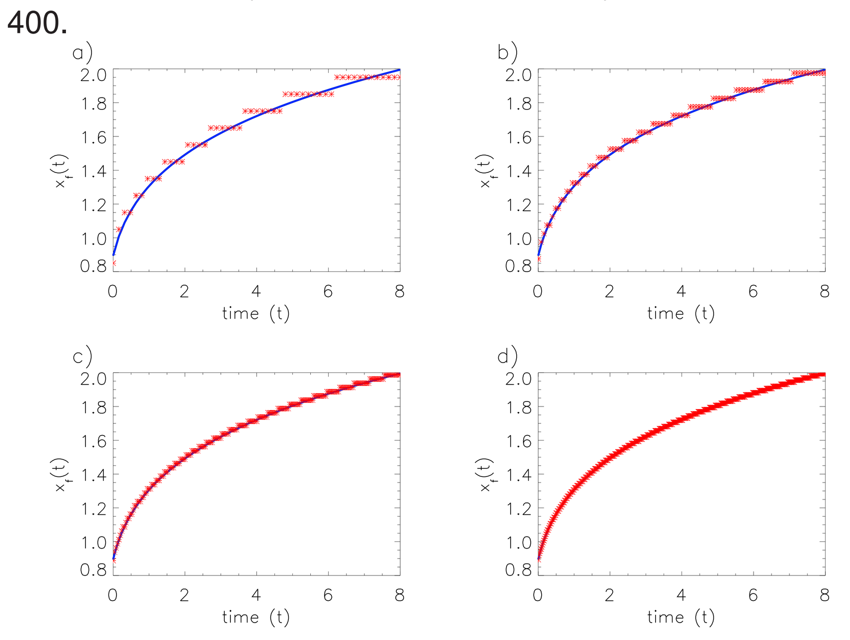


Exact solutions & STS approximations:

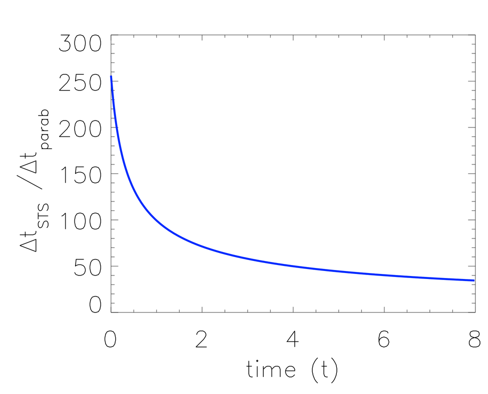
Exact solution $T(x,t)$ (solid line) and STS approximation T_j^n (asterisks) with resolution $N_x = 40$, $N_t = 100$ at time: a) $t=1$, b) $t=2$, c) $t=4$, d) $t=8$.



Wavefront location $x_f(t)$ (solid line) and STS approximation (asterisks) with resolution : a) $N_x = 20$, $N_t = 50$, b) $N_x = 40$, $N_t = 100$, c) $N_x = 80$, $N_t = 200$, d) $N_x = 160$, $N_t = 400$.



The STS solutions correctly capture the time evolution of the thermal wavefront.



As we increase the resolution, the STS wavefront locations converge to the corresponding exact locations. We plot the ratio of the super time-step (Δt_{sts}) to the parabolic time-step (Δt_{parab}) for Case d).

Average $\Delta t_{sts}/\Delta t_{parab} = 60$.

Therefore, **we obtain substantial computational gains with STS.**

Model & Hydrostatic Equilibrium

1D field-aligned model:

$$\frac{\partial\rho}{\partial t} + v\frac{\partial\rho}{\partial s} = -\rho\frac{\partial v}{\partial s},$$
$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial s} = -\frac{\partial P}{\partial s} - \rho g(s),$$
$$\rho\frac{\partial\epsilon}{\partial t} + \rho v\frac{\partial\epsilon}{\partial s} = -P\frac{\partial v}{\partial s} + \frac{\partial}{\partial s}\left(\kappa_0 T^{5/2}\frac{\partial T}{\partial s}\right) - \rho^2\chi T^\alpha + H(s).$$

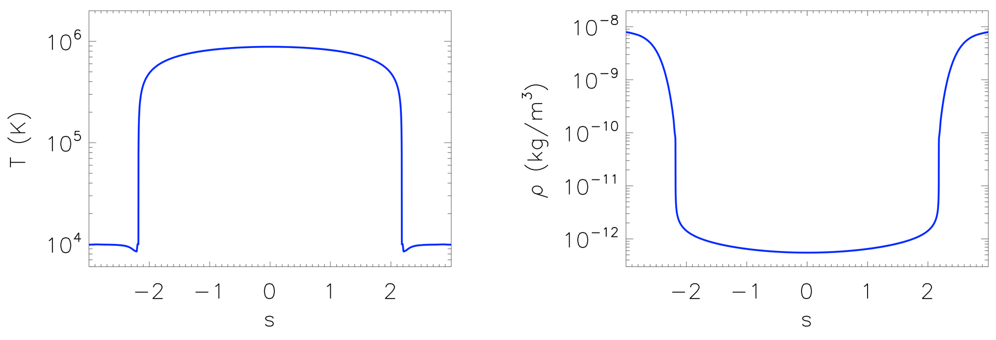
Solved using Lare1D code (Arber et al. 2001) with STS to treat thermal conduction.

Hydrostatic equilibrium:

60Mm loop.

$H_{bg} = 10^{-5} Jm^{-3}s^{-1}$.

T and ρ as functions of position along the loop.



Chromospheric Evaporation

Include coronal heating source:

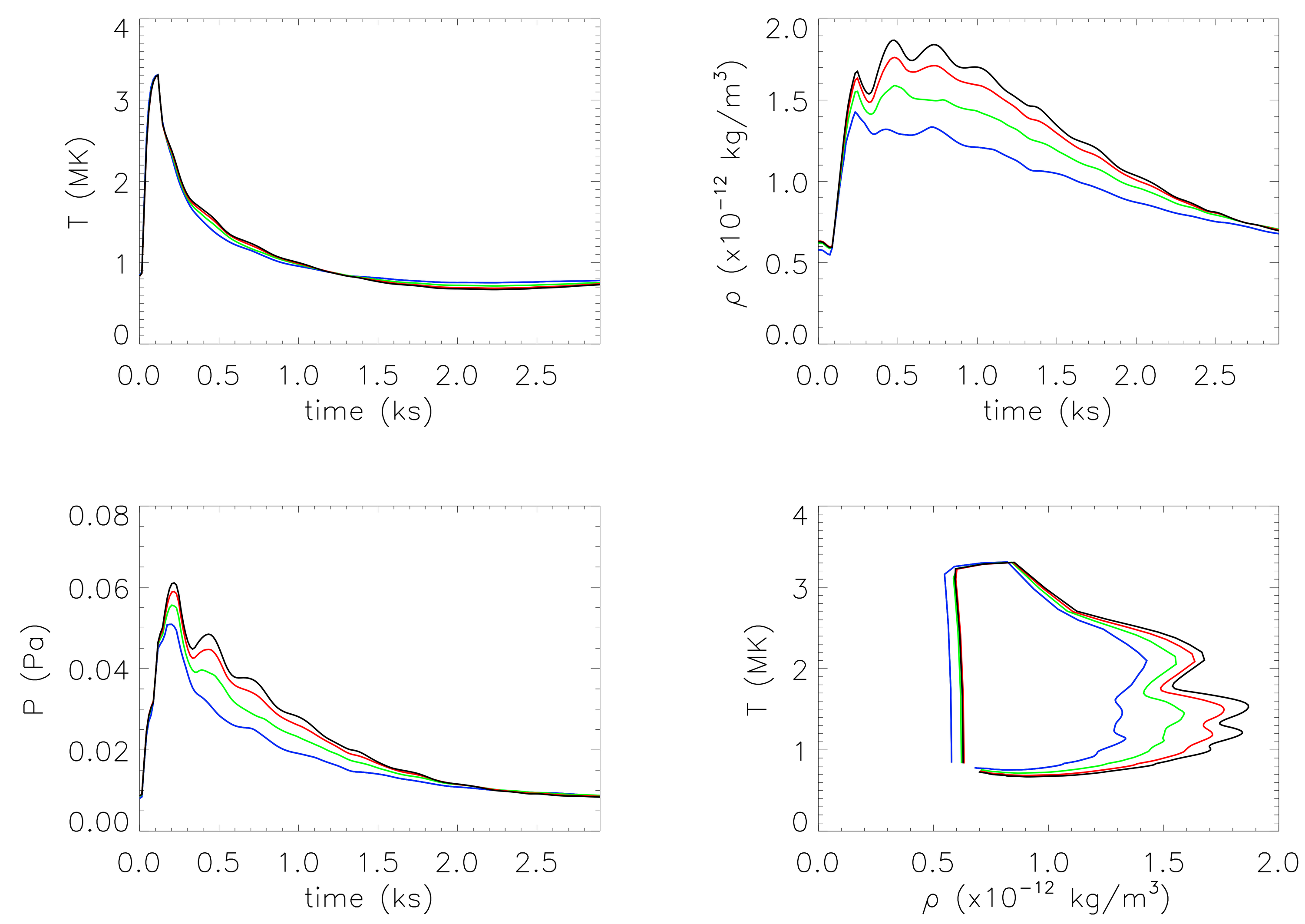
$H = 10^{-3} Jm^{-3}s^{-1}$.

$\tau_H = 120s$.

Re-run at different resolutions.

Run	N_s	Resolution	$\frac{\Delta\bar{t}_{sts}}{\Delta t_{parab}}$	$\bar{s}$	$\frac{\tau_{exp}}{\tau_{sts}}$
B	512	$\approx 120km$	5.6	3.8	1.0
G	1024	$\approx 60km$	12.2	5.7	2.1
R	2048	$\approx 30km$	25.5	7.4	4.5
Bl	4096	$\approx 15km$	52.6	11.2	~10.0

Coronal averages:



The coronal averages confirm the result presented by Bradshaw and Cargill (2013), that the main effect of insufficient resolution is on the coronal density while the coronal temperature is more weakly dependent.

At 15km resolution we take a super time-step (Δt_{sts}) which is on average 52 times larger than the parabolic time-step (Δt_{parab}). For this resolution level we estimate that we obtain a 10 fold gain in computational efficiency with Super Time Stepping over explicit time-step sub-cycling. Therefore, as we increase the resolution, with Super Time Stepping methods, **we obtain substantial computational gains because we are able to relax the prohibitive time-step restrictions**.

Conclusions

Super Time Stepping methods correctly capture the time evolution of thermal wavefronts while using a relaxed time- step and limited number of stages.

For chromospheric evaporation models, Super Time Stepping methods have the potential to fully resolve the temperature and density profiles without prohibitive time-step restrictions.

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