Anomalous resistivity in solar and space plasmas

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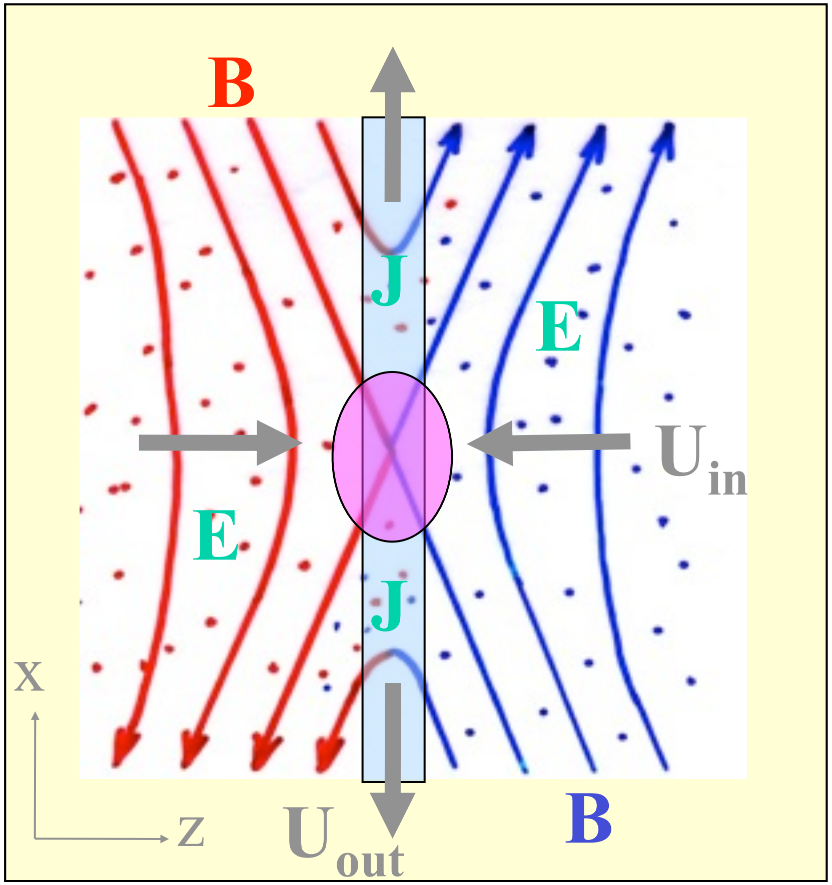
Reconnection in Collisionless plasma



- Reconnection at MHD scale requires violation of frozen-in field condition:

$$\underline{E} + \underline{v} \times \underline{B} = \frac{m_e}{ne^2}\frac{\partial \underline{J}}{\partial t} + \frac{m_e}{ne^2}\left(\underline{v}\underline{J} + \underline{J}\underline{v}\right) - \frac{1}{ne}\nabla \times \underline{p_e} + \frac{1}{ne}\left(\underline{J} \times \underline{B}\right) + \frac{m_e}{ne^2}\nu_e \underline{J}$$

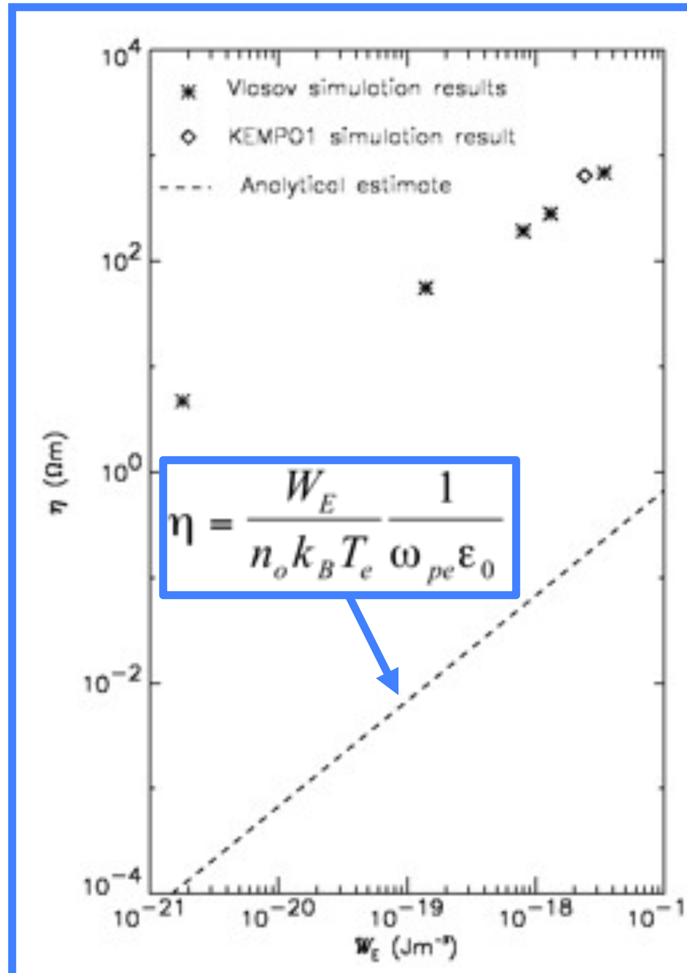
- Kinetic-scale wave turbulence can scatter particles to generate anomalous resistivity at MHD scale [Davidson and Gladd, 1975] :

$$\eta = -\frac{1}{\varepsilon_o \omega_{pe}^2 p_e}\frac{\partial p_e}{\partial t} = \frac{1}{\varepsilon_o \omega_{pe}^2 J}\frac{\partial J}{\partial t}$$

- How does anomalous resistivity depend on MHD variables (n, T, $\mathbf{J}$)?

Anomalous Resistivity due to Ion-Acoustic Waves (Petkaki et al., JGR, 2003,2006)

- Resistivity from Wave-Particle interactions is important in collisionless plasmas
- We have studied resistivity from Current Driven Ion-Acoustic Waves (CDIAW)
  - Used 1D1V Electrostatic Vlasov Simulations
  - Realistic space plasma conditions i.e. $T_e \sim T_i$, Maxwellian and Lorentzian distribution function
  - Found substantial resistivity at quasi-linear saturation



(Watt et al., GRL, 2002)

Why study Ion-Acoustic (IA) Waves?

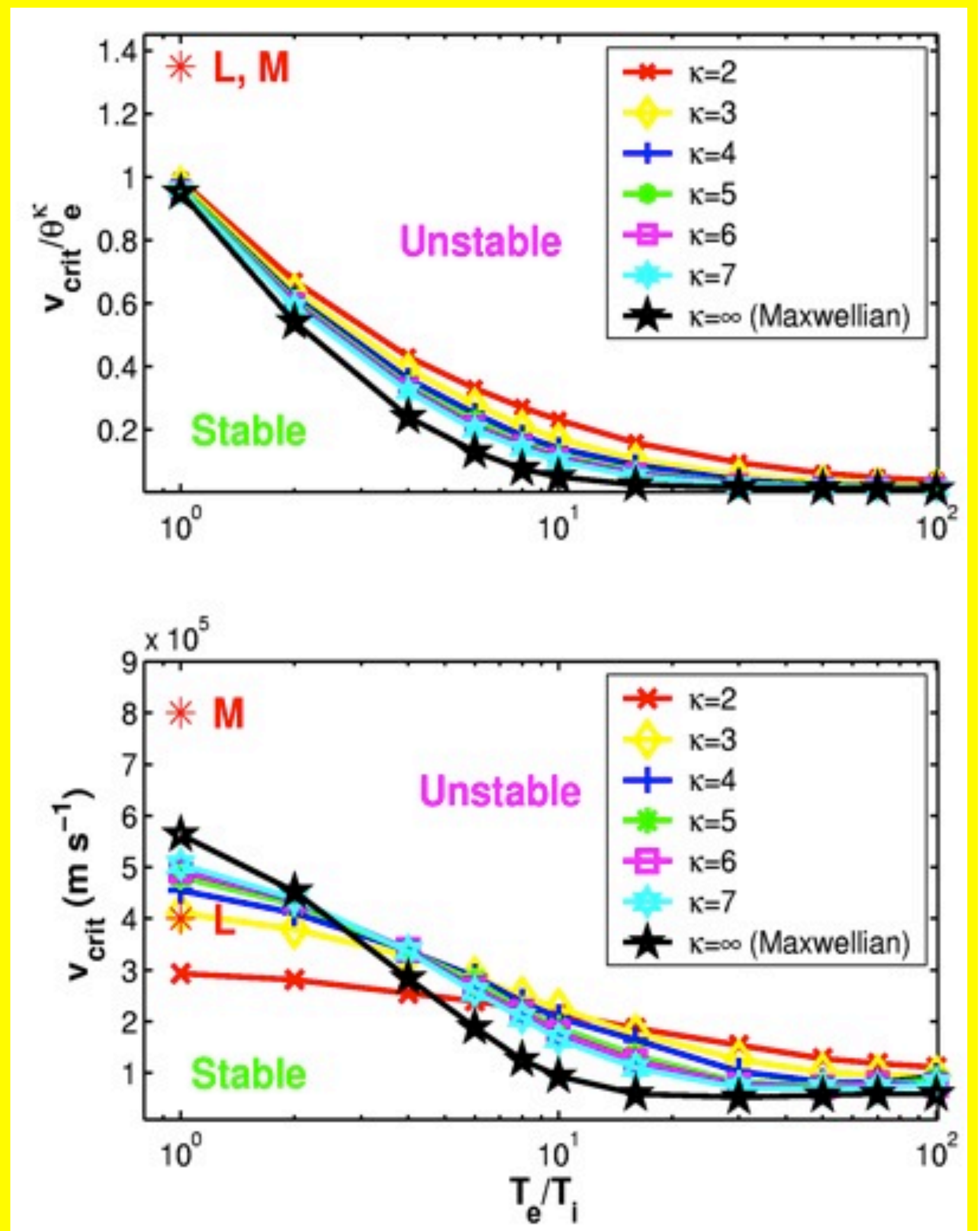
- Ion-acoustic waves are measured in many regions of space plasma (magnetopause, ionosphere), and in laboratory plasma experiments - indicates the need to study them in more detail for a range of plasma parameters.
- Analytical estimates and simulations of the resistivity due to current-driven ion-acoustic waves have concentrated on the regime where electron temperature far exceeds ion temperature. Not the case in solar and space plasmas.
- A Maxwellian plasma with similar electron and ion temperatures, needs a large current to excite unstable ion-acoustic waves. Less for Lorentzian (kappa).

Petkaki et al., **JGR,** 2003

Critical Electron Drift Velocity Normalized to

$$\theta = \left(\frac{\kappa - 3/2}{\kappa} \frac{2k_B T}{m} \right)^{1/2}$$

$M_i = 1836 m_e$



Ion-acoustic anomalous resistivity for space plasmas conditions, for low $T_e/T_i < 4$, Lorentzian DF.

- A Lorentzian DF enables significant anomalous resistivity for conditions where none would result for a Maxwellian DF.
- At wave saturation, the anomalous resistivity for a Lorentzian DF can be an order of magnitude higher than that for a Maxwellian DF, even when the drift velocity and current density for the Maxwellian case are larger.
- The anomalous resistivity resulting from ion acoustic waves in a Lorentzian plasma is strongly dependent on the electron drift velocity, and can vary by a factor of ~ 100 for a 1.5 increase in the electron drift velocity.
- Resistivity I) Corona = 0.1 Ω m, II) Magnetosphere = 0.001 Ω m

Petkaki et al., **JGR,** 2003

Evolution of Vlasov Simulation

- 1D1V and electrostatic with periodic boundary conditions.
- Plasma species e, i modelled with $f\,(z, v, t)$ on discrete grid
- The B = 0 in the current sheet, but curl B = $\mu_0 c^2$J.
- Second-Order Splitting Upwind Method (Petkaki, 2005)

Governing equations

The Vlasov Equation in 1D, no $\mathbf{B}$ in z

$$\frac{\partial f_\alpha}{\partial t} + v_z \frac{\partial f_\alpha}{\partial z} + \frac{q_\alpha}{m_\alpha} E_z \frac{\partial f_\alpha}{\partial v_z} = 0 \qquad (1)$$

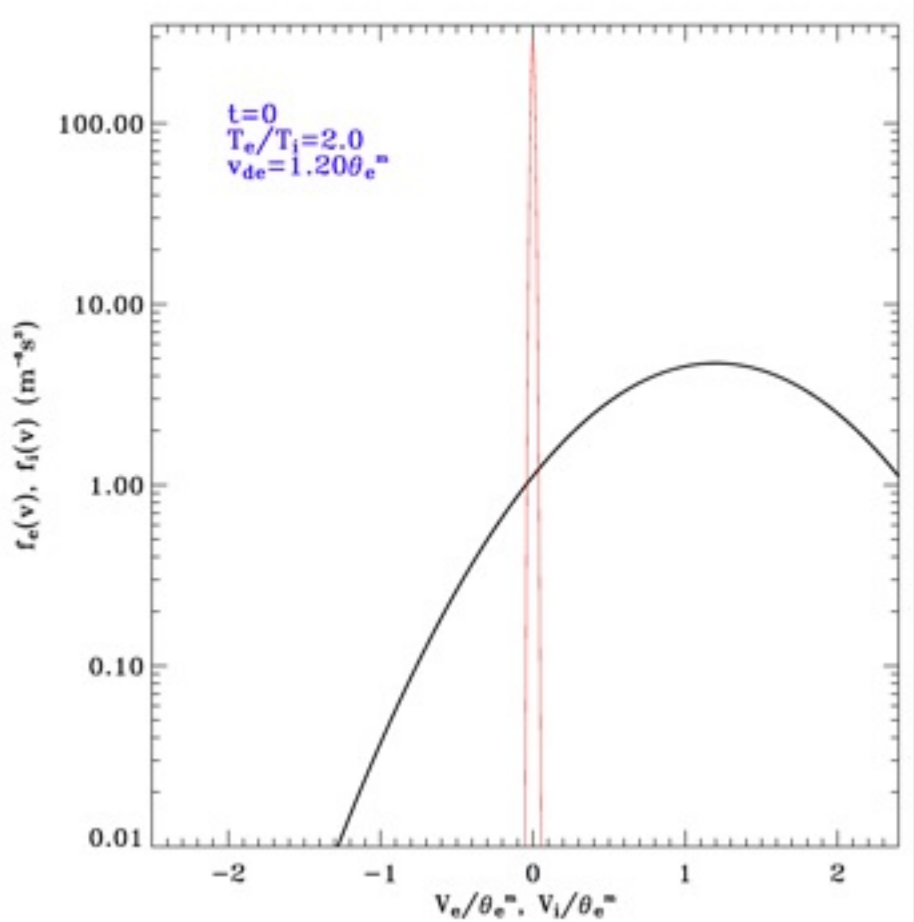
Ampère's Law is used to integrate E_z forward in time;

$$\frac{\partial E_z}{\partial t} = -\mu_0 c^2 J_z \qquad (2)$$

where

$$J_z = \sum_\alpha q_\alpha \int_{-\infty}^{\infty} v_z f_\alpha dv_z \qquad (3)$$

Real Mass Vlasov Simulation Initial Conditions



- CDIAW- drifting electron and ion distributions
- Apply white noise Electric field

$$E_1(z,0)=\sum_{n=1}^{N} E_{tf}\sin\left(k_n z+\varphi\right)$$

$$E_{tf}=\left(\frac{2k_B T_e}{\varepsilon_0\lambda_{De}^3}\right)^{1/2}$$

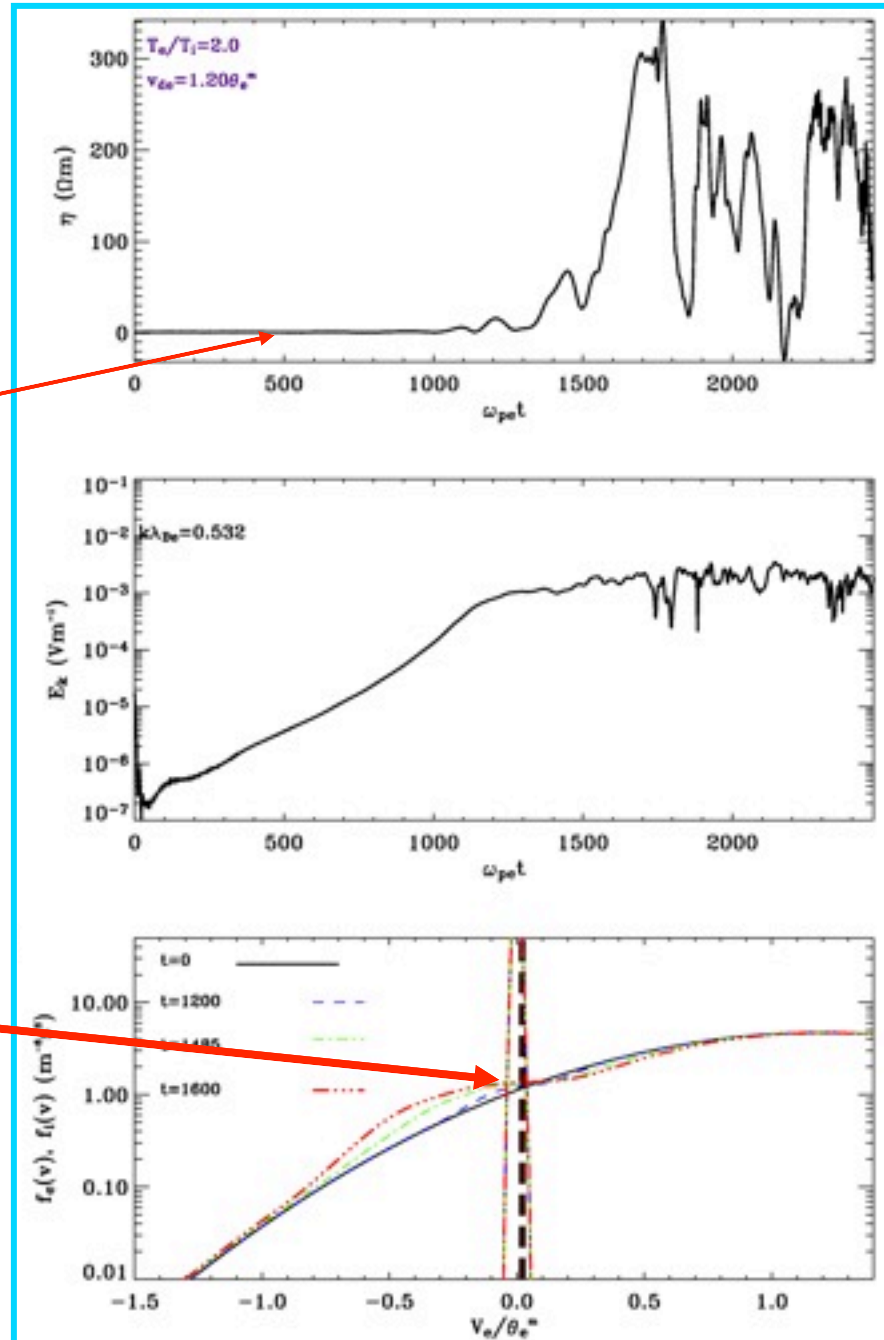
- f close to zero at the edges
- *Maxwellian*
- Drift Velocity - V_{de} = 1.2 x θ
- (θ = $(2T/m)^{1/2}$)
- M_i=1836.15 m_e
- T_i=1 eV, T_e = 2 eV
- n_i=n_e = 7 x 10^6 /m^3
- N_z = 529, N_{ve} = 3729, N_{vi} = 307

Real Mass Ratio Simulations

$$\eta = -\left(\frac{1}{\omega_{pe}^{2}\varepsilon_{0}\dot{j}}\right)\frac{1}{p_e}\frac{\partial p_e}{\partial t}$$

- Evolution from linear to quasi-linear saturation to nonlinear
- Distribution function changes
- Plateau formation at linear resonance

Petkaki, and Freeman , **ApJ,** 2008

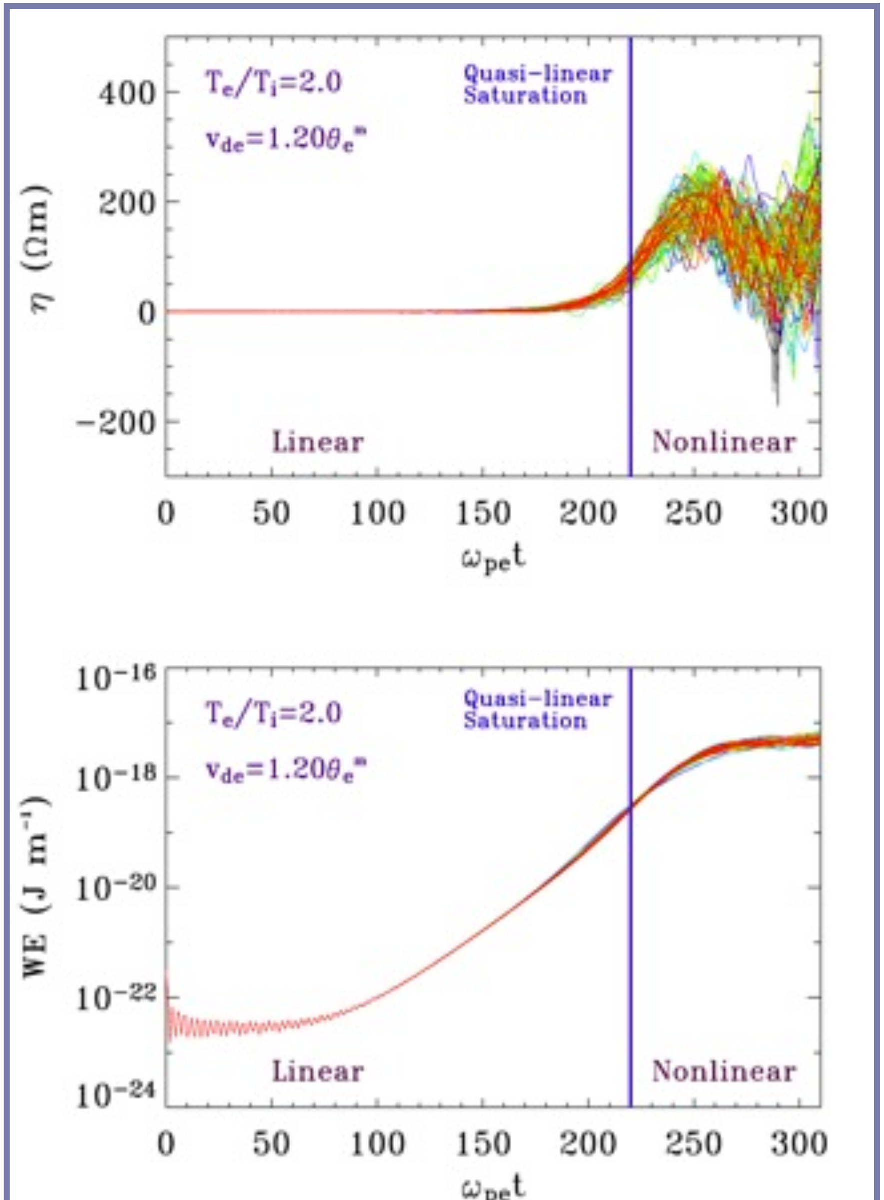


Vlasov Simulations of the Ion-Acoustic Instability

- Ensembles of 10 Vlasov Simulations with real mass ratio of the current driven ion-acoustic instability with identical initial conditions except for the initial phase of noise field
- Explore dependence on the electron-to-ion drift velocity, and on electron to ion temperature ratio
- Variations of the resistivity value observed in the quasilinear and nonlinear phase
- Timescale of variations consistent with electron bounce motion in real mass ratio Vlasov simulations
- The probability distribution of resistivity values Gaussian in linear, quasilinear, nonlinear phase
- A well-bounded uncertainty can be placed on any single estimate of resistivity, e.g., at quasi-linear saturation, nonlinear regime
- Estimation at quasi-linear saturation provides underestimation of resistivity

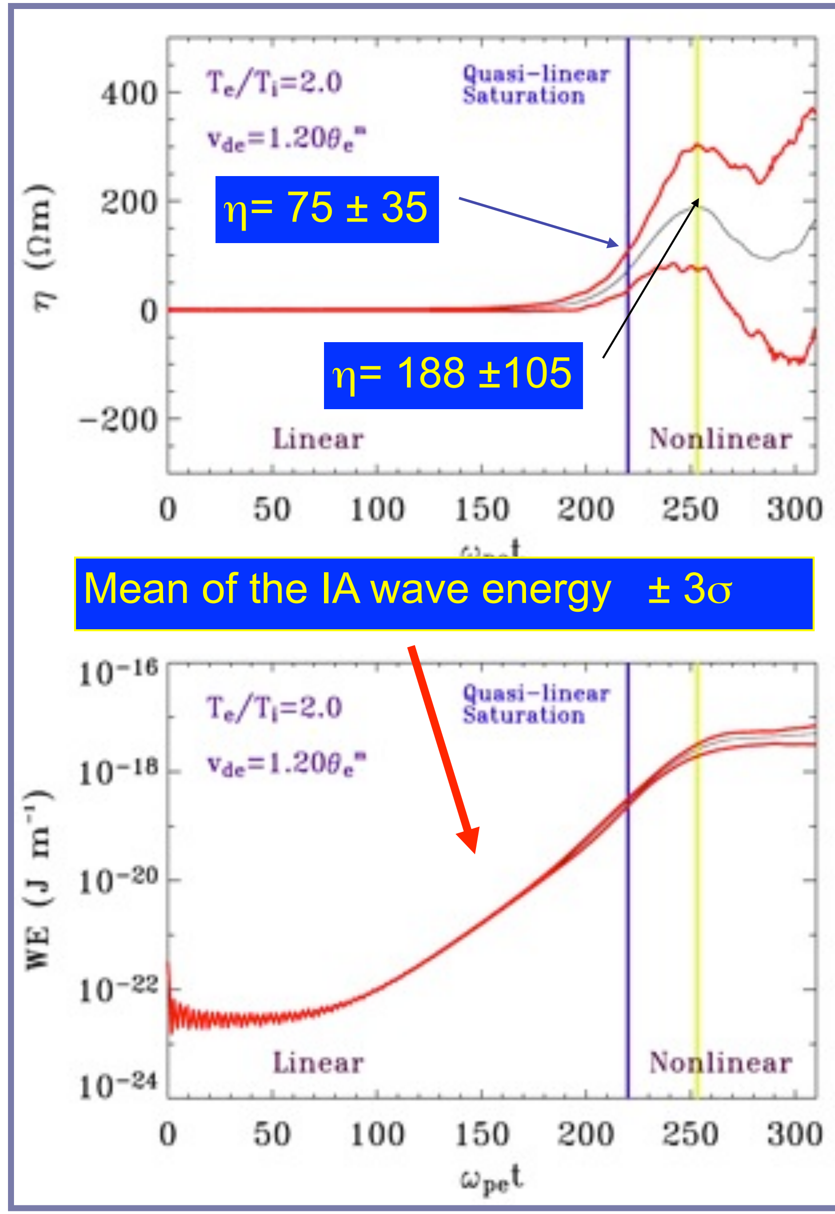
Petkaki, and Freeman , **ApJ,** 2008

Superposition of the time evolution of 104 Vlasov Simulations

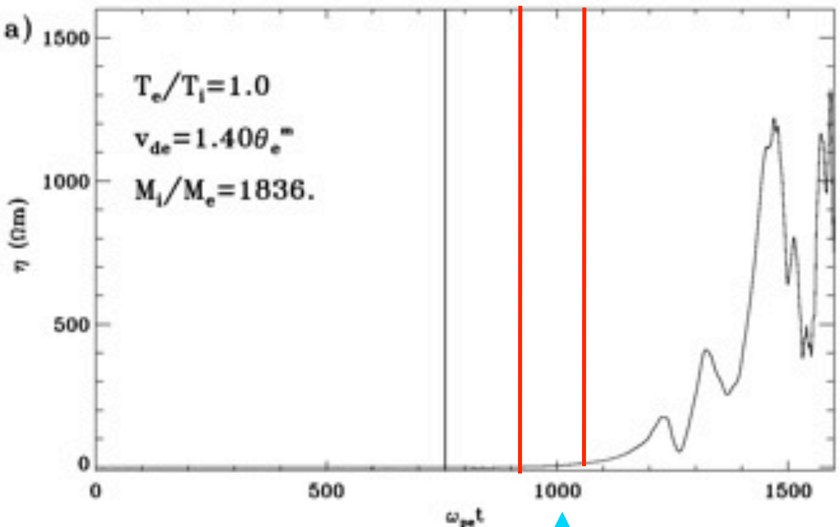


(Petkaki et al., JGR, 2006)

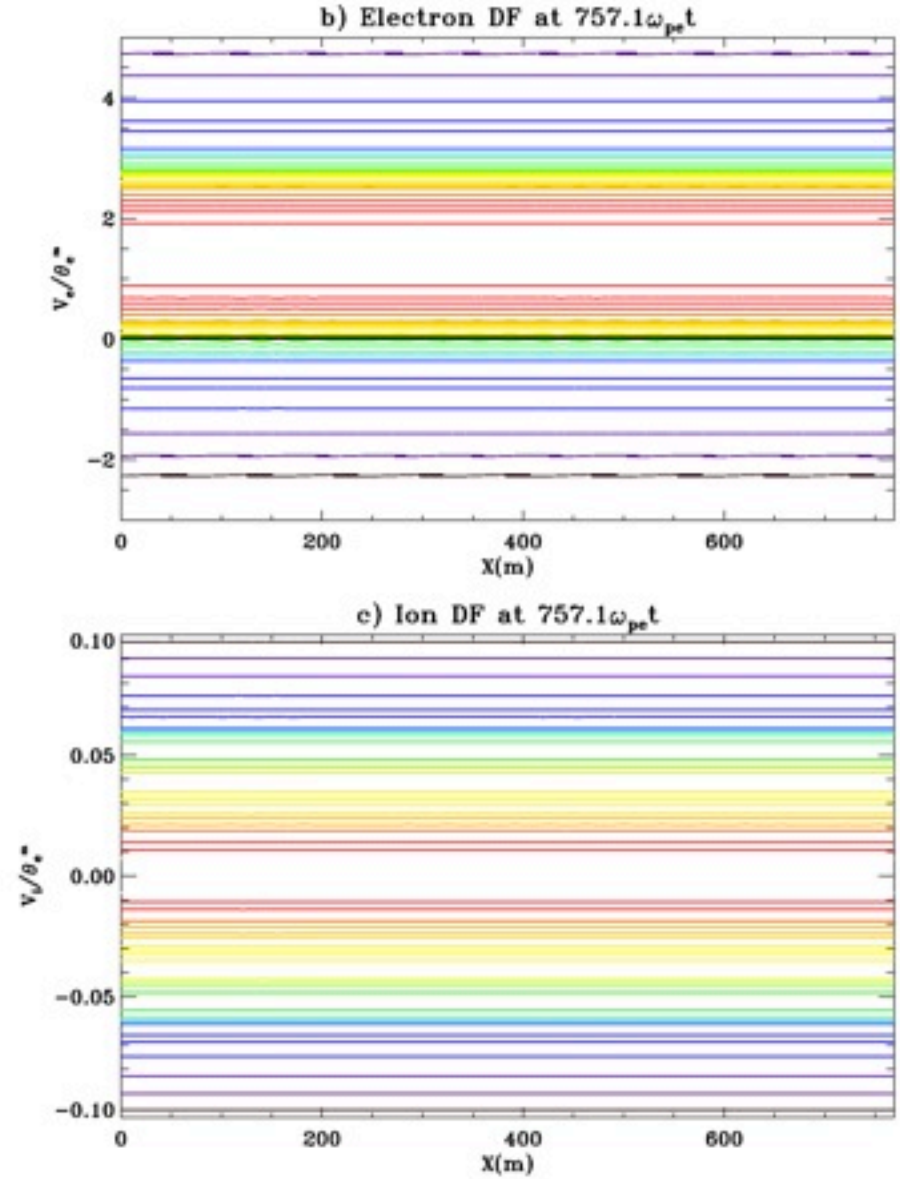
Mean of the IA resistivity (η) $\pm 3\sigma$



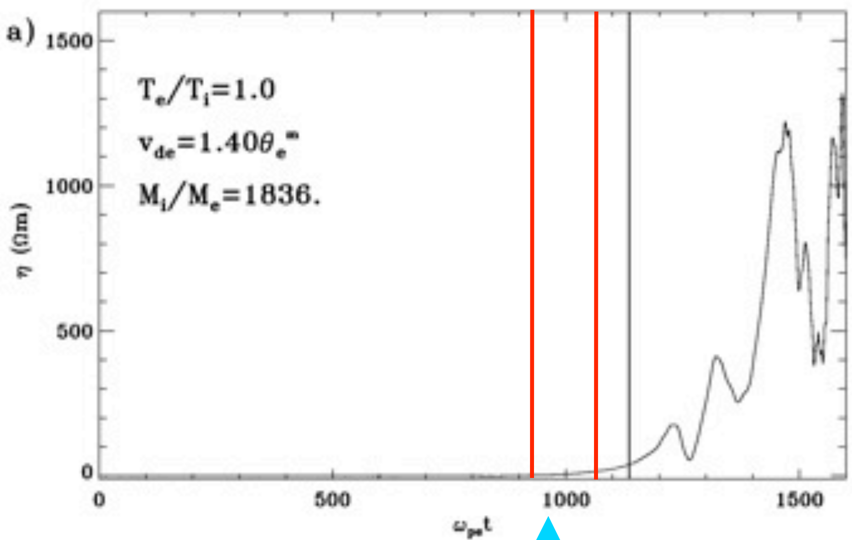
Time Evolution of Distribution



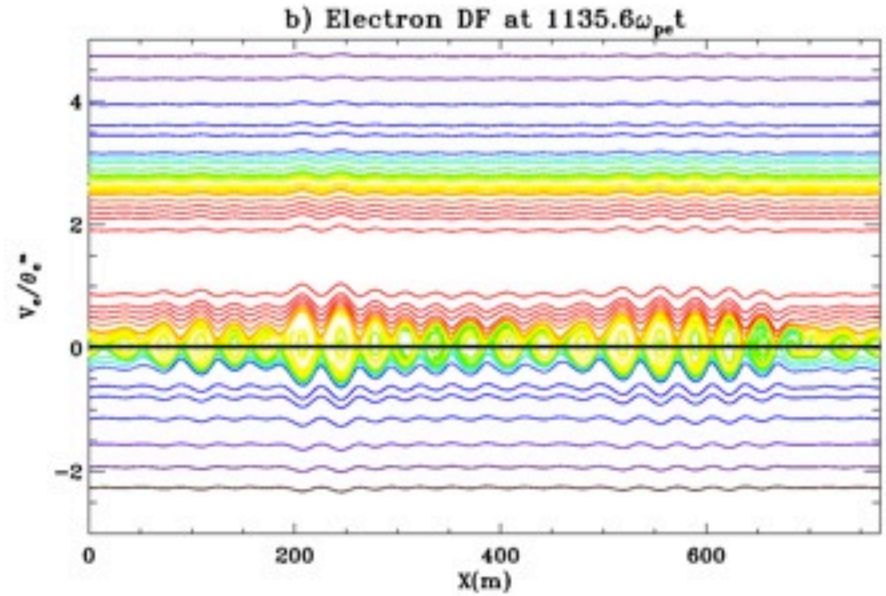
Quasi-linear saturation

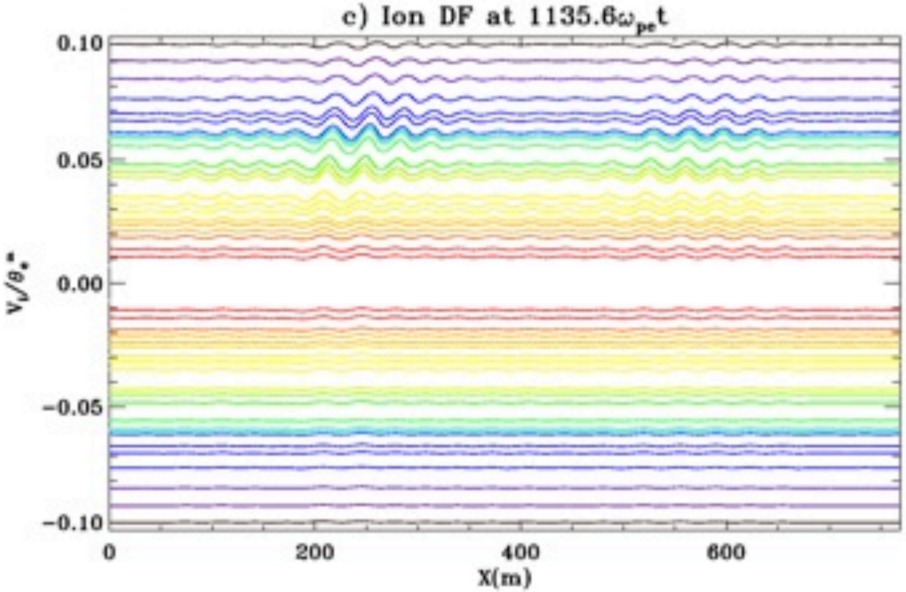


Time Evolution of Distribution

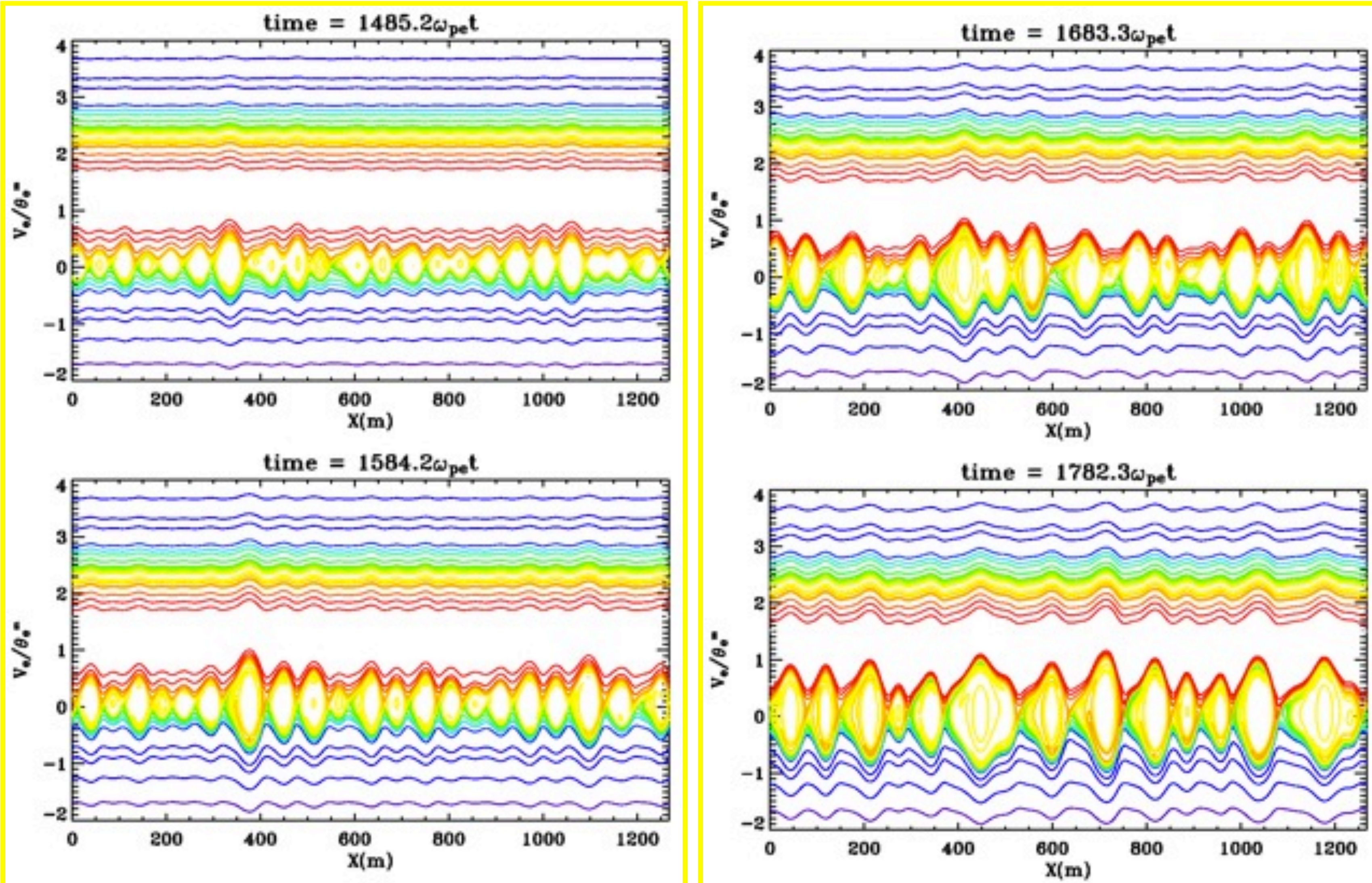


Quasi-linear saturation





Time Evolution of Electron Distribution Function



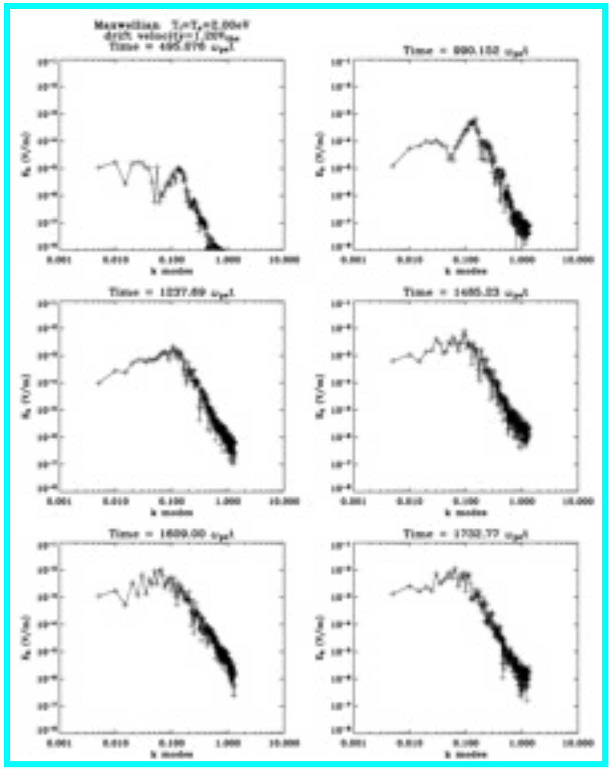
Real Mass Ratio Simulations

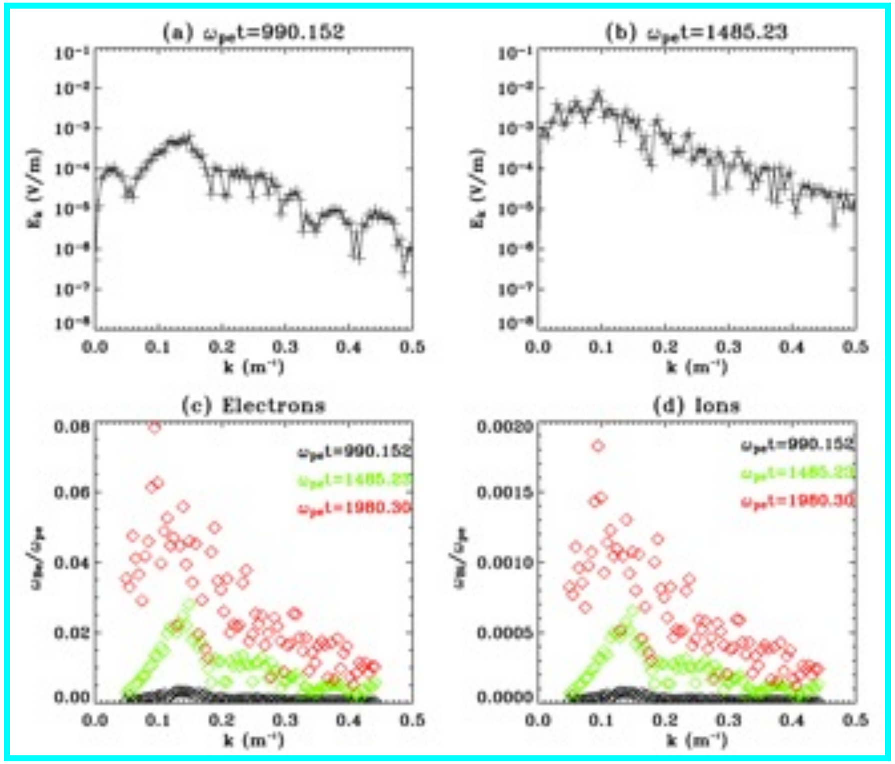
Electron and Ion Bounce Frequencies

- Calculate Electron and Ion bounce frequencies using

$$\omega_{b\alpha} = \sqrt{\left\| \frac{q_\alpha k E_k}{m_\alpha} \right\|} \qquad \alpha \ni e,i$$

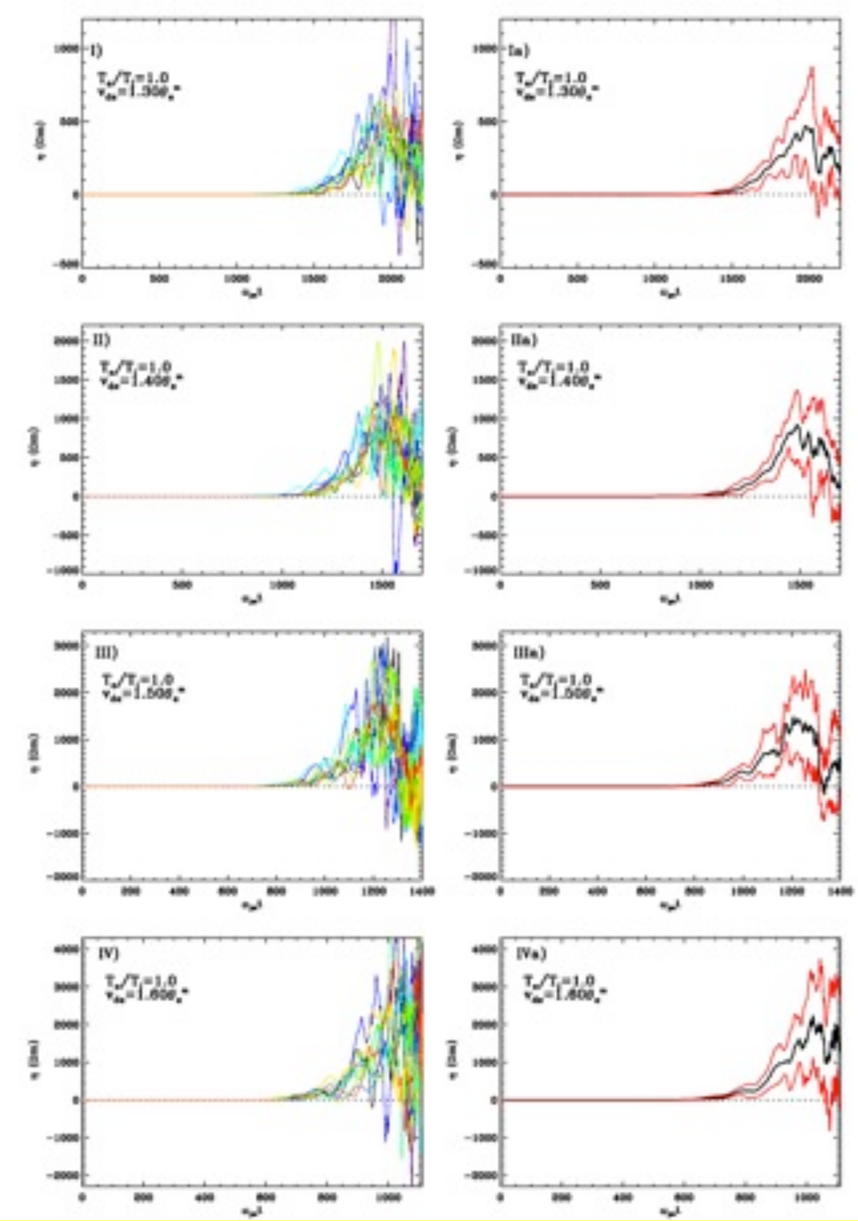
- Compare with Fluctuations in Anomalous Resistivity



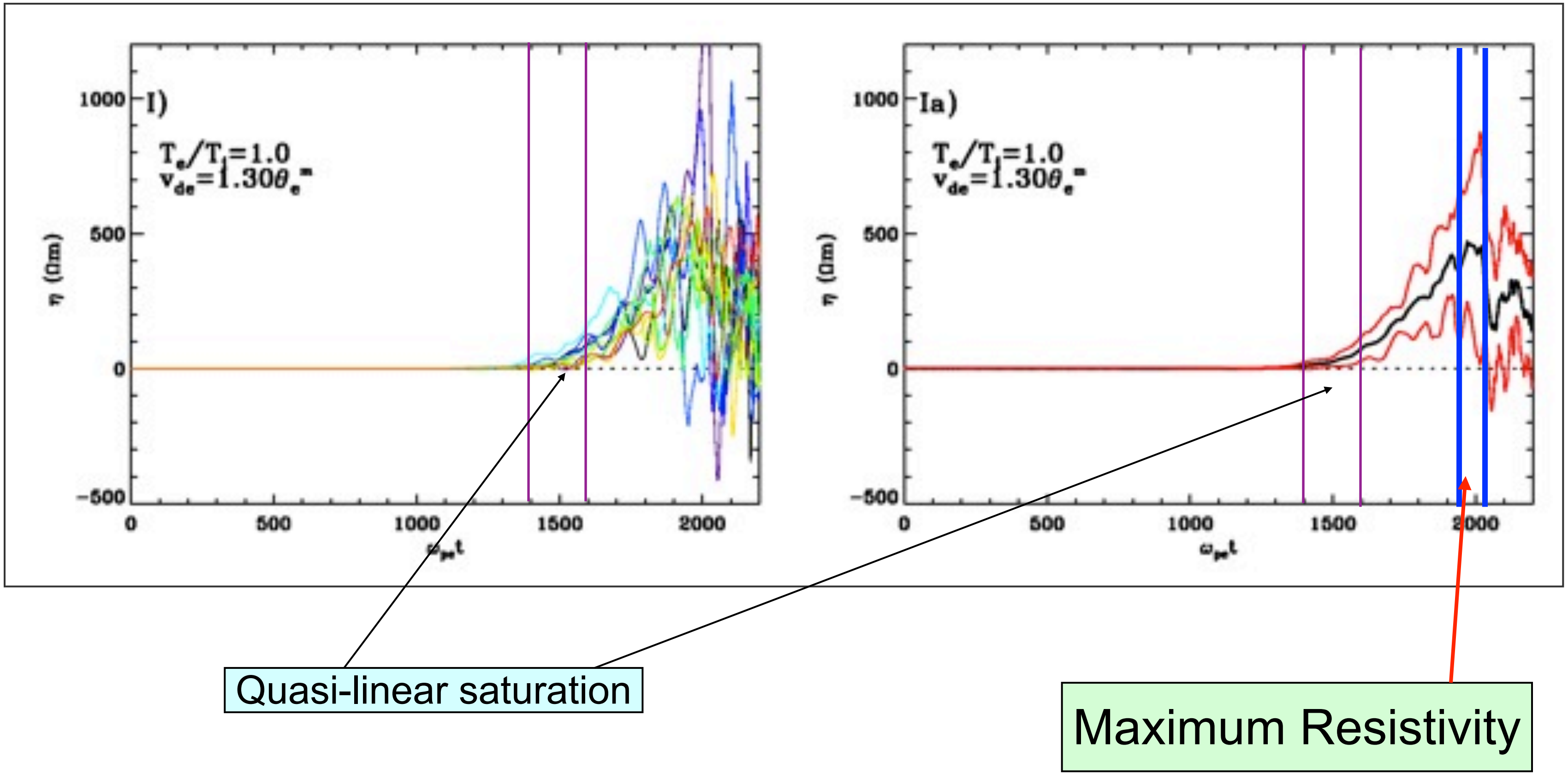


(Petkaki & Freeman, ApJ, 2008)

- CDIAW- drifting electron and ion distributions
- Apply white noise Electric field
- F close to zero at the edges
- *Maxwellian*
- Drift Velocity - $V_{de} = 1.3 – 1.6\ \theta$ ($\theta = (2T/m)^{1/2}$)
- M_i=1836.15 m_e
- T_i=1 eV, T_e = 1 eV
- n_i=n_e = 7 x 10^6 /m^3
- N_z = 559, N_{ve} = 3371, N_{vi} = 339
- N_z = 511, N_{ve} = 3545, N_{vi} = 365
- N_z = 498, N_{ve} = 3305, N_{vi} = 393
- N_z = 486, N_{ve} = 3141, N_{vi} = 421

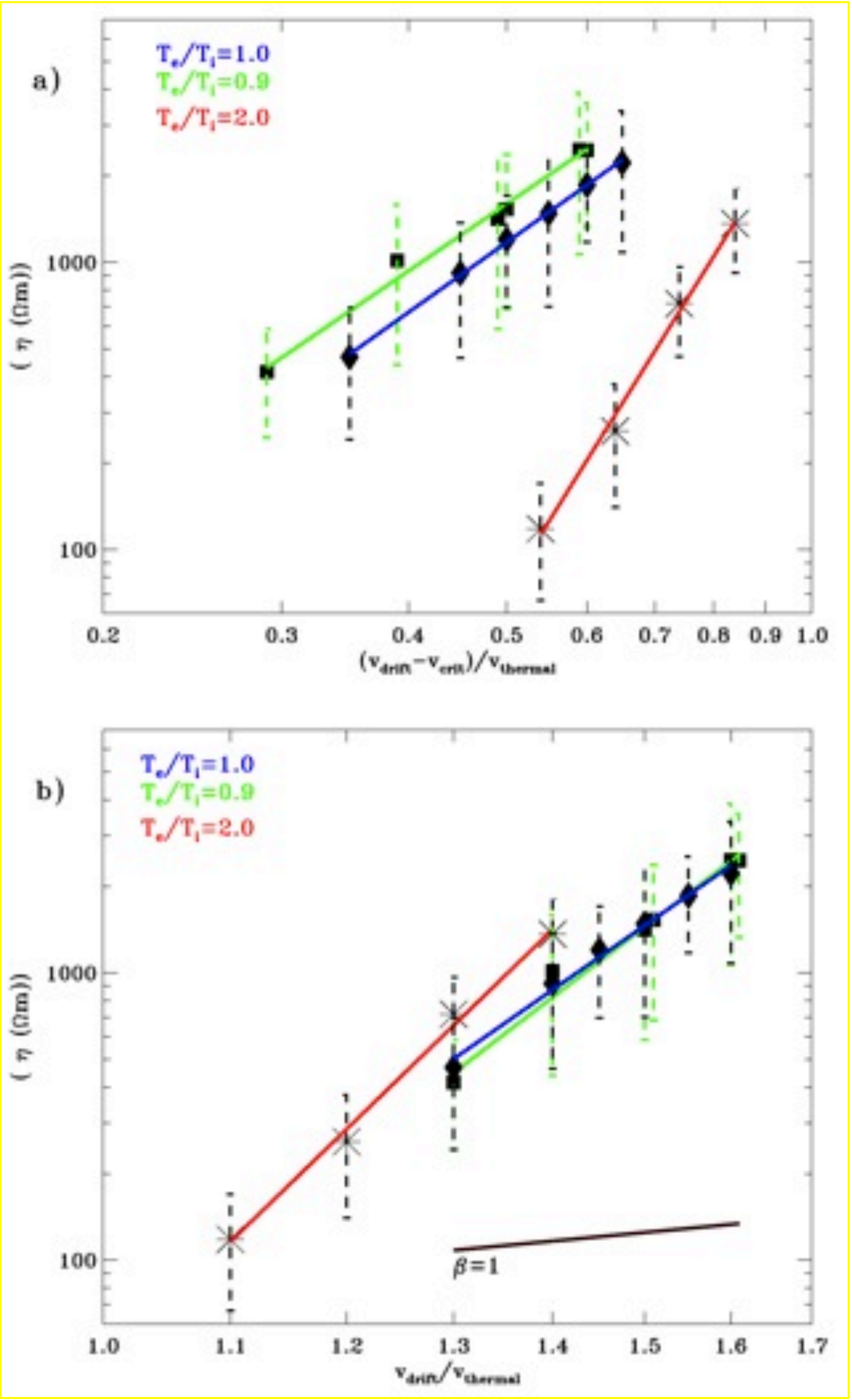


Ensembles of 10 Vlasov simulations

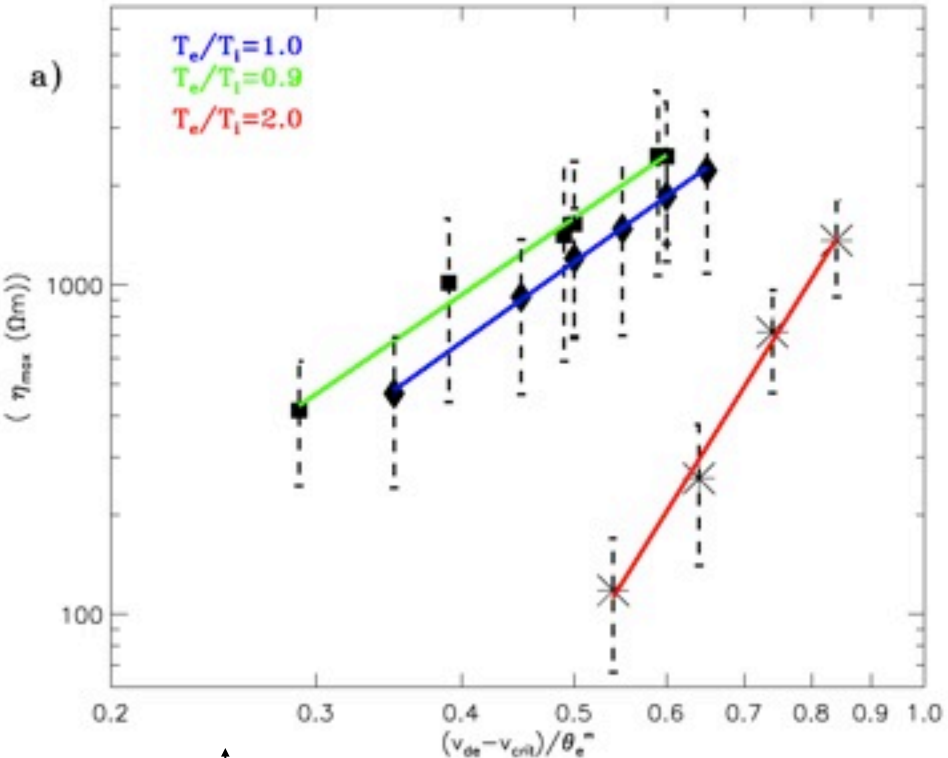


(Petkaki & Freeman, ApJ, 2008)

- Classic Ion-acoustic resistivity linear function of V_{drift}
- Use maximum of η averaged over an ensemble of 10 Vlasov simulations
- **0.9 ≤ T_e / T_i ≤ 2,** temperature ratios in the terrestrial magnetosphere, the solar corona and in solar flares.
- Vlasov simulations reveal nonlinear dependence on the electron to ion drift velocity – current

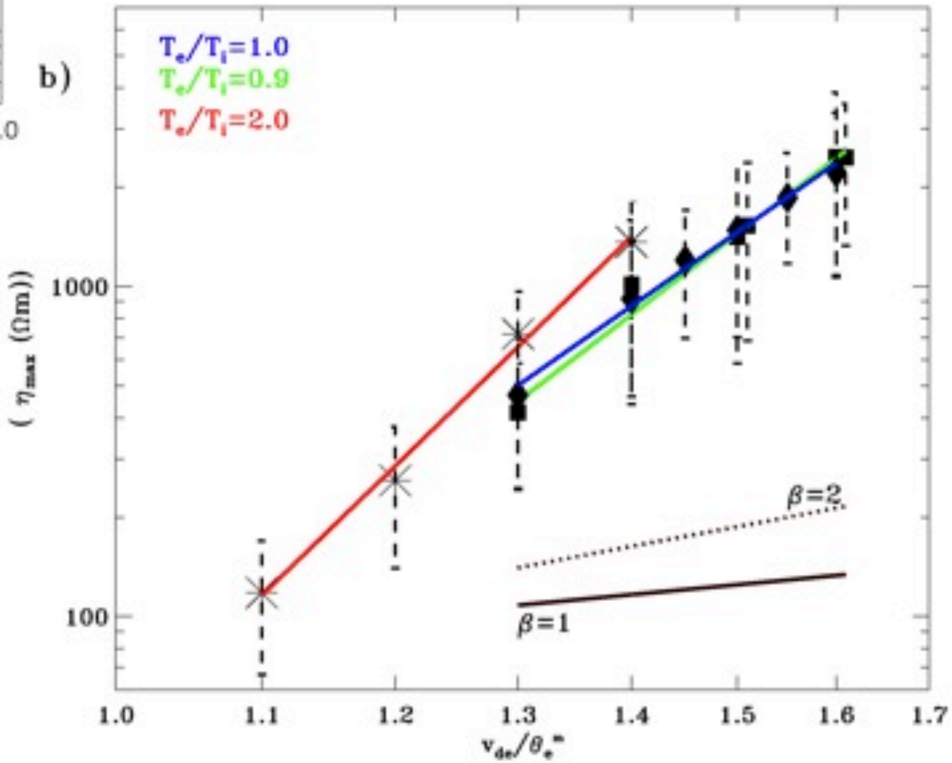


(Petkaki & Freeman, ApJ, 2008)



$$\eta_{max} = 10^d \left(\frac{u_{de}}{\theta_e^m} \right)^\beta$$

β ~ **8 - 10**



$$\eta_{max} = 10^c \left(\frac{u_{de} - u_{crit}}{\theta_e^m} \right)^a$$

α ~ **2.5 - 6**

Conclusions

1. Investigated the dependence of anomalous resistivity for the case of the ion-acoustic instability on the ratio of electron to ion drift velocity – physical range of drift velocities.
2. Explored the low temperature ratio range of $0.9 \leq T_e / T_i \leq 2$ close to temperature ratios in the terrestrial magnetosphere, the solar corona and in solar flares.
3. Anomalous resistivity is measured at the maximum of anomalous resistivity averaged over an ensemble of 10 Vlasov simulations

- Anomalous resistivity is a power law function of the normalized electron drift velocity, $v_{de} / \theta^e{}_m$ approximately with exponent is $\alpha \sim 8 - 10$.
- Dependence is considerably stronger that the linear dependence by linear theory.
- Anomalous resistivity is a power law function of the normalized drift velocity $(v_{de} - v_{crit}) / \theta^e{}_m$ power law exponent $\beta \sim 2.5 - 6$.
- Stronger dependence than linear or quadratic dependence used in simulations.

References

Petkaki P., Watt C.E.J., Horne R., Freeman M., JGR, 108, A12, 1442, doi:10.1029/2003JA010092, 2003

Watt C.E.J., Horne R. Freeman M., Geoph. Res. Lett., 29, doi:10.1029/2001GL013451, 2002

Petkaki P., Freeman M., Kirk T., Watt C.E.J., Horne R., JGR, 111, doi:10.1029/2004JA010793, 2006

Petkaki P., Freeman M., The Astrophysical Journal, 686, 686-693, 2008

Tables

$T_e/$	0.9	1.0	2.0
c	3.929	3.827	3.57
α	2.41	2.52	5.68

$T_e/$	0.9	1.0	2.0
d	1.72	1.85	1.634
β	8.18	7.47	10.38

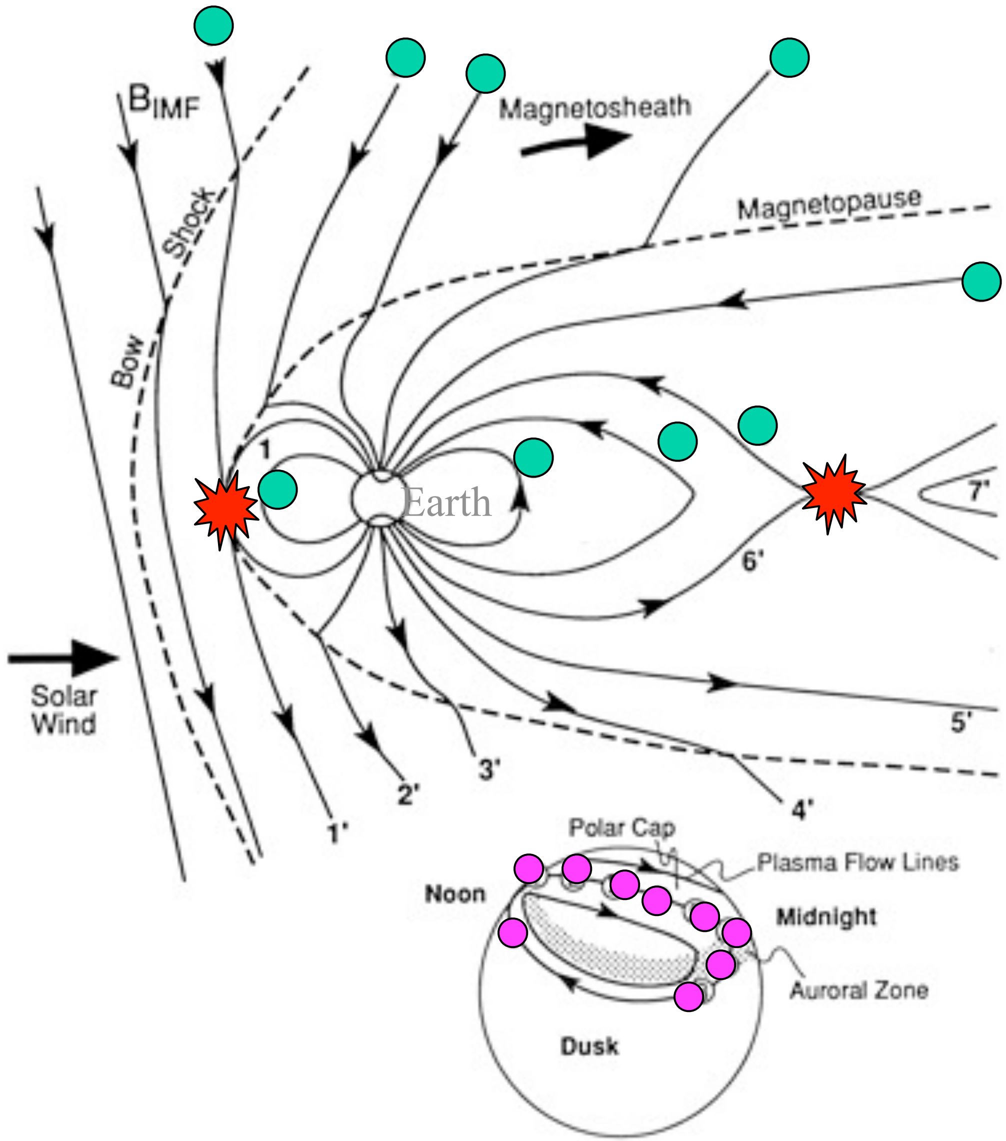
Reconnection and Geospace

Geospace is the only space environment in which magnetic reconnection can be observed both

- In-situ (locally) by spacecraft
- Remotely from ground (globally)

Reconnection between interplanetary magnetic field and geomagnetic field at magnetopause

Drives plasma convection cycle involving reconnection in the magnetotail.



Courtesy of Mervyn Freeman

Ion-Acoustic Resistivity Post-Quasilinear Saturation

- Resistivity from Wave-Particle interactions is important in Collisionless plasmas
- We have studied resistivity from Current Driven Ion-Acoustic Instability using Vlasov Simulations
  - Realistic plasma conditions i.e. $T_e \sim T_i$, Maxwellian
  - Found substantial resistivity at quasi-linear saturation (saturation of fastest growing mode)
- What happens after quasi-linear saturation
- We investigate the non-linear evolution of the ion-acoustic instability and its resulting anomalous resistivity by examining the properties of two statistical ensembles of Vlasov simulations.
- Resistivity after saturation also important
  - Behaviour of resistivity highly variable
- Ensemble of simulation runs – probability distribution of resistivity values, study its evolution in time
  - Evolution of each individual simulation in the nonlinear regime is very sensitive to initial noise field
  - Require Statistical Approach
- 104 ensemble run on High Performance Computing Edinburgh

Evolution of Vlasov Simulation

- 1-D and electrostatic with periodic boundary conditions.
- Plasma species α modelled with $f_\alpha(z, v, t)$ on discrete grid
- The B = 0 in the current sheet, but curl B = $\mu_0 c^2$J.
- Second-Order Splitting Upwind Method (Petkaki, 2005)

Governing equations

The Vlasov Equation in 1D, no **B** in z

$$\frac{\partial f_\alpha}{\partial t} + v_z \frac{\partial f_\alpha}{\partial z} + \frac{q_\alpha}{m_\alpha} E_z \frac{\partial f_\alpha}{\partial v_z} = 0 \quad (1)$$

Ampère's Law is used to integrate E_z forward in time;

$$\frac{\partial E_z}{\partial t} = -\mu_0 c^2 J_z \quad (2)$$

where

$$J_z = \sum_\alpha q_\alpha \int_{-\infty}^{\infty} v_z f_\alpha dv_z \quad (3)$$

Finite Difference Equations

Using the splitting upwind method the forward finite difference in space, time derivative $(\partial f_{i,j}^{n,n}/\partial t)$ where ^{n,n}, denotes n^{th} timestep, in space i and velocity space j

$$\frac{\partial f_{i,j}^{n,n}}{\partial t} = -v_j \left(\frac{f_{(i+m),j}^{n,n} - f_{i+m-2,j}^{n,n}}{2\Delta z} \right) \quad (1)$$

where m=(1-s), with s=sign(v_j). Integrate forward in space for time $\Delta t/2$

$$f_{i,j}^{n+1/2,n} = f_{i,j}^{n,n} + \frac{\Delta t}{2} \frac{\partial f_{i,j}^{n,n}}{\partial t} \quad (2)$$

Use $f_{i,j}^{n+1/2,n}$ to evaluate the forward finite difference in v-space

$$\frac{\partial f_{i,j}^{n+1/2,n}}{\partial t} = -\frac{q_\alpha}{m_\alpha} E_i^{n+1/2} \left(\frac{f_{i,(j+m)}^{n+1/2,n} - f_{i,j+m-2}^{n+1/2,n}}{2\Delta v_z} \right) \quad (3)$$

where m=(1-r), with r=sign($\frac{q_\alpha}{m_\alpha} E_i^{n+1/2}$). Integrate for time Δt in v-space

$$f_{i,j}^{n+1/2,n+1} = f_{i,j}^{n+1/2,n} + \Delta t \frac{\partial f_{i,j}^{n+1/2,n}}{\partial t} \quad (4)$$

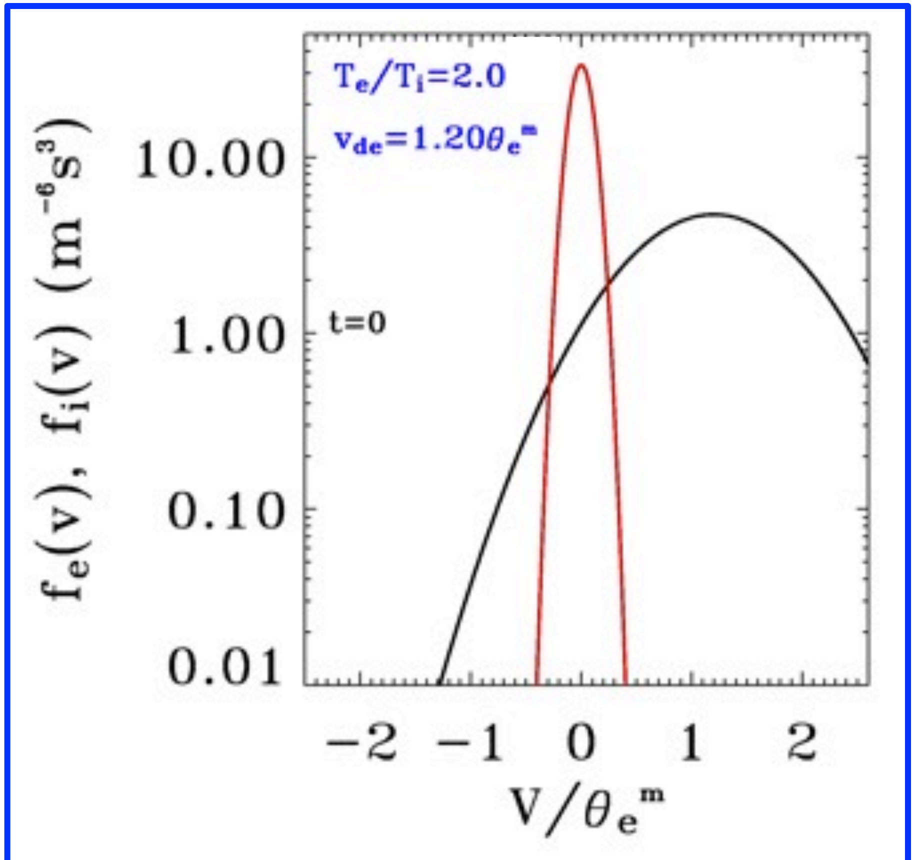
Use $f_{i,j}^{n+1/2,n+1}$ to evaluate the forward difference in space at the intermediate position $(n+1/2, n+1)$

$$\frac{\partial f_{i,j}^{n+1/2,n+1}}{\partial t} = -v_j \left(\frac{f_{(i+m),j}^{n+1/2,n+1} - f_{i+m-2,j}^{n+1/2,n+1}}{2\Delta z} \right) \quad (5)$$

where m=(1-s). Integrate for time $\Delta t/2$ in space :

$$f_{i,j}^{n+1,n+1} = f_{i,j}^{n+1/2,n+1} + \frac{\Delta t}{2} \frac{\partial f_{i,j}^{n+1/2,n+1}}{\partial t} \quad (6)$$

Vlasov Simulation Initial Conditions



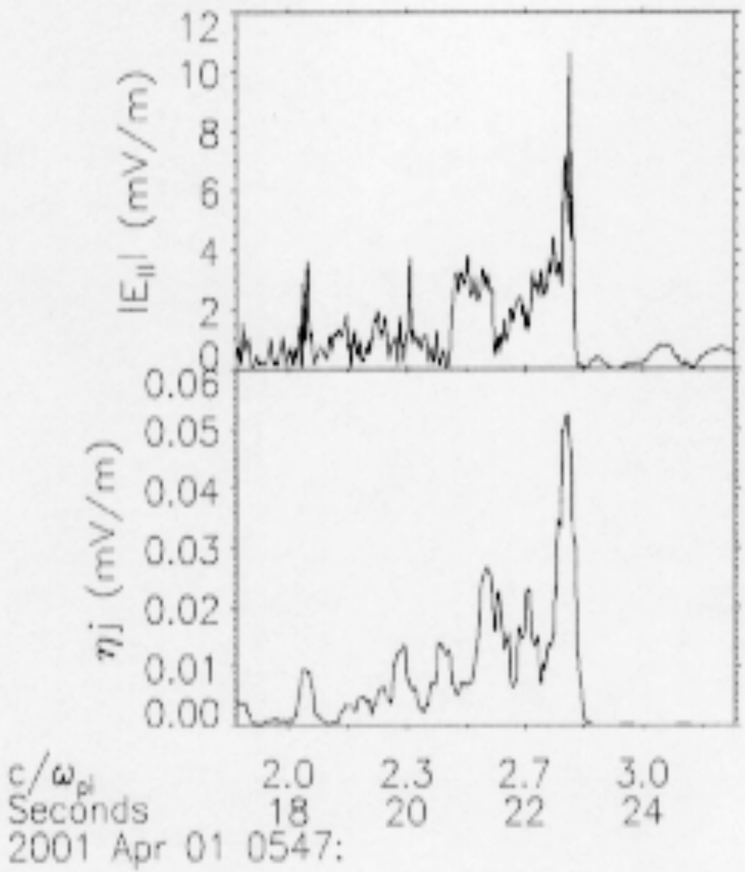
- **CDIAW- drifting electron and ion distributions**
- **Apply white noise Electric field**

$$E_1(z,0) = \sum_{n=1}^{N} E_{tf} \sin(k_n z + \varphi)$$

$$E_{tf} = \left(\frac{2k_B T_e}{\varepsilon_0 \lambda_{De}^3}\right)^{1/2}$$

- **f_α close to zero at the edges**
- ***Maxwellian***
- **Drift Velocity - V_{de} = 1.2 x θ**
- **(θ = $(2T/m)^{1/2}$)**
- **M_i=25 m_e**
- **T_i=1 eV, T_e = 2 eV**
- **n_i=n_e = 7 x 10^6 /m^3**
- **N_z = 642, N_{ve} = 891, N_{vi} = 289**

Reconnection in Collisionless plasma



- Reconnection at MHD scale requires violation of frozen-in field condition.
- Kinetic-scale wave turbulence can scatter particles to generate anomalous resistivity.
- Change in electron momentum p_e contributes to electron inertial term [Davidson and Gladd, 1975] with effective resistivity given by

$$\eta = -\frac{1}{\varepsilon_o \omega_{pe}^2 p_e} \frac{\partial p_e}{\partial t} = \frac{1}{\varepsilon_o \omega_{pe}^2 J} \frac{\partial J}{\partial t}$$

- Broad band waves seen in crossing of reconnecting current sheet [Bale et al., Geophys. Res. Lett., 2002].
- The Measured Electric Field is more than 100 times the analytically estimated due to Lower Hybrid Drift Instability

Discussion

- Ensemble of 104 Vlasov Simulations with reduced mass ratio of the current driven ion-acoustic instability with identical initial conditions except for the initial phase of noise field
- Ensemble of 10 Vlasov Simulations with real mass ratio of the current driven ion-acoustic instability as before
- Variations of the resistivity value observed in the quasilinear and nonlinear phase
- Timescale of variations consistent with electron and proton bounce motion in reduced mass ratio Vlasov simulations.
- Timescale of variations consistent with electron bounce motion in reduced mass ratio Vlasov simulations
- The probability distribution of resistivity values Gaussian in Linear, Quasilinear, Non-linear phase
- A well-bounded uncertainty can be placed on any single estimate of resistivity, e.g., at quasi-linear saturation
- Estimation at quasi-linear saturation provides underestimation of Resistivity