

Atomic Data: their role in interpreting observations of the solar corona / TR

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Further readings

Spectroscopic diagnostics (X-rays, EUV):

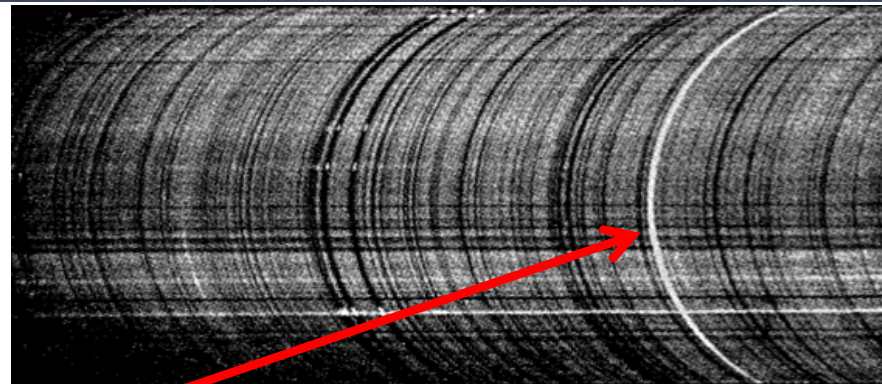
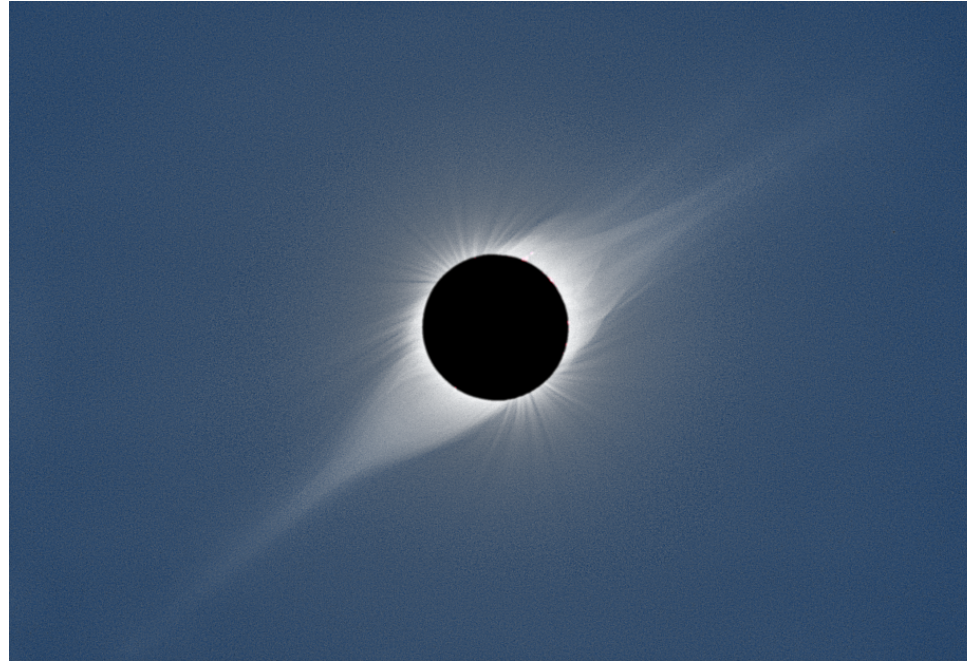
- Living Review in Solar Physics (Del Zanna & Mason, 2018)
- CHIANTI user guide (Del Zanna+)
- More advanced level: Part-III lectures (Del Zanna, in prep.)

- Phillips, Feldman, Landi, 2008, Ultraviolet and X-ray Spectroscopy of the Solar Atmosphere (Cambridge University press)
- Gabriel & Mason, 1982, in Applied Atomic Collision Physics, Atmospheric Physics and Chemistry, Massey, Bates eds.
- Dere & Mason, 1981, in Solar Active Regions: A Monograph from Skylab Solar Workshop III, eds. F. Q. Orrall
- Mariska, 1992 (The solar transition region, Cambridge University Press)

- Mason & Monsignori Fossi, 1994, A&A Rev., 6, 123

- Del Zanna (PhD thesis, 1999)
- Del Zanna, Landini & Mason, 2002, A&A, 385, 968
- Del Zanna & Mason, The Sun as a star, Springer 2012

Solar corona



Spectrum of the
solar corona
from Lyot

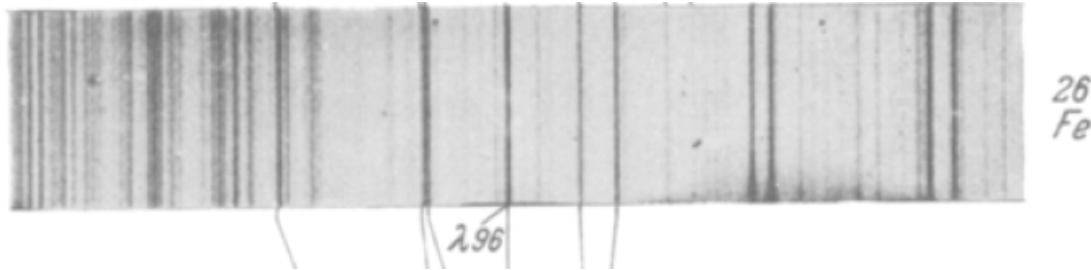
‘Coronium’ green emission line first observed in 1869 by Young and Harkness at 5303 Å. Red line observed later at 6374.6 Å

Fe X

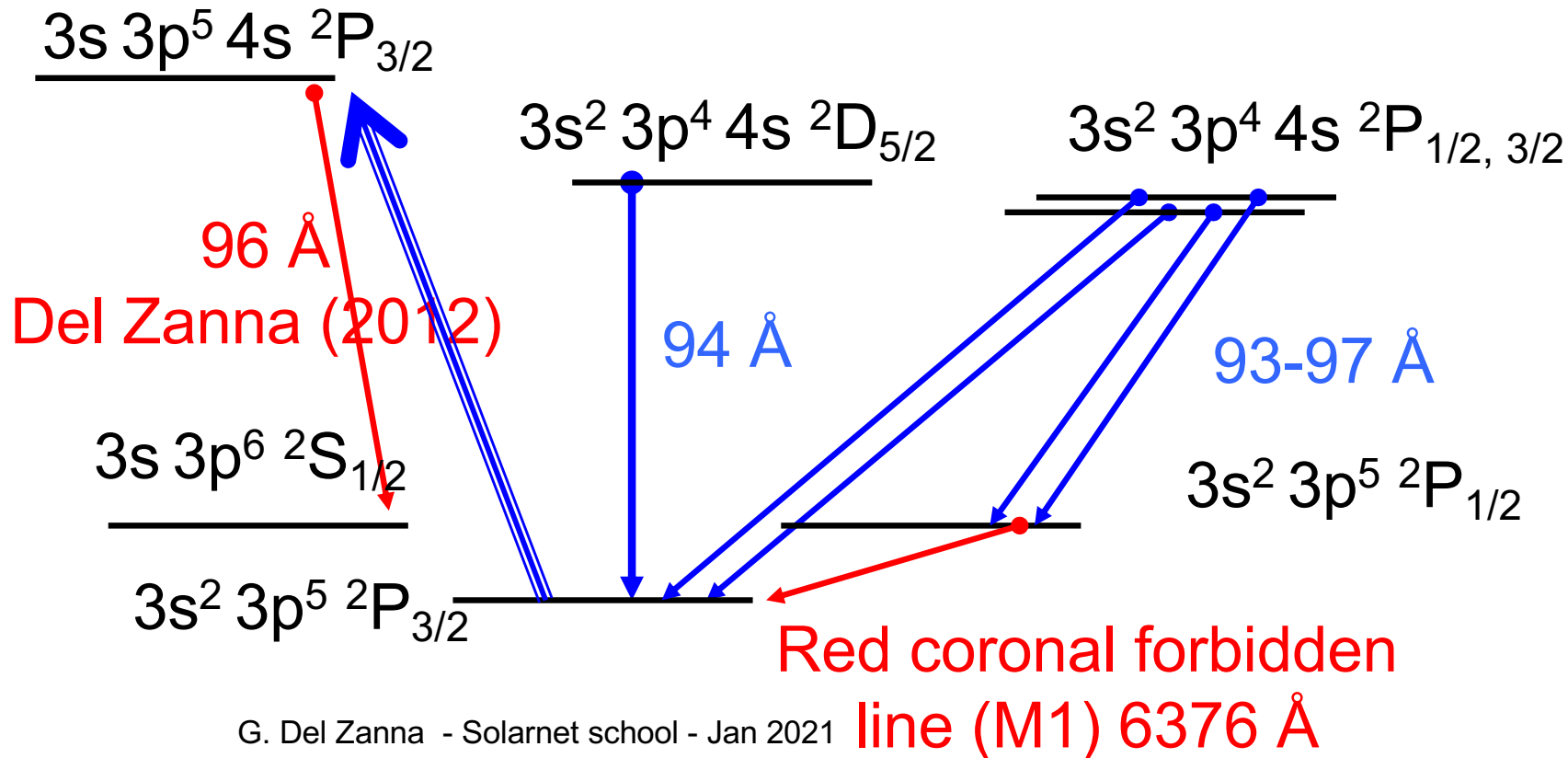
Zur Kenntnis der ClI-ähnlichen Spektren Cl I, A II, K III,
Ca IV, Ti VI, V VII, Cr VIII, Mn IX, Fe X und Co XI.

Von Bengt Edlén in Upsala.

Mit 5 Abbildungen. (Eingegangen am 21. November 1936.)



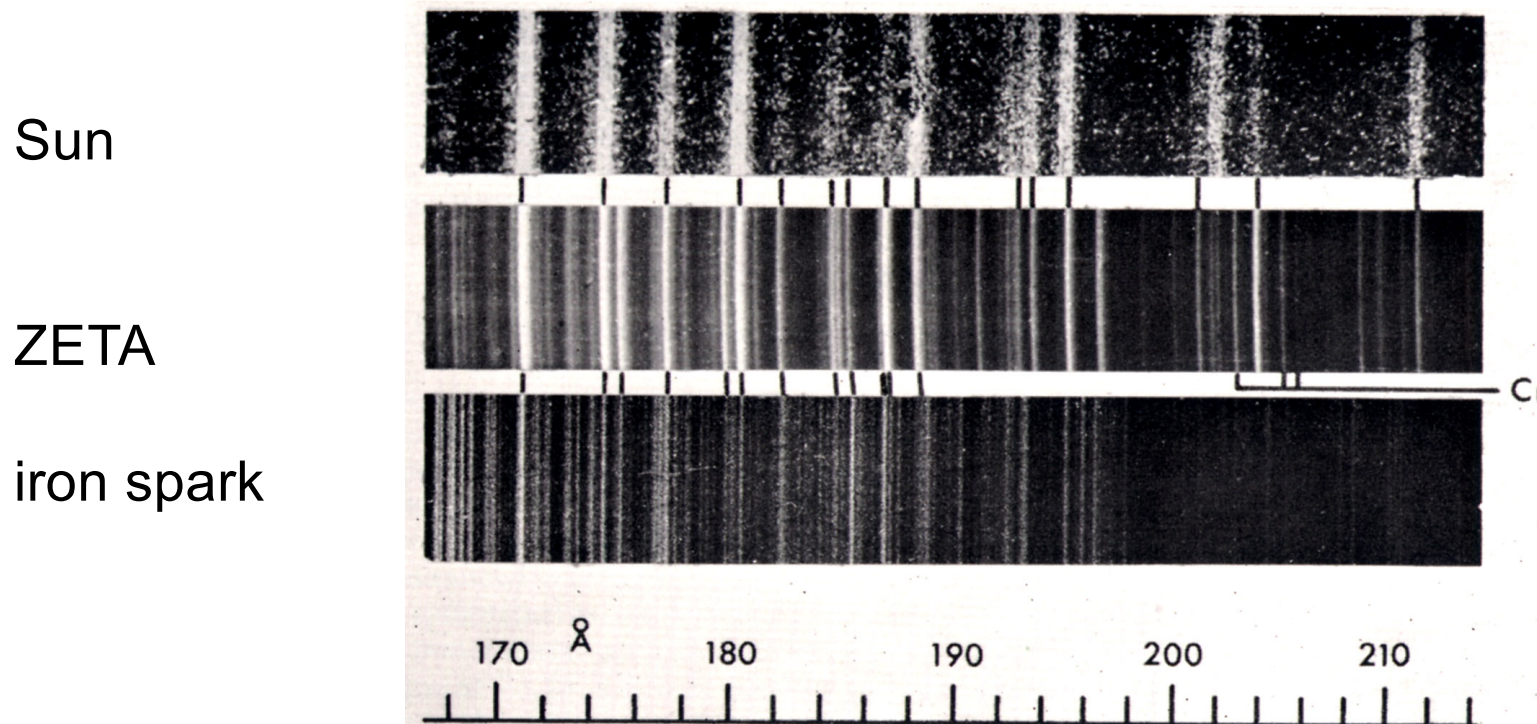
Grotrian (1939) suggested that the red line was due to Fe X, based on Edlen X-ray work.



Solar corona in the EUV

1958: First EUV spectra of the Sun (Violet & Rense) using rockets. First identifications only in 1965

comparison of fusion machines (ZETA) with solar spectra

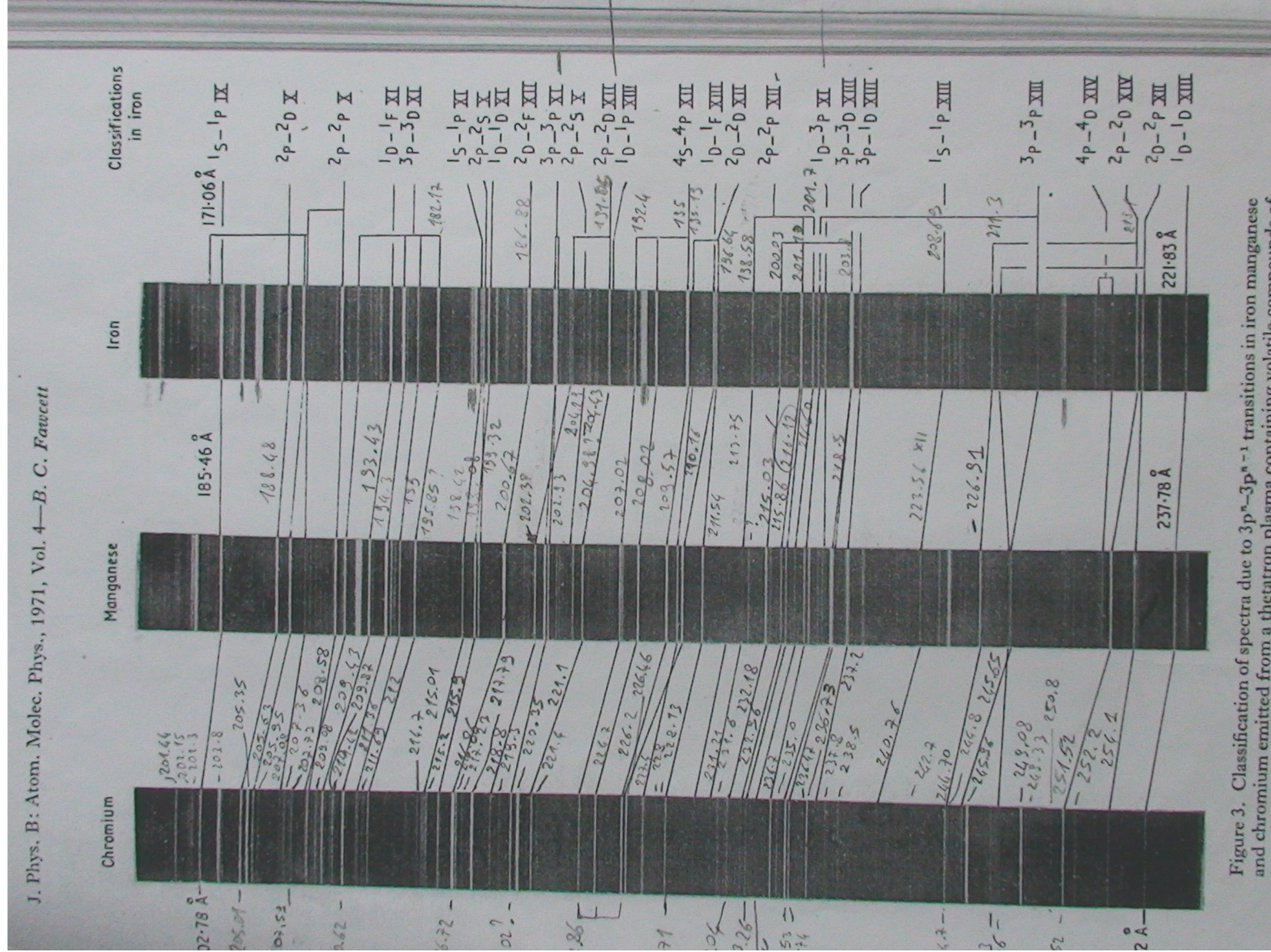


Brian Fawcett, Alan Gabriel and Carole Jordan (1965)

$3p^n - 3p^{n-1}3d$ transitions in Fe IX to Fe XIV

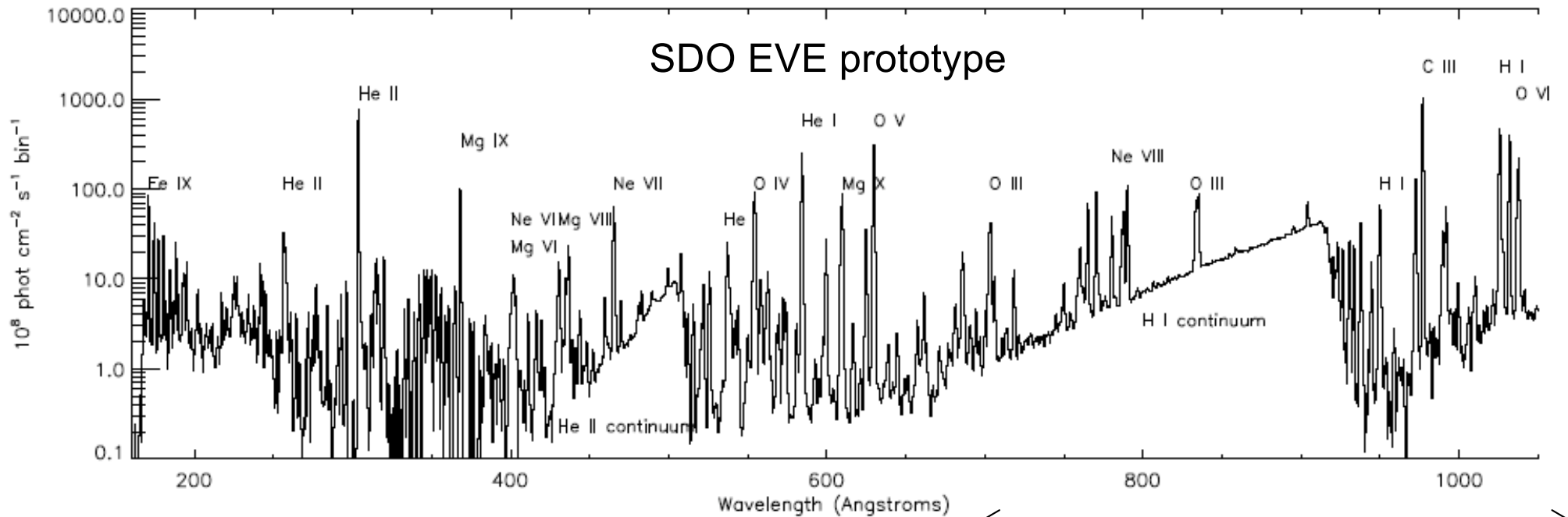
Line identifications are still on-going..

See Del Zanna 'benchmarking' papers



The EUV

SDO EVE prototype

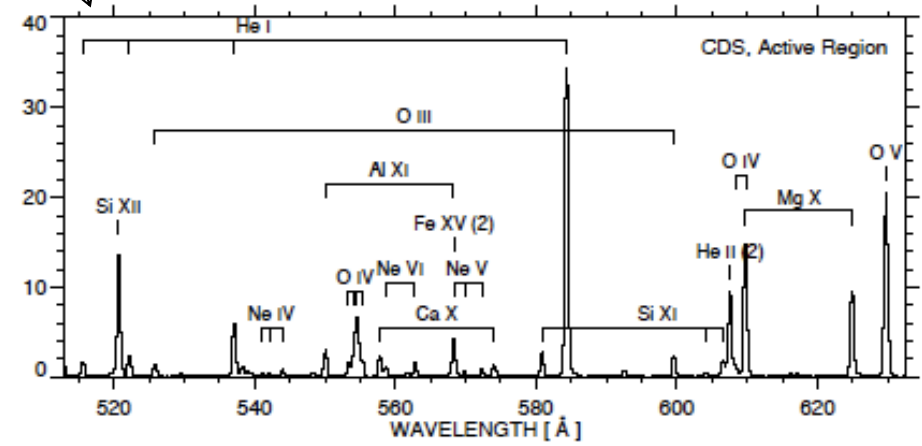
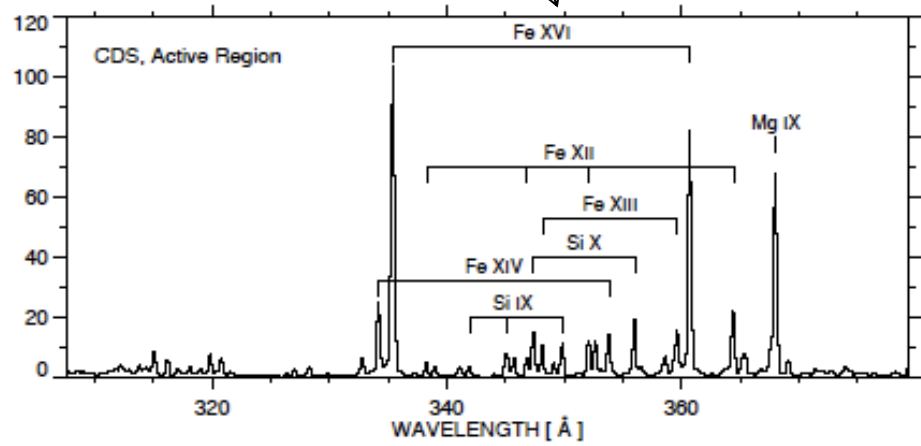


↔ ↔
Hinode/EIS

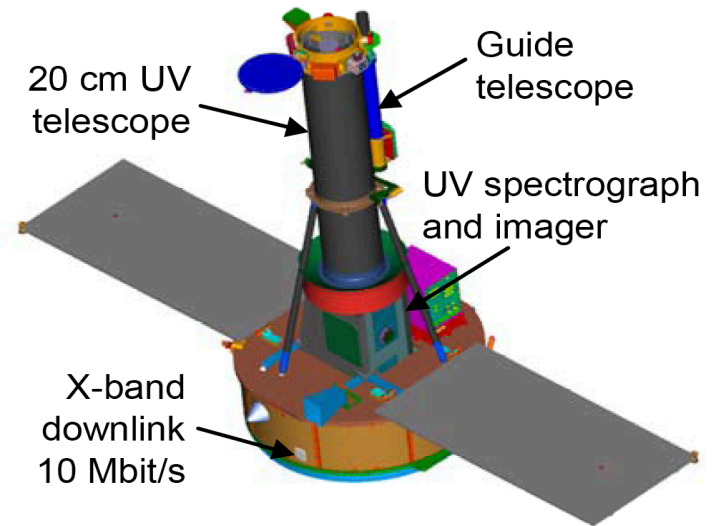
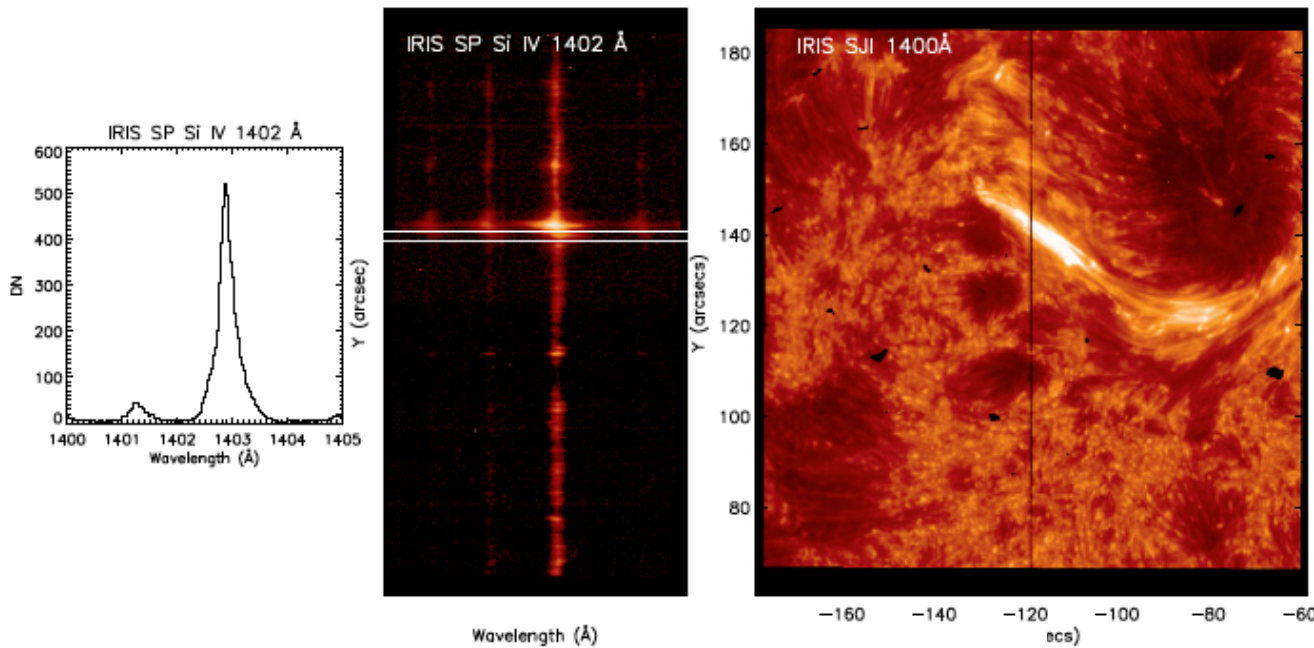
↔ ↔
SOHO CDS

↔ ↔
SOHO SUMER, UVCS

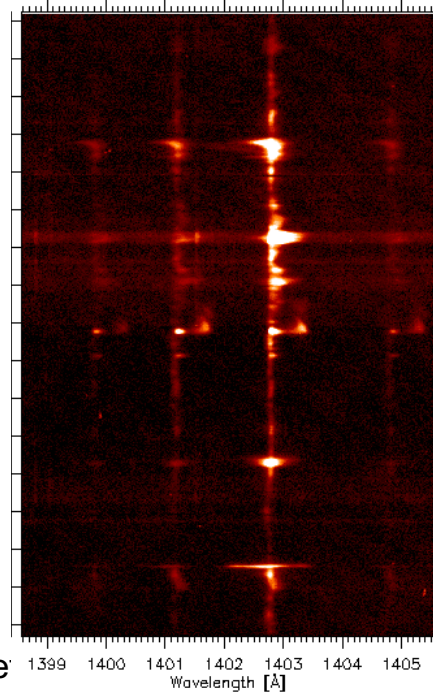
↔ ↔
SoHO SPICE, IRIS



IRIS FUV2: TR Lines



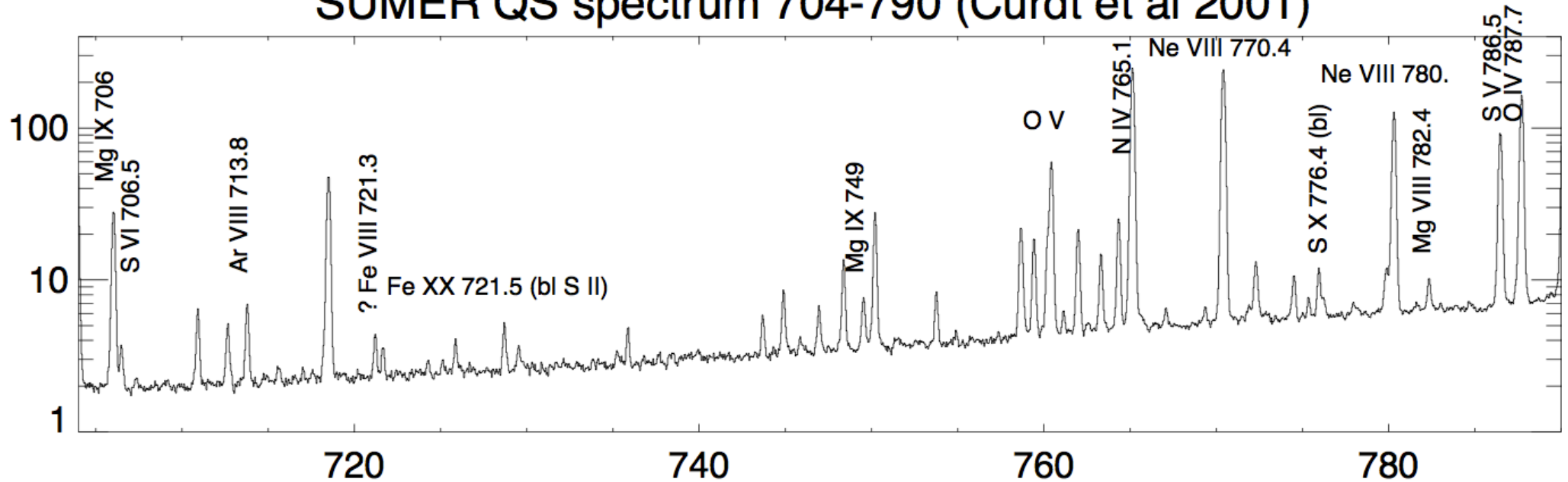
Also in the FUV1
C I, O I, C II
formed in the
photosphere /
chromosphere



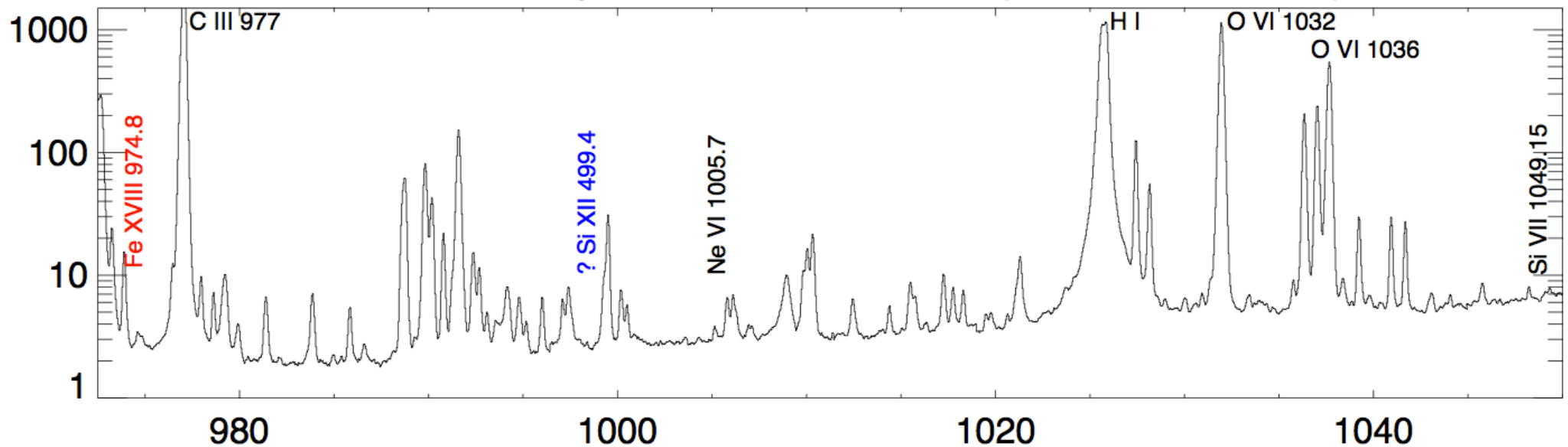
ion	λ [Å]	transition	levels
Si IV	1393.76	$3s \ ^2S_{1/2} - 3p \ ^2P_{3/2}$	1 - 3
Si IV	1402.77	$3s \ ^2S_{1/2} - 3p \ ^2P_{1/2}$	1 - 2
O IV	1397.20	$2s^2 2p \ ^2P_{1/2} - 2s 2p^2 \ ^4P_{3/2}$	1 - 4
O IV	1399.77	$2s^2 2p \ ^2P_{1/2} - 2s 2p^2 \ ^4P_{1/2}$	1 - 3
O IV	1401.16	$2s^2 2p \ ^2P_{3/2} - 2s 2p^2 \ ^4P_{5/2}$	2 - 5
O IV	1404.78	$2s^2 2p \ ^2P_{3/2} - 2s 2p^2 \ ^4P_{3/2}$	2 - 4
S IV	1398.04	$3s^2 3p \ ^2P_{1/2} - 3s 3p^2 \ ^4P_{3/2}$	1 - 4
S IV	1404.81	$3s^2 3p \ ^2P_{1/2} - 3s 3p^2 \ ^4P_{1/2}$	1 - 3
S IV	1406.02	$3s^2 3p \ ^2P_{3/2} - 3s 3p^2 \ ^4P_{5/2}$	2 - 5

Solar Orbiter SPICE wavelengths

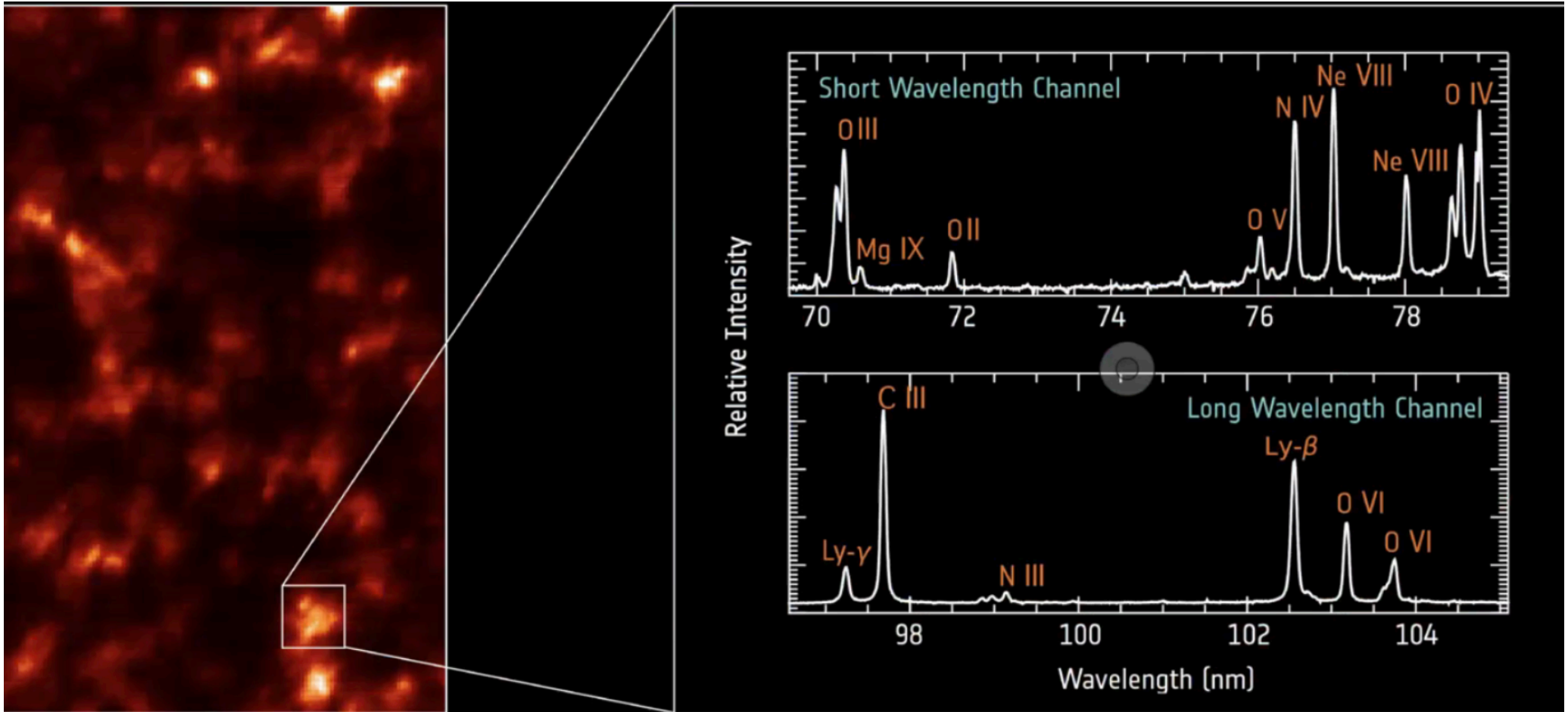
SUMER QS spectrum 704-790 (Curdt et al 2001)



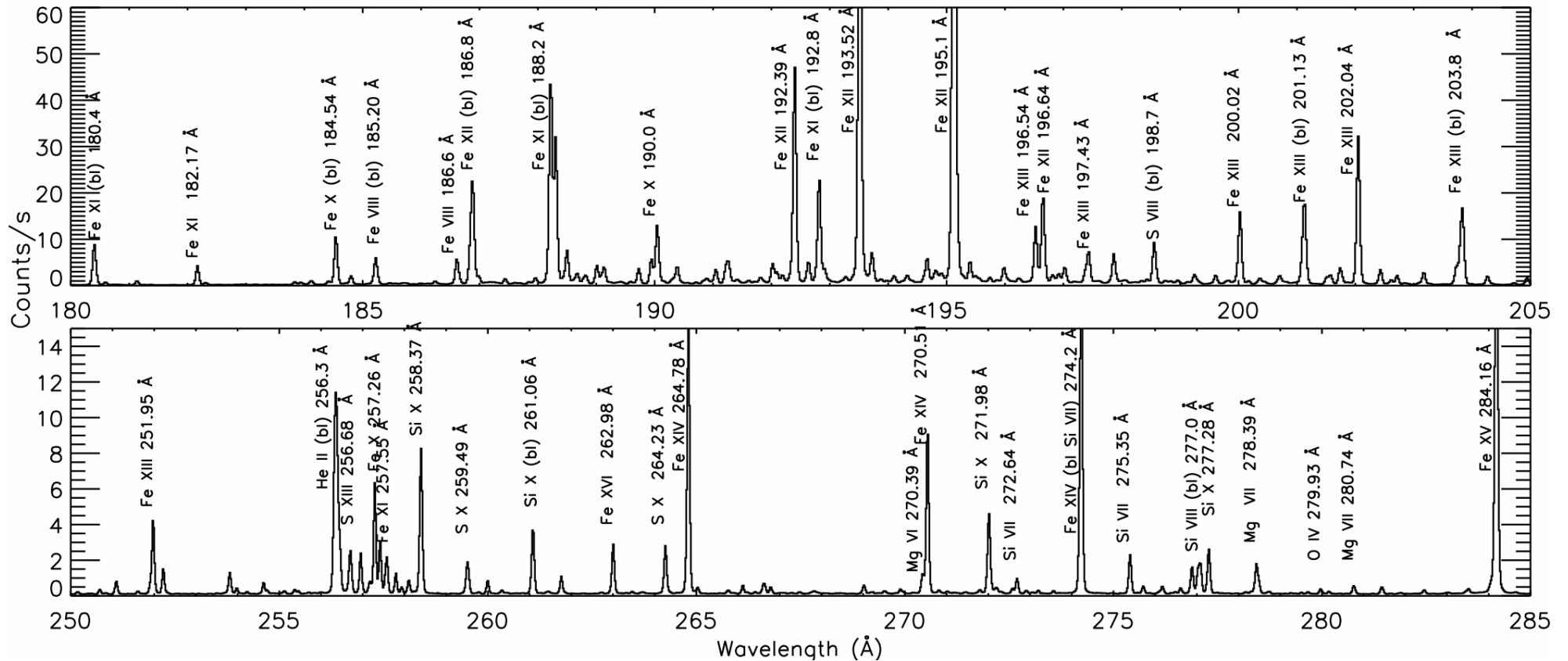
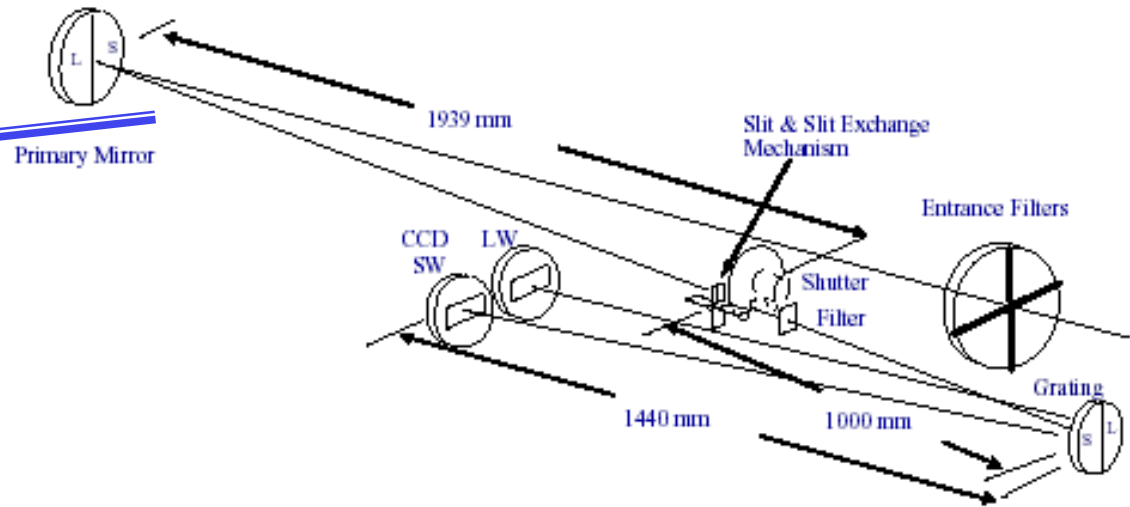
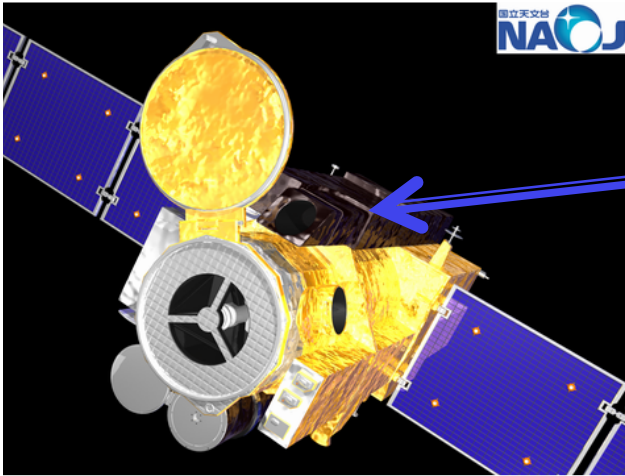
SUMER QS spectrum 972.5-1050 (Curdt et al 2001)

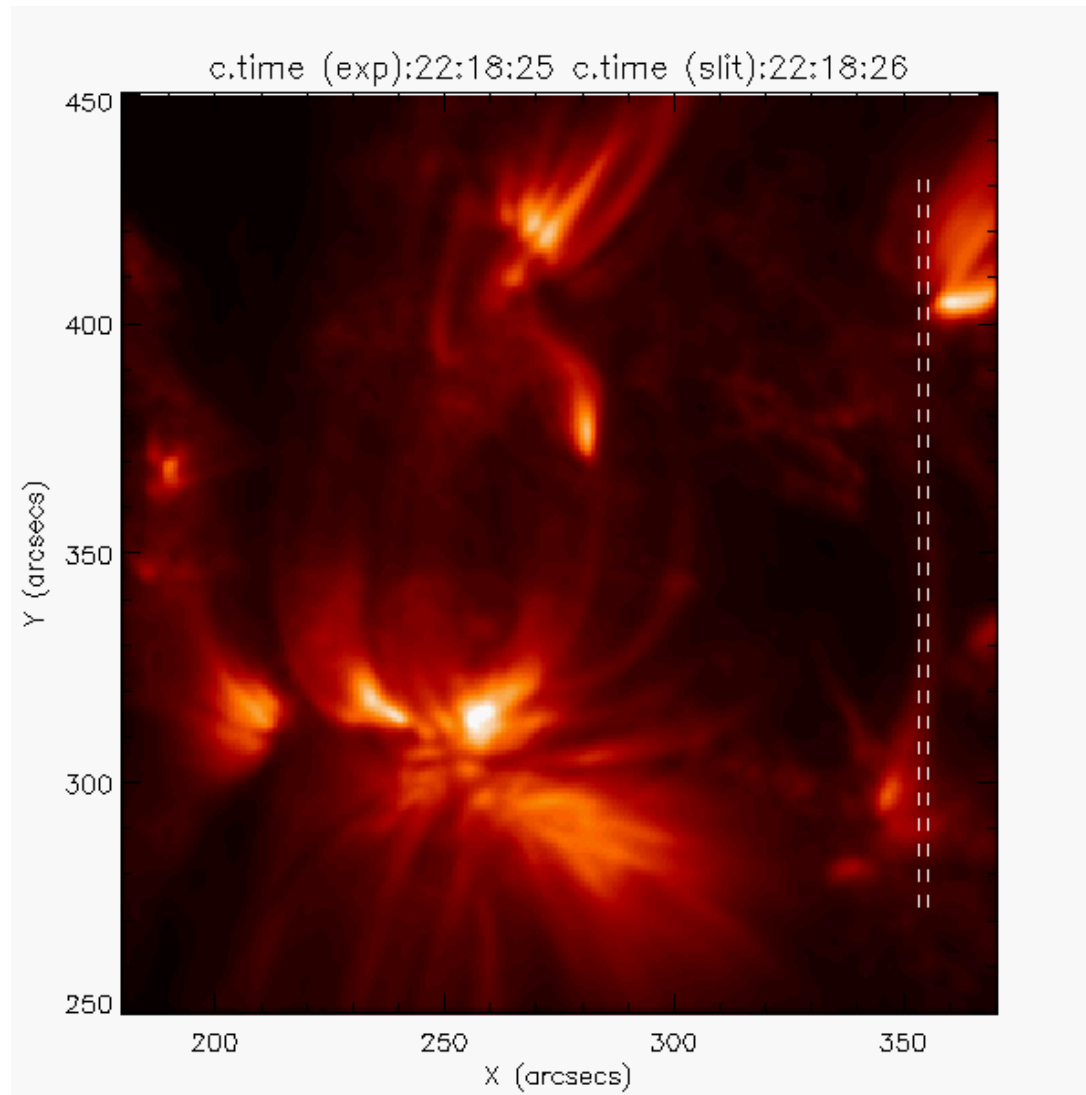


SPICE data (first light)



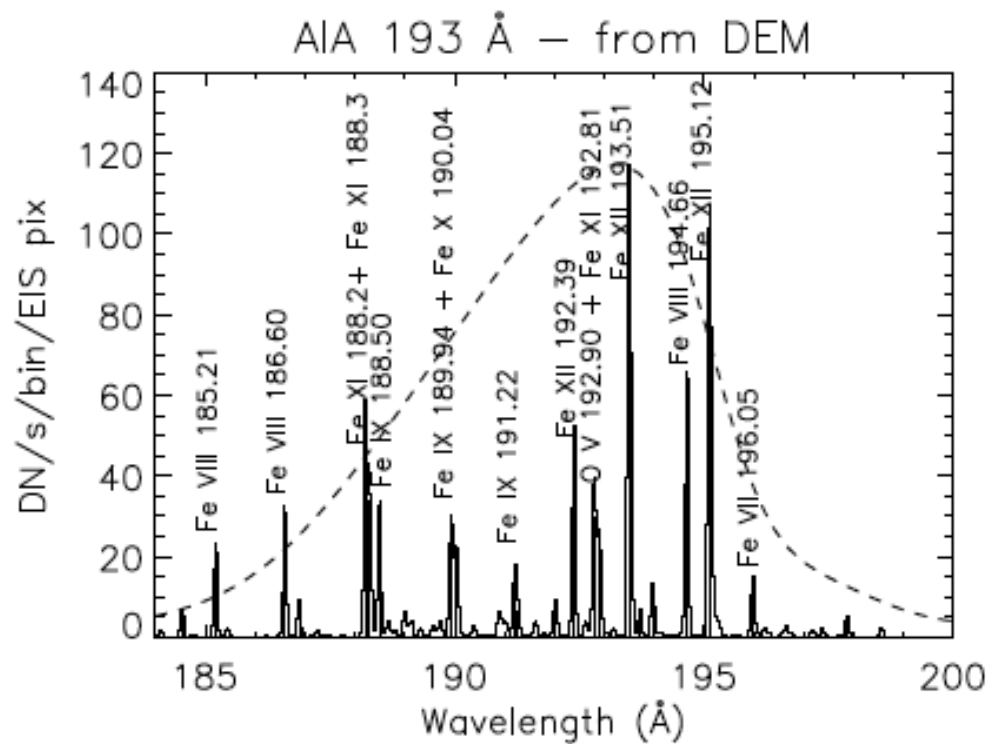
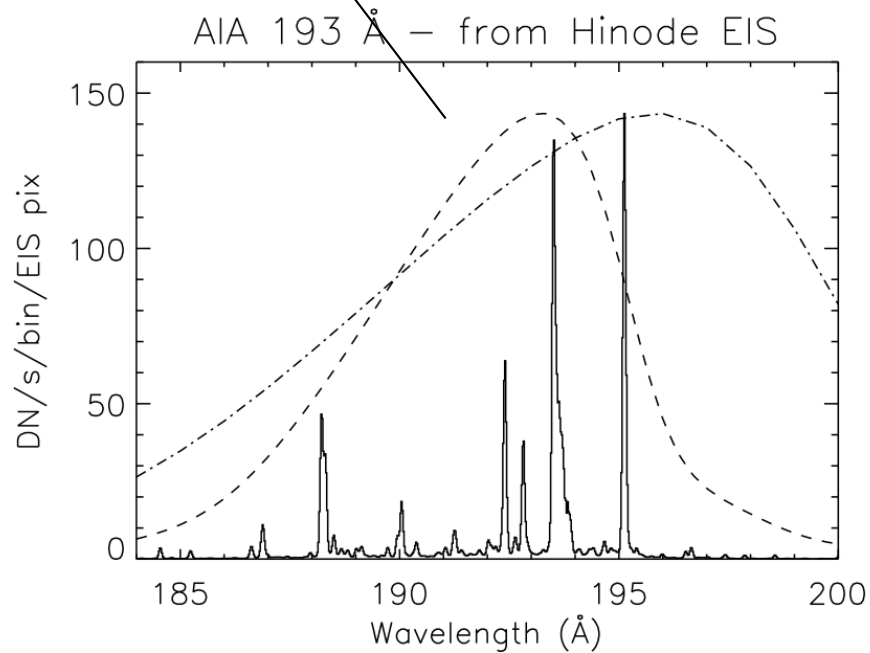
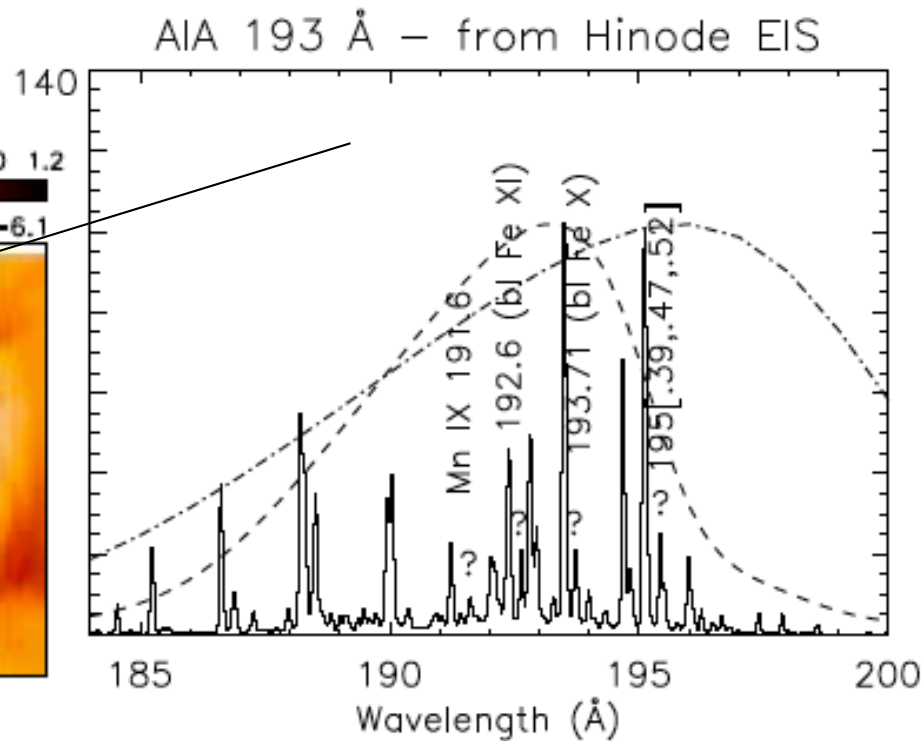
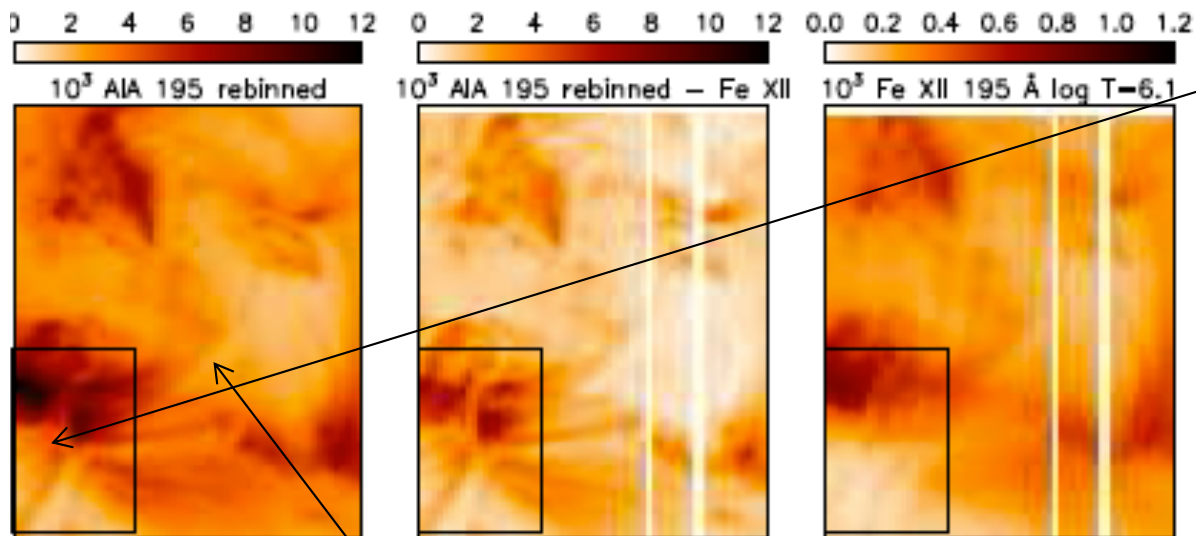
Hinode EUV imaging spectrometer (EIS)



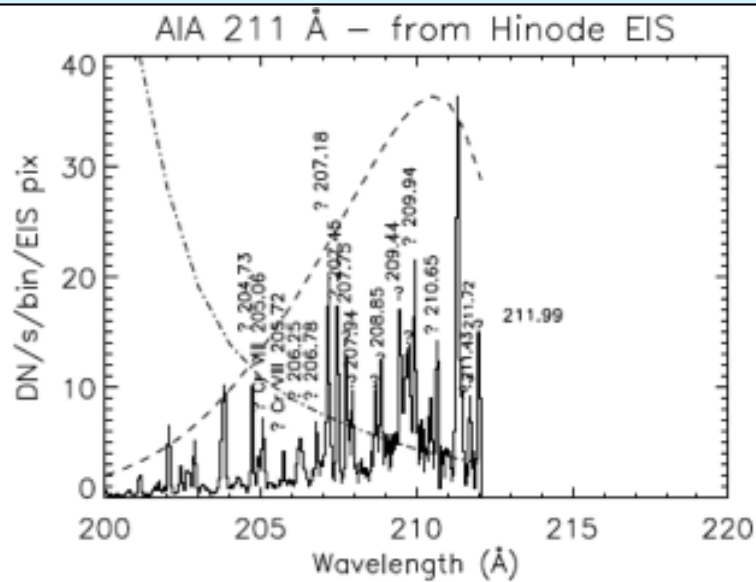
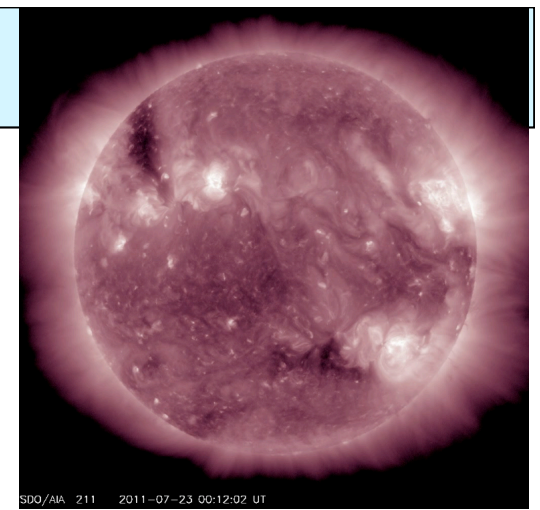


SDO/AIA vs. Hinode/EIS
Del Zanna et al. (2011)

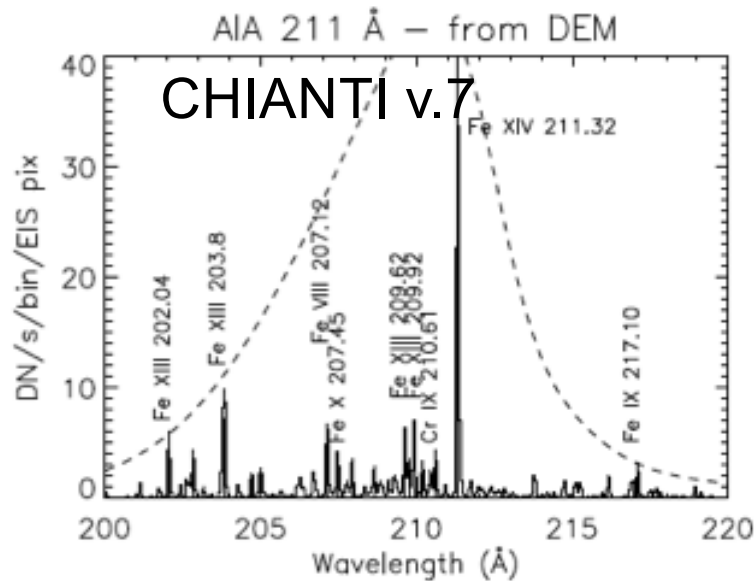
SDO AIA 193



AIA EUV 211 band



A lot of cool/unidentified lines in AIA bands (Del Zanna+2011).



EBIT laboratory spectra showed many unidentified lines

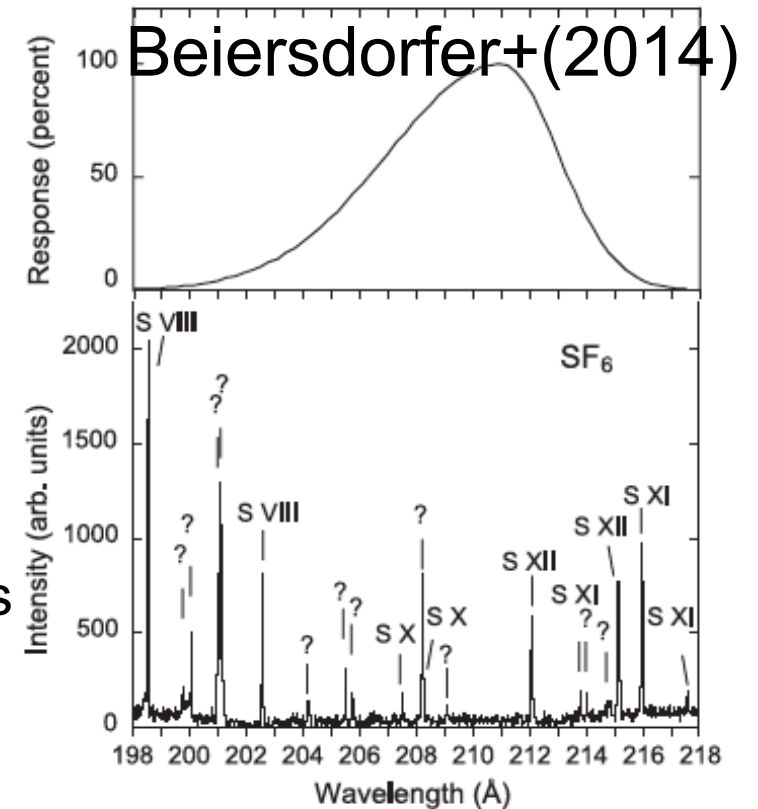
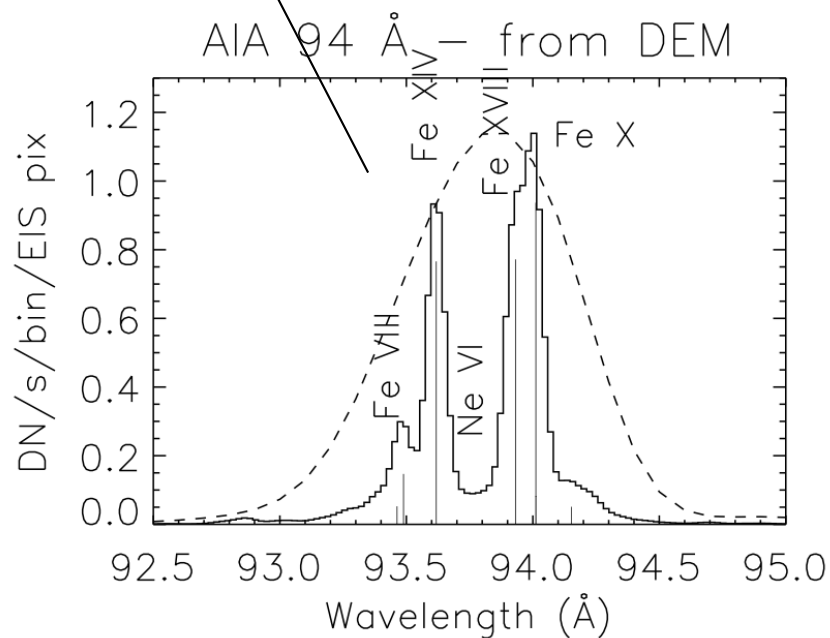
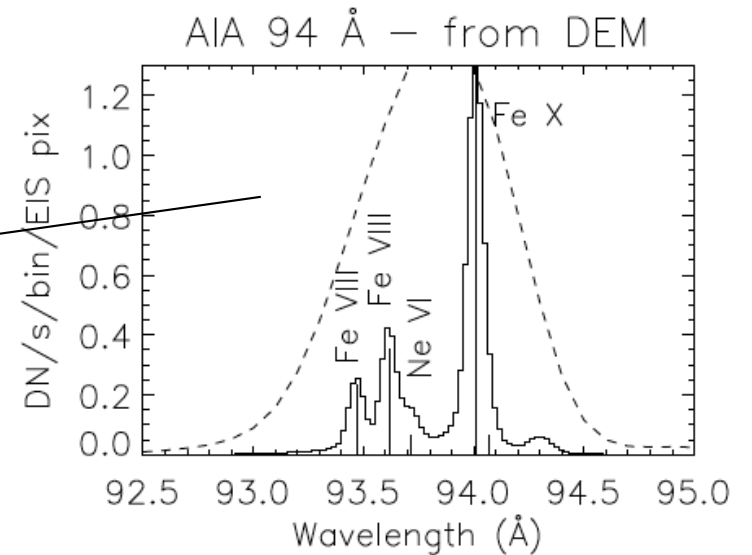
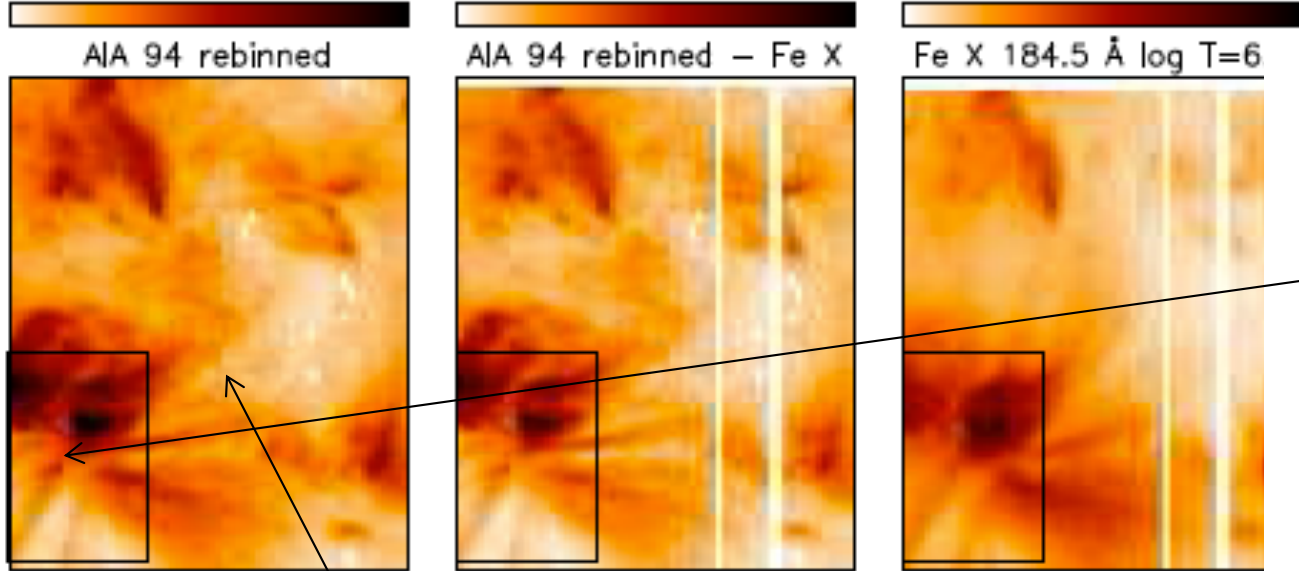


Figure 3. EUV spectrum of fluorine and sulfur produced by injecting SF₆ into the EBIT-I electron beam ion trap covering the spectral range λ 198– λ 218. The electron beam energy was 1000 eV, but the spectrum differs very little from observations at 600 eV. Identified spectral features are labeled by the corresponding spectrum. Unidentified features are labeled by a question mark. The response of the SDO/AIA λ 211 channel is indicated on top.

SDO AIA 94

0 10 20 30 40 50 60 0 10 20 30 40 50 60 0 50 100 150 200 250 300 350

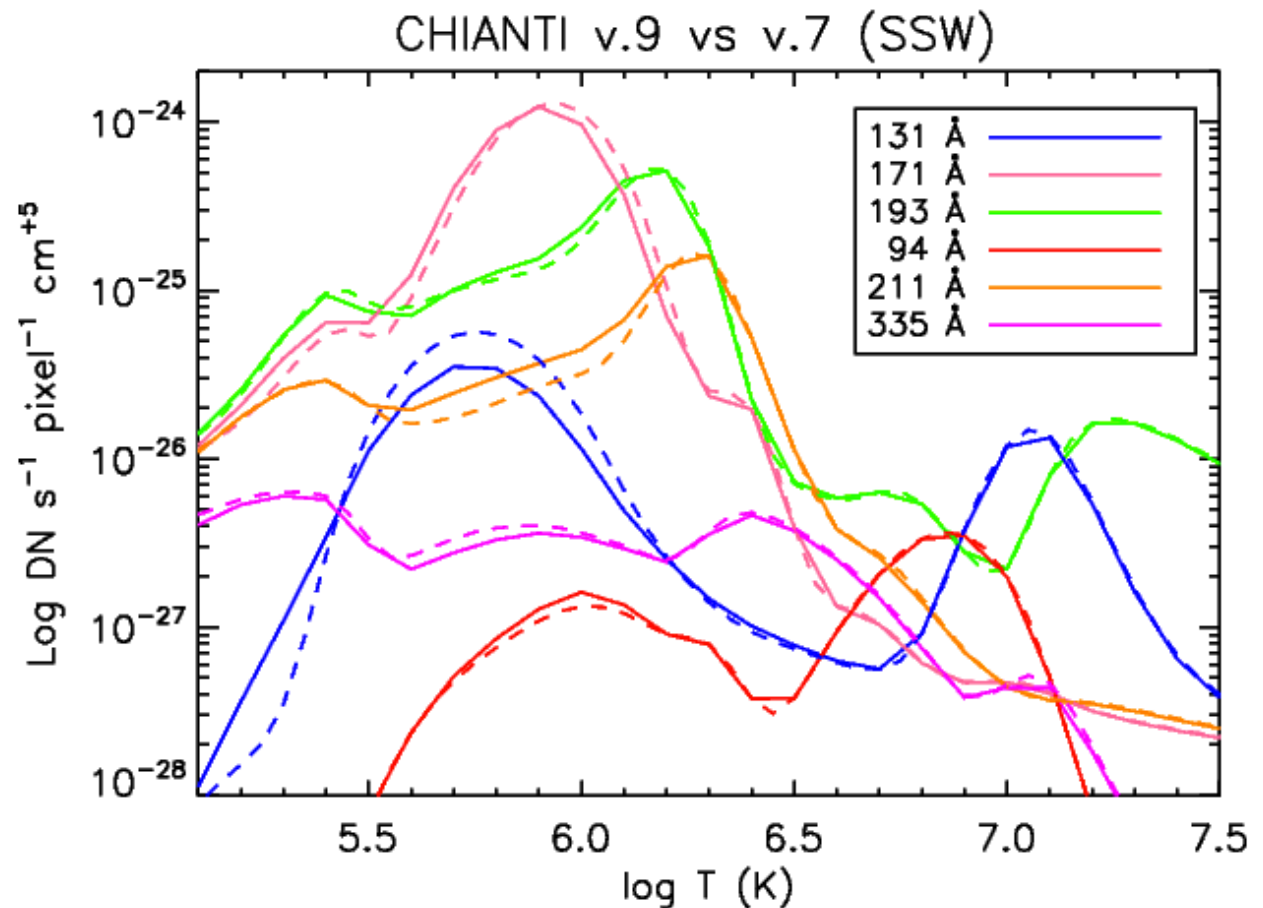


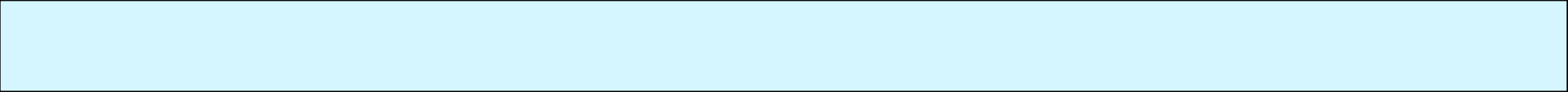
- Fe X 94.0: first proper calculation by Del Zanna et al. (2012)
- Fe XIV 93.6: new ID in Del Zanna (2012)
- Fe VIII, Fe IX: O'Dwyer et al. (2011)
- Fe XVIII 93.9 line: 6MK

CHIANTI AIA responses (v.9)

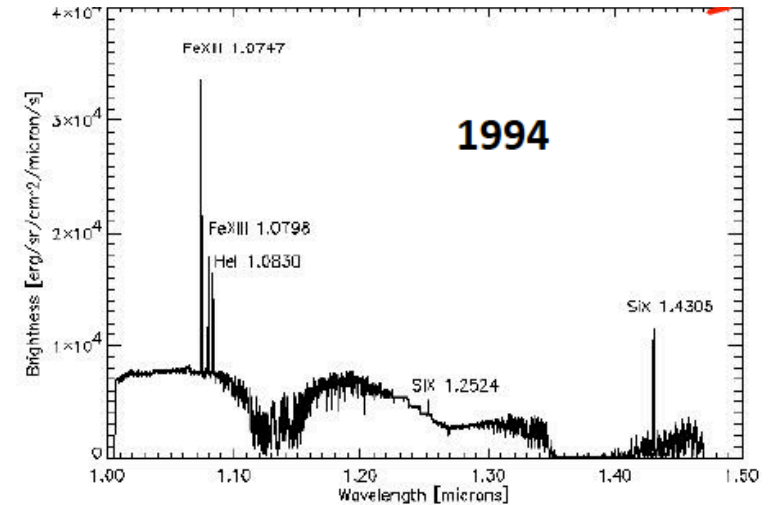
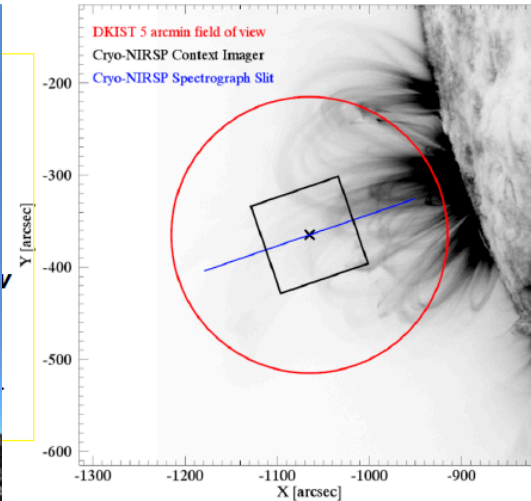
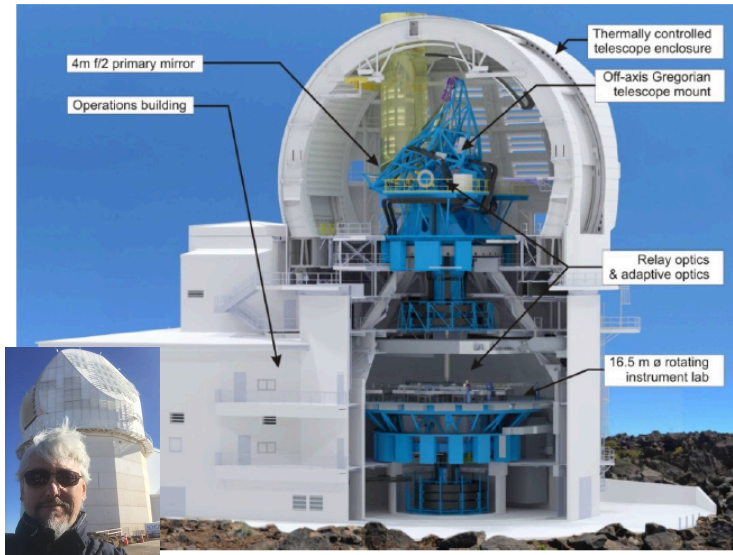
```
AIA_RESP= ch_aia_resp( '20100522', pressure=1e15, $  
abund_name=!xuvtop+'/abundance/sun_coronal_1992_feldman_ext.abund', $  
ioneq_name= !xuvtop+'/ioneq/chianti.ioneq',/verbose )
```

- Bands are multi-thermal.
- Off-band contribution
- Degradation of the bands difficult to assess.
- Responses depend on chemical abundances



- 
- High-resolution line spectroscopy is featuring in many future missions and proposals.
 - Main instrument is EUVST.

DKIST



Cryo-NIRSP Spectropolar.

Fe XIII λ 10747 ; Log(T) \sim 6.22
 Fe XIII λ 10797 ; Log(T) \sim 6.22
 He I λ 10830 ; Log(T) \sim 4*
 Si X λ 14300 ; Log(T) \sim 6.13
 Si IX λ 39350 ; Log(T) \sim 6.04

Cryo-NIRSP Context Imager

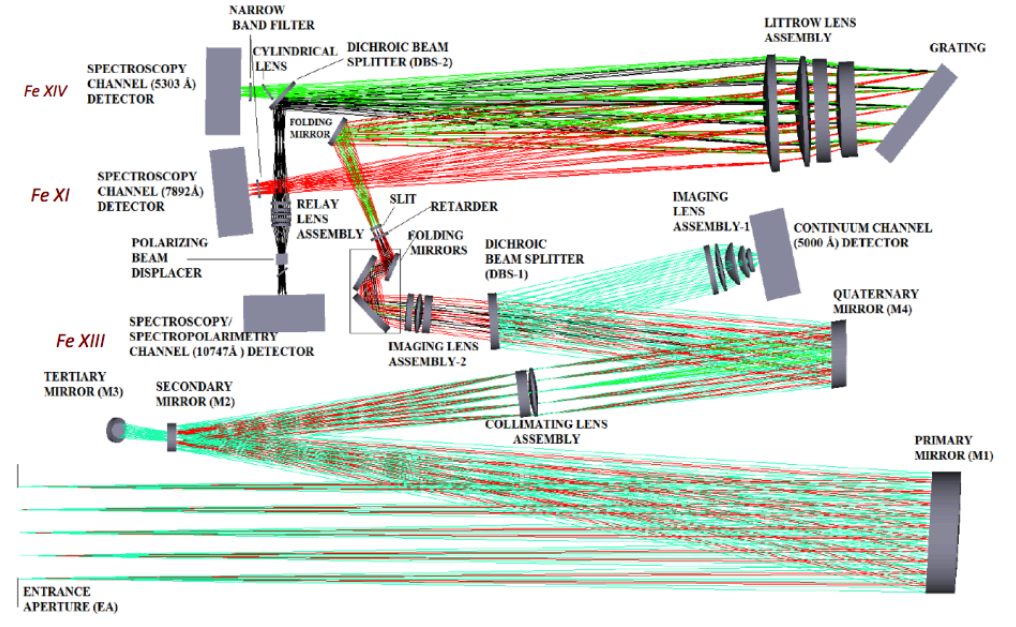
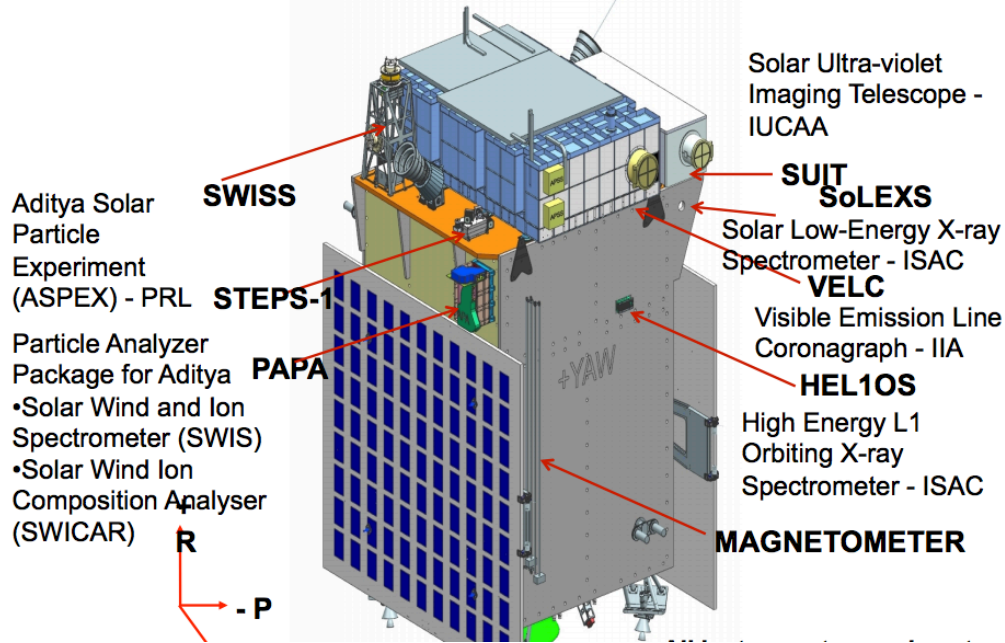
Fe XIII λ 10747 ; Log(T) \sim 6.22
 He I λ 10830 ; Log(T) \sim 4*
 Si IX λ 39340 ; Log(T) \sim 6.04

DKIST Cryo-NIRSP will measure up to 1.5 R_{sun}:

1. coronal Ne (Fe XIII ratio and others),
2. non-thermal widths
3. Te (approx)
4. coronal B
5. in the future chemical abundances in coronal 1-3 MK plasma (Del Zanna & DeLuca 2018).

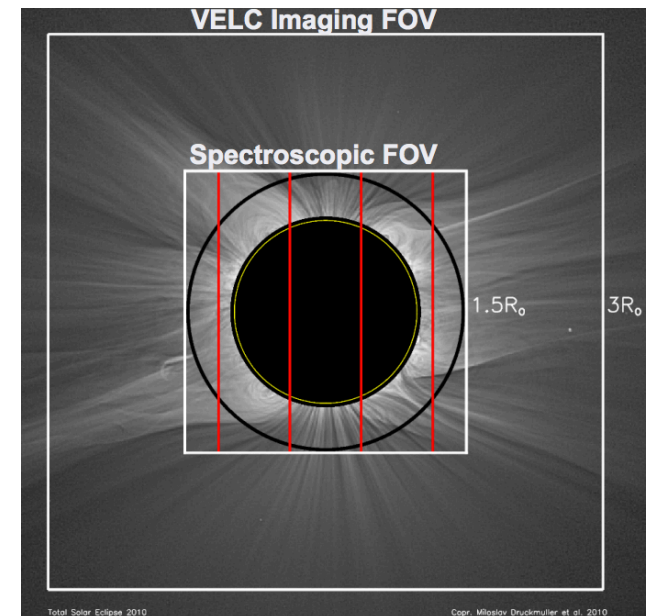
Aditya VELC

Payload STOWED VIEW OF ADITYA-L1



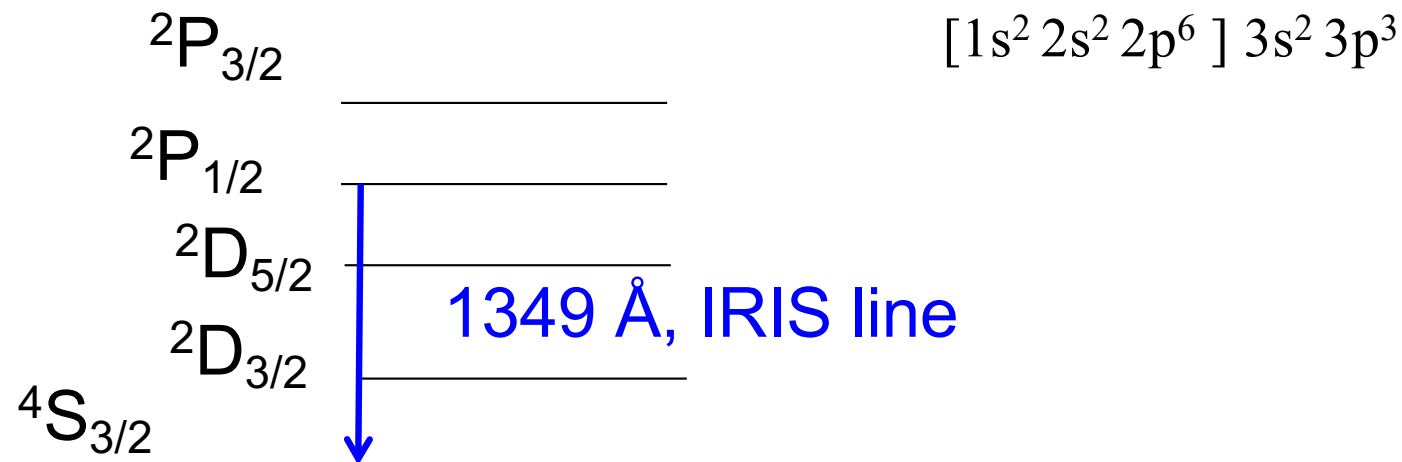
Spectroscopic mode !
 FOV: 1.05 – 1.5 R_{\odot} (3 R_{\odot} imaging)
 Spatial resolution: 5''
 Cadence: 1-5s
 Fe XIV (green), Fe XI (7892 Å),
 Fe XIII (NIR), WL continuum
 Spectropolarimetry in Fe XIII
 → **magnetic field**

Images:
 D. Banerjee



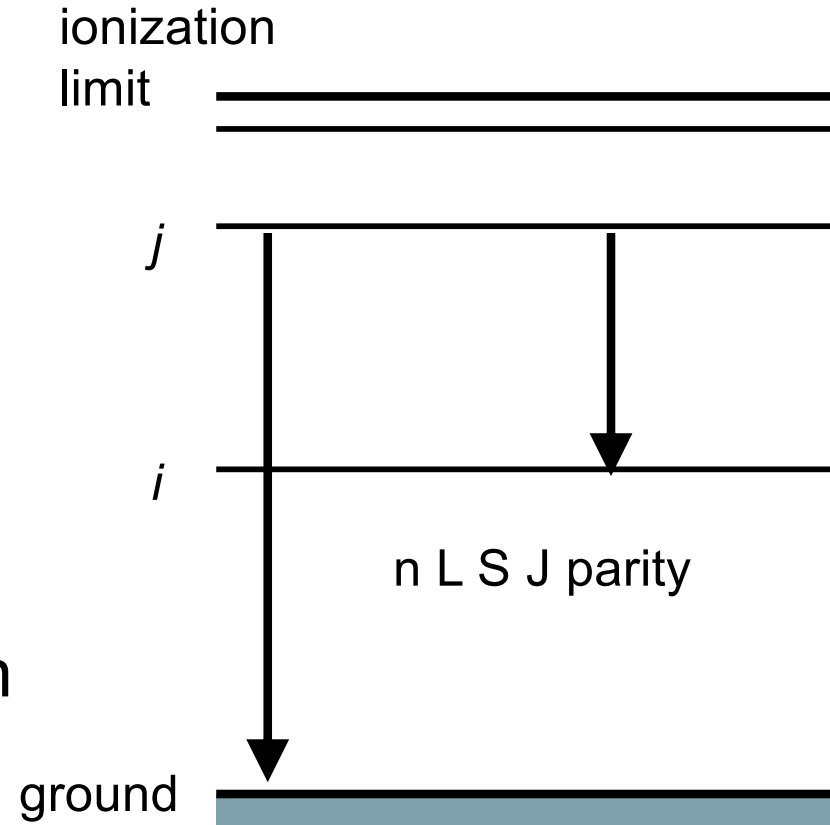
Level designation

- Electrons occupy nl orbitals
- L = orbital angular momentum, represented by a letter: $S=0$, $P=1$, $D=2$, $F=3$...
- S = spin angular momentum
- J = total angular momentum
- S and J take values 0 , $1/2$, 1 , $3/2$, 2 , ...
- Notation is $^{2S+1}L_J$
- E.g., for Fe XII, the ground configuration is



Line emission

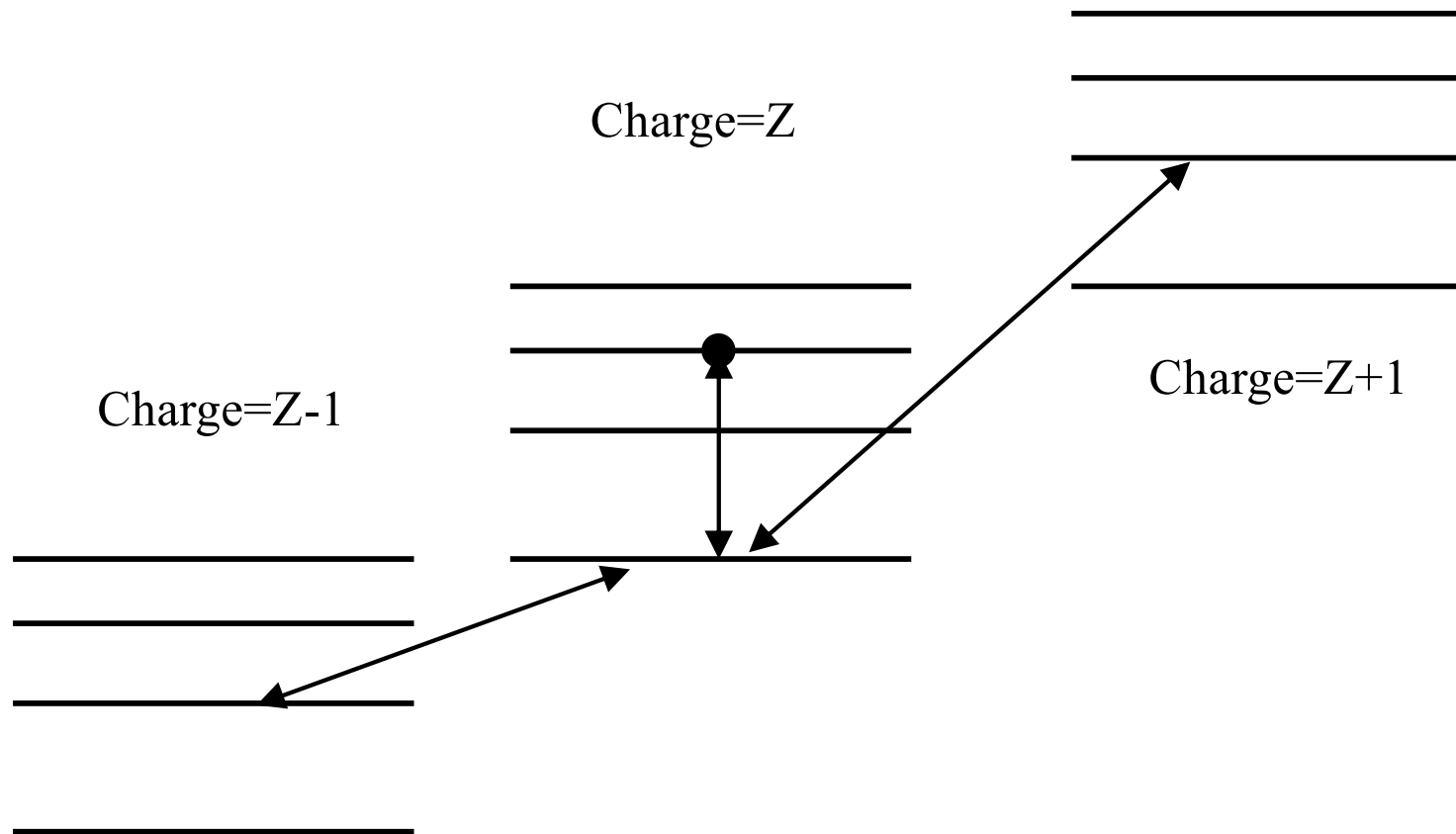
- Number of photons = $N_j A_{ji}$
- A_{ji} is Einstein's probability for spontaneous decay
- N_j fraction of ions in the state j
- Two main types
 - forbidden ($A_{ji} < 10^3 \text{ s}^{-1}$)
 - allowed ($A_{ji} > 10^5 \text{ s}^{-1}$)
- $\Delta J = 0, \pm 1$ and $\Delta L = 0, \pm 1$
- $\Delta S = 1$ intercombination have smaller A_{ji}
- All transitions *within* a configuration are forbidden
- If a level has no allowed transitions then it is *metastable*



Ionization vs. excitation

Ionizations/recombinations occur on timescales of 1-100s

Dipole-allowed lines decay in $\sim 10^{-10}$ s. Forbidden ones in 10^{-4} s or longer



Usual to treat separately excitation / ionization, but collisional-radiative models (CRM) are actually necessary in many cases.

Most codes calculate ion balance assuming population is all in ground states.

Line intensities

In optically-thin plasmas line intensities are proportional to:

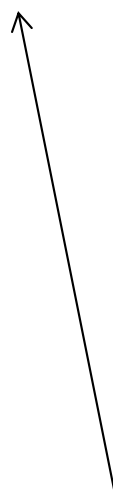
$$I \sim n_j A_{ji} = \frac{N_j(X^{+m})}{N(X^{+m})} A_{ji} \frac{N(X^{+m})}{N(X)} \frac{N(X)}{N(H)} \frac{N(H)}{N_e} N_e$$

Level population
(Ne, Te)

A-value

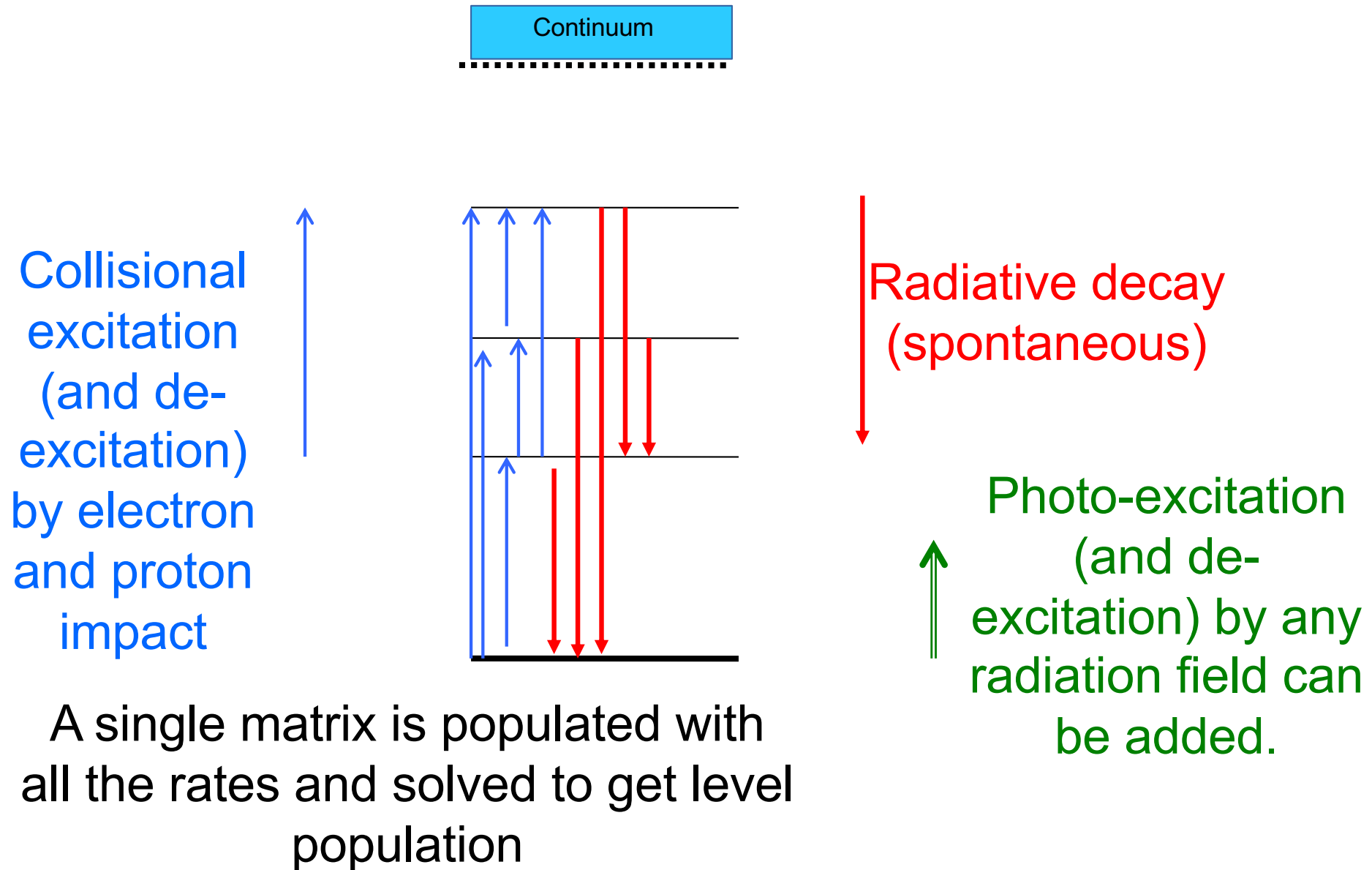
Ion abundance
(Te, Ne)

El. abundance



The Hydrogen/electron density depends on the elemental abundances relative to H. For the solar corona, H, He are fully ionised and the ratio is about 0.8--0.9.

CHIANTI population modelling before v.9



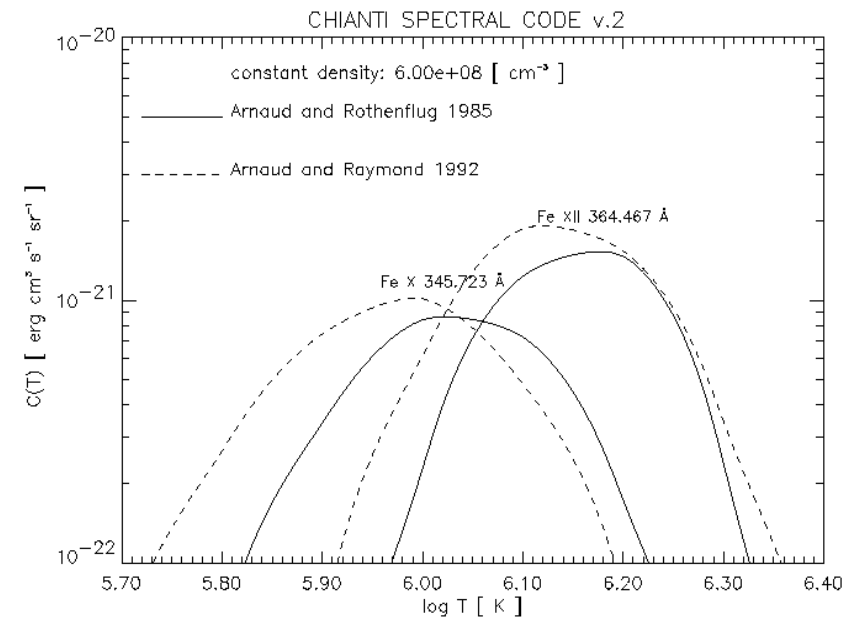
Contribution function

Spectral line intensity:

$$I(\lambda_{ij}) = \frac{h\nu_{ij}}{4\pi} \int N_j A_{ji} dh = \int Ab(X)C(T, \lambda_{ij}, N_e)N_e N_H dh \quad [\text{ergs cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}]$$

$$C(T, \lambda_{ij}, N_e) = \frac{h\nu_{ij}}{4\pi} \frac{A_{ji}}{N_e} \frac{N_j(X^{+m})}{N(X^{+m})} \frac{N(X^{+m})}{N(X)} \quad [\text{ergs cm}^{+3} \text{ s}^{-1}]$$

Note that in the literature there are various definitions of contribution functions. Aside from having values in either photons or ergs, sometime the factor π is not included. Sometimes a value of 0.83 for $N(H)/N(e)$ is assumed and included. Sometimes the element abundance factor is also included.



Atomic data for astrophysics

CALCULATION:

UK **APAP Network** <http://www.apap-network.org/>
(STFC funded): main ion atomic data provider
for fusion and astrophysics (PI: Badnell, Strathclyde)

BENCHMARK:

EUV line identifications and benchmark

DISTRIBUTION:

CHIANTI (www.chiantidatabase.org)

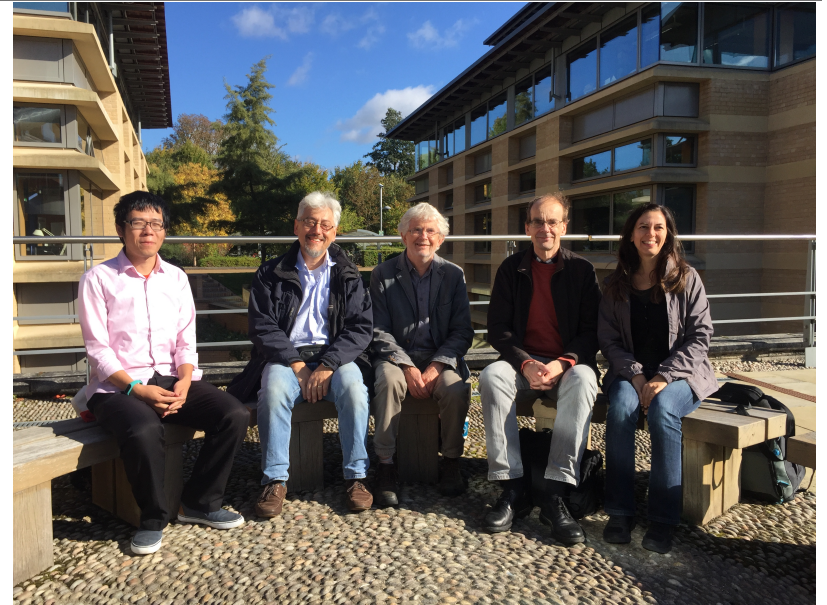
CHIANTI Google user group

IDL suite (stand-alone, SSW)

Chiantipy (Dere) in Python

CHIANTI-VIP (Del Zanna) in Python

Latest version 10: Del Zanna+2021



Atomic physics

Most theory and programs developed over a long period of time with significant contributions in the UK from UCL (Seaton's group), QUB (e.g. Burke, Hibbert), Cambridge (Burgess), and a few groups in Europe.

Atomic structure theory and codes received significant contributions from C. Froese Fisher and I. Grant (Oxford).

Atomic physics went out of fashion long ago and soon we will go back to the middle ages.



Mike Seaton (UCL)
Alan Burgess (Cambridge)

Collisional excitation (CE) and de-excitation

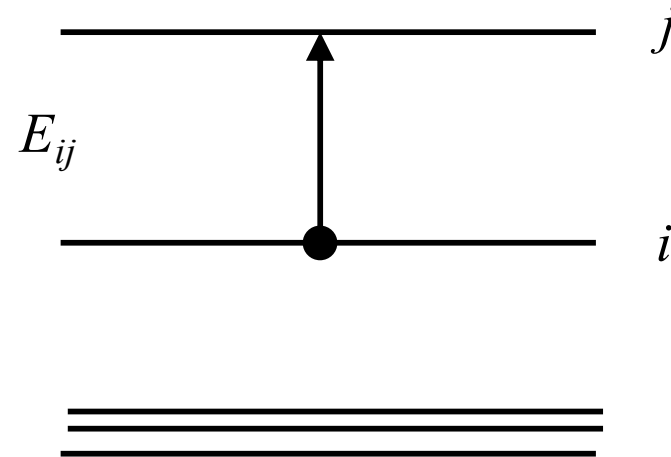
The main processes affecting the level populations within one ion are scattering with particles.

Most abundant particles in the quiescent corona are thermal electrons, protons, alpha.

Electron collisions are the main excitation process in the low corona.

Proton collisions are also important when level separation is small.

Within CHIANTI, a matrix with all the upward and downward rates (no bf) is solved to obtain the level populations (since v.9 I introduced a two-ion CRM)



Collision strength

The number of transitions from level i to level j due to electron collisions per unit volume and time is given by $N_i N_e C_{ij}$ where C_{ij} is the rate:

$$C_{ij}^e = \int_{v_0}^{\infty} v \sigma_{ij}(v) f(v) dv$$

with v_0 the velocity corresponding to the threshold energy for the transition and the cross section normally written as a function of a symmetric adimensional quantity, function of the kinetic energy of the colliding electron, called the **collision strength**

$$\sigma_{ij} = \frac{\pi a_0^2 E_H}{g_i E} \Omega_{ij}(E)$$

a_0 the first Bohr orbit radius for the hydrogen, g_i the statistical weight of the level i , E_H the ionization energy for H I (13.6 eV). Assuming a Maxwellian distribution function f :

$$C_{ij}^e = 8.63 \cdot 10^{-6} \frac{\Upsilon_{ij}}{T_e^{1/2} g_i} \exp\left(-\frac{E_{ij}}{kT_e}\right)$$

Collision strength

The Maxwellian-averaged collision strength:

$$\Upsilon_{ij} = \int_0^\infty \Omega_{ij} \exp\left(-\frac{E_j}{kT_e}\right) d\left(\frac{E_j}{kT_e}\right)$$

E_j is the energy of the scattered electron relative to the energy of the final state of the ion.

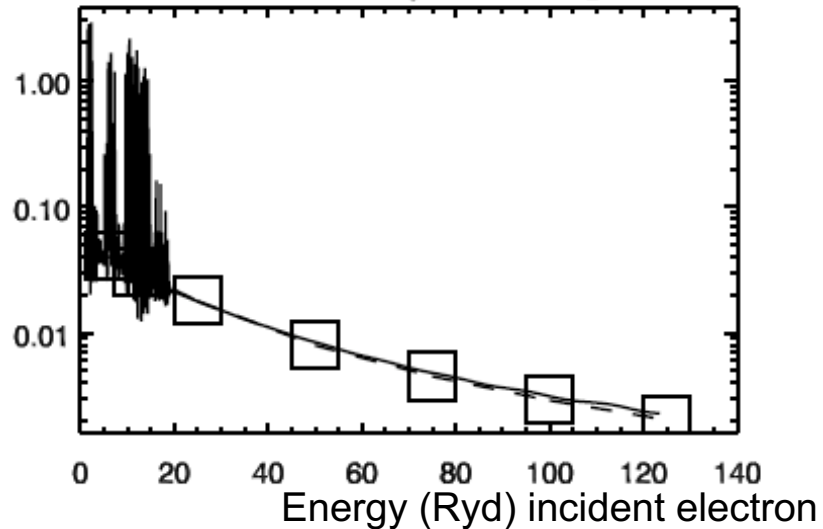
The electron de-excitation rate is obtained with the principle of detailed balance:

$$C_{ji}^e = \frac{g_i}{g_j} C_{ij}^e \exp\left(\frac{E_{ij}}{kT_e}\right)$$

Collision strength- example: Mg IX

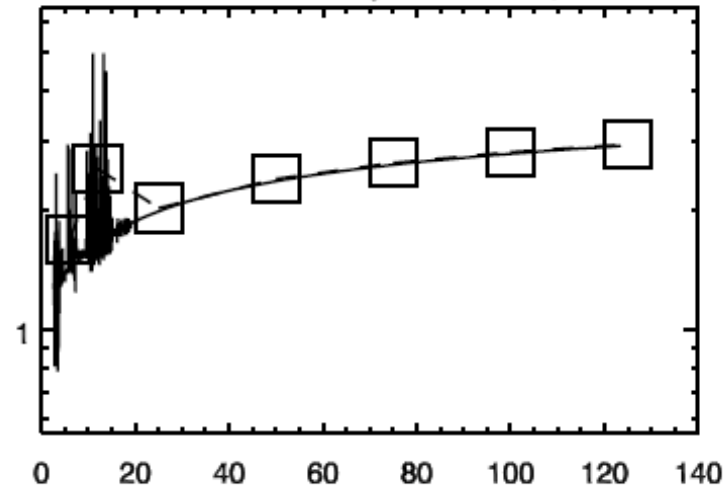
intercombination

1 - 4 $2s^2\ ^1S_0 - 2s\ 2p\ ^3P_2$



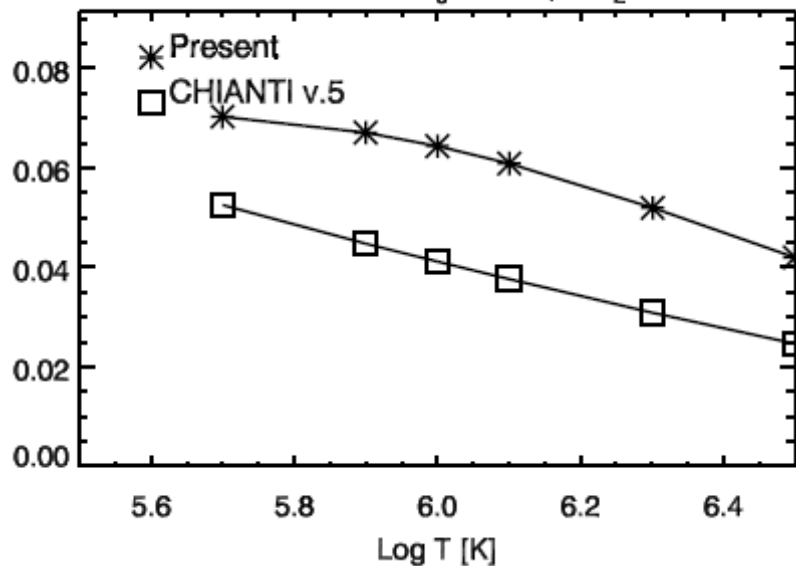
allowed

1 - 5 $2s^2\ ^1S_0 - 2s\ 2p\ ^1P_1$

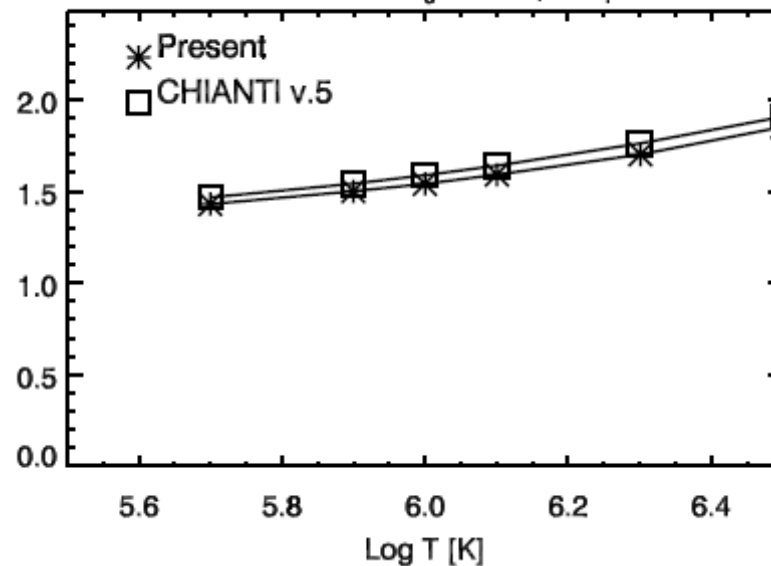


Coll.
strength

1 - 4 $2s^2\ ^1S_0 - 2s\ 2p\ ^3P_2$



1 - 5 $2s^2\ ^1S_0 - 2s\ 2p\ ^1P_1$



Maxwellian
avg. Coll.
strength

CHIANTI data

Currently, for each ion at least ascii files:

fe_12.elvlc	Energy levels (theoretical, observed), level descriptions
fe_12.wgfa	Transition probabilities, gf values, theoretical, observed wavelengths
fe_12.scups	Maxwellian-averaged e- collision strengths on a scaled domain

- + proton rates only for a few levels and ions.

Non-Maxwellian e- (NMED)

Non-thermal particles are also present, especially during flares.

Most codes in astrophysics assume Maxwellian particles.

There is evidence that in solar active regions non-Maxwellian electron distributions (NMED) are present (Lorincik+2020).

Within CHIANTI, there is a way to treat NMED assuming sums of Maxwellians

$$f(E; a_i) = \sum_i a_i f_M(E, T_i)$$

$$\begin{aligned} C_{jk} &= \int_{E_{jk}}^{\infty} Q_{jk} v f(E; a_i) dE \\ &= \sum_i a_i \int_{E_{jk}}^{\infty} Q_{jk} v f_M(E, T_i) dE \\ &= \sum_i a_i C_{jk}(T_i) \end{aligned}$$

The collisional excitation rate is the sum.
The coefficients a_i are passed via a keyword

Alternatively, if you assume a k-distribution you can use the KAPPA package (being updated for CHIANTI v.8) or ask for cross-sections.

Level population and coronal model

Without
proton CE:

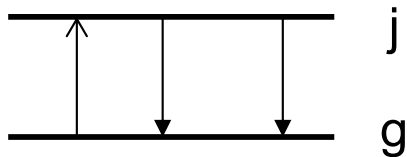
$$\begin{aligned} \frac{dN_j}{dt} = & \sum_{k>j} N_k A_{k,j} + \sum_{k>j} N_k N_e C_{k,j}^d + \sum_{i<j} N_i N_e C_{i,j}^e + \sum_{i<j} N_i B_{i,j} J_{\nu_{i,j}} \\ & + \sum_{k>j} N_k B_{k,j} J_{\nu_{k,j}} - N_j \left(\sum_{i<j} A_{j,i} + N_e \sum_{i<j} C_{j,i}^d + N_e \sum_{k>j} C_{j,k}^e \right. \\ & \left. + \sum_{i<j} B_{j,i} J_{\nu_{j,i}} + \sum_{k>j} B_{j,k} J_{\nu_{k,j}} \right) \end{aligned}$$

In a two-level ion model (e.g. a ground state g and an excited state j) and neglecting photoexcitation:

Two-level ion

$$N_g N_e C_{gj}^e = N_j (N_e C_{jg}^e + A_{jg})$$

so the relative population of the level j is

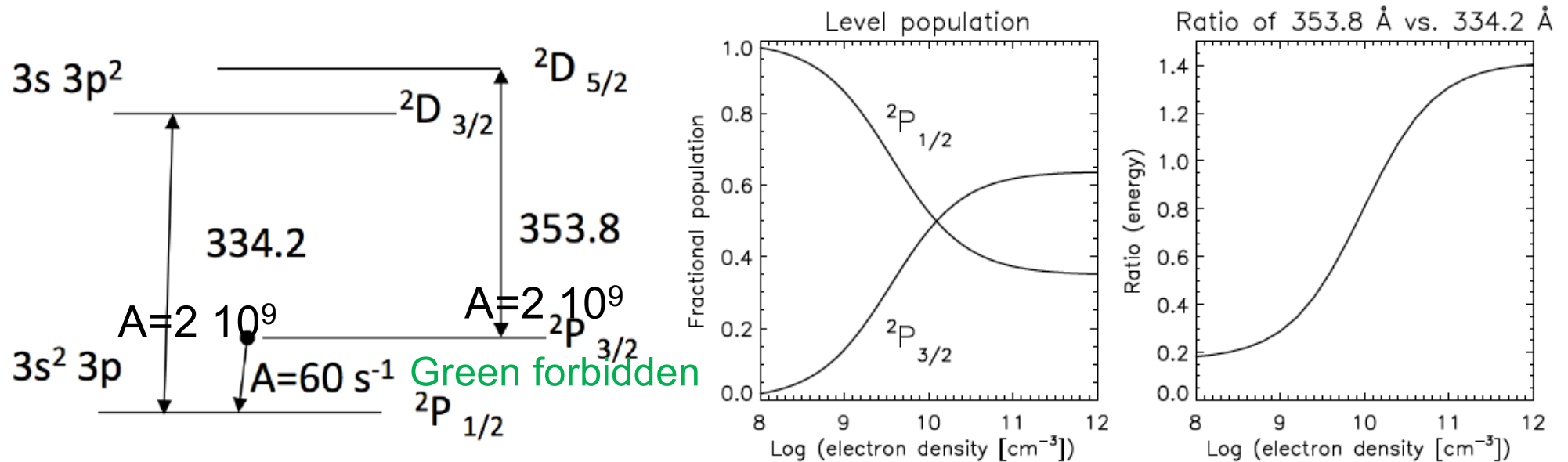


$$\frac{N_j}{N_g} = \frac{N_e C_{gj}^e}{N_e C_{jg}^e + A_{jg}} \quad (15)$$

i.e. depends strongly on the relative values between the radiative rate A_{jg} and the collisional de-excitation term $N_e C_{jg}^e$.

Metastable levels- electron density diagnostics

Density diagnostics involve metastable levels. Best are ratios of allowed lines



CHIANTI: IDL>plot_populations, 'fe_14',2e6

The number of photons emitted by the transition per second and per ion is:

$$A_{jg} \frac{N_j}{N_g} = A_{jg} \frac{C_{gj}^e}{C_{jg}^e} \left(1 + \frac{A_{jg}}{N_e C_{jg}^e} \right)^{-1} = \frac{g_j}{g_g} \exp \left(\frac{-\Delta E_{gj}}{kT_e} \right) A_{jg} \left(1 + \frac{A_{jg}}{N_e C_{jg}^e} \right)^{-1}$$

At low densities the intensity is then proportional to the density but independent of the A value: $N_e C_{gj}^e$. The emission is directly related to the collision.

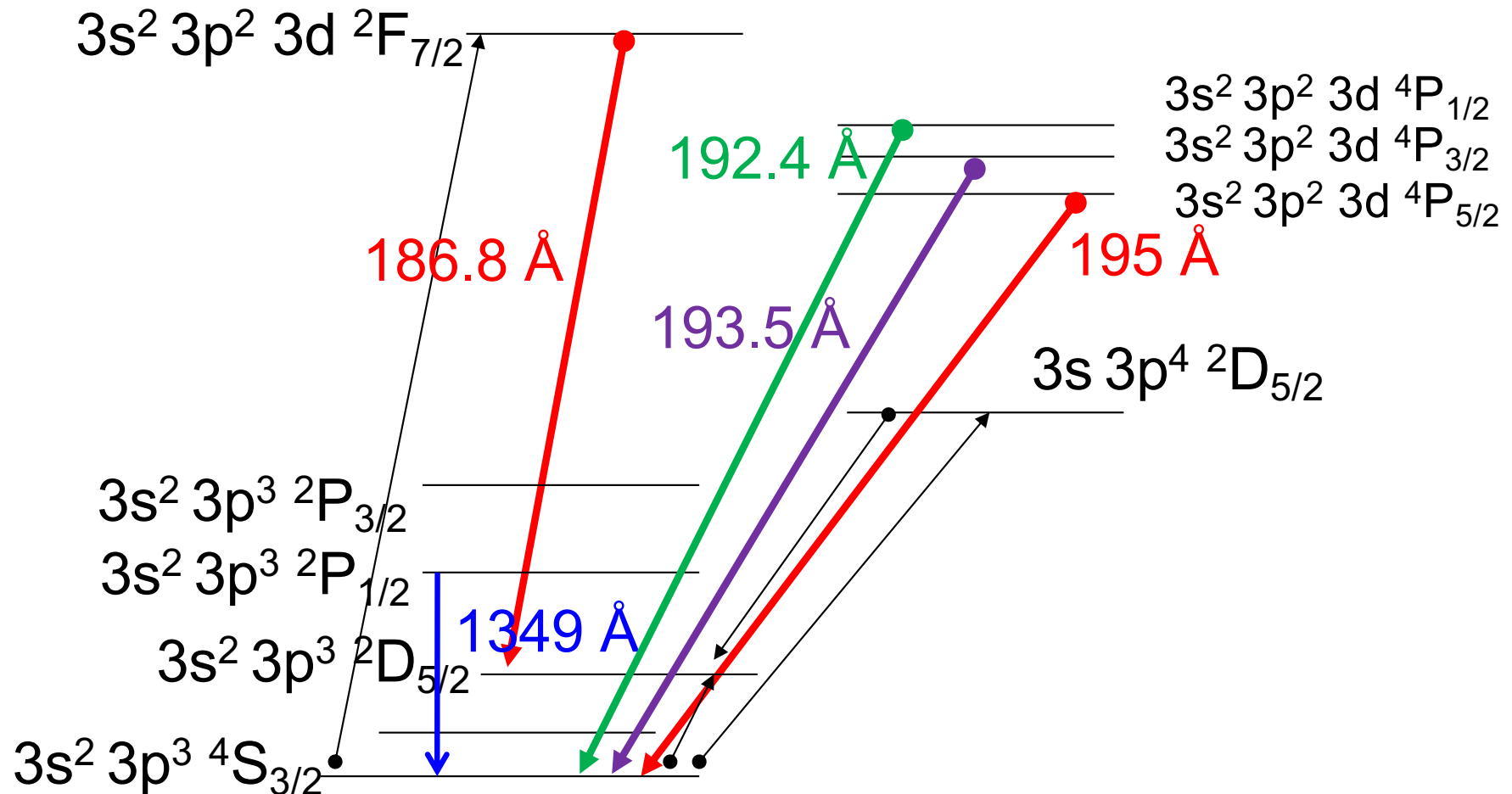
At high densities the intensity is

$$A_{jg} \frac{g_j}{g_g} \exp \left(\frac{-\Delta E_{gj}}{kT_e} \right),$$

Fe XII

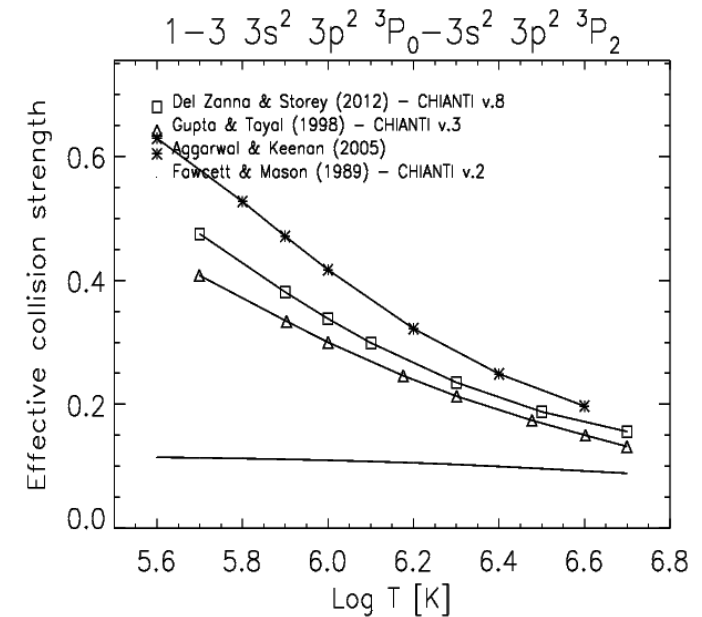
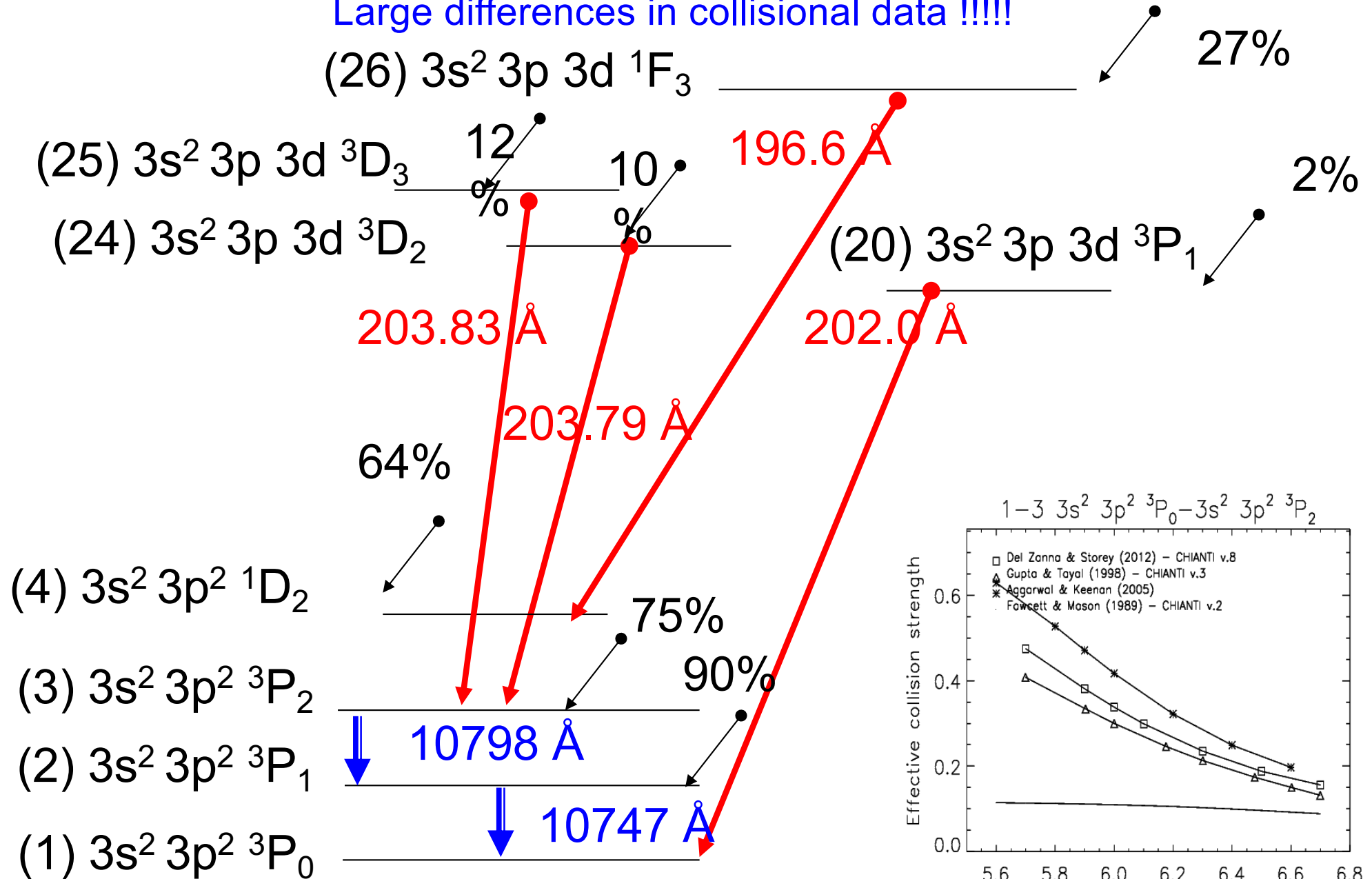
The 186/195 is a good density diagnostic for Hinode EIS. The populations of the metastable levels are hard to calculate.

Del Zanna (2012): the population of the $3s^2 3p^3 \ ^2D_{5/2}$ is 50 % higher than previous models (Storey et al. 2005; Del Zanna & Mason 2005).



Fe XIII – log Ne=8

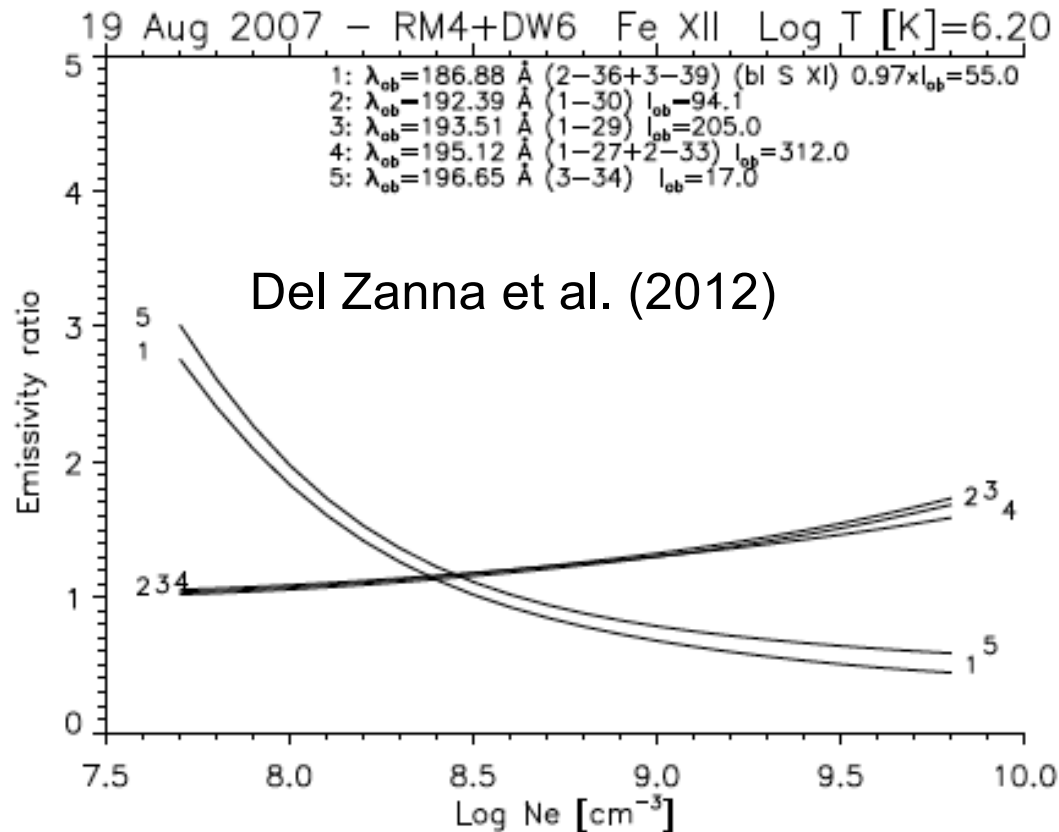
Complex cascading effects (Yu, Del Zanna+2018; Del Zanna & Mason 2018).
 Large differences in collisional data !!!!!



Measuring densities

When more than one line ratio is available, a convenient way is the emissivity ratio method (Del Zanna et al. 2004)

$$F_{ji}(N_e, T_e) = C \frac{I_{ob} N_e}{N_j(N_e, T_e) A_{ji}}$$



IRIS densities

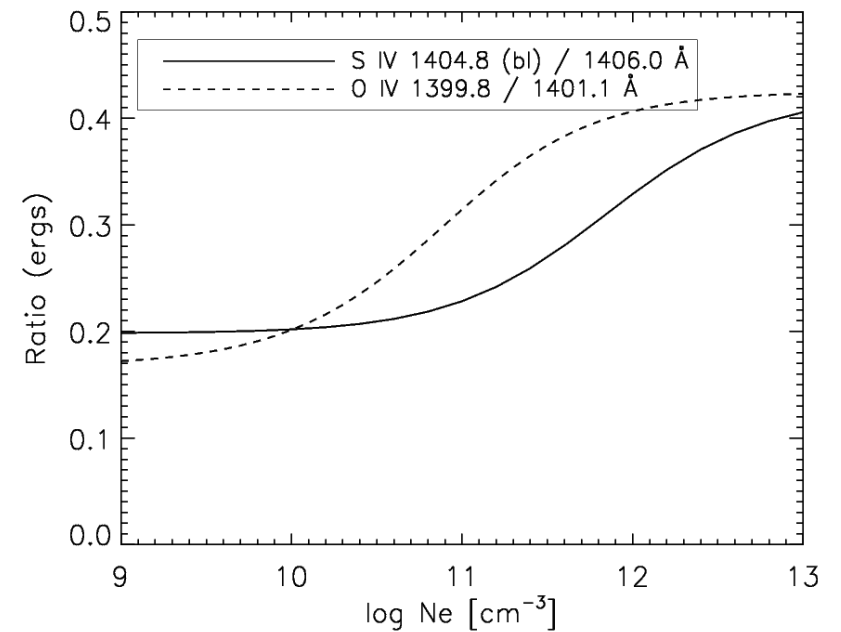
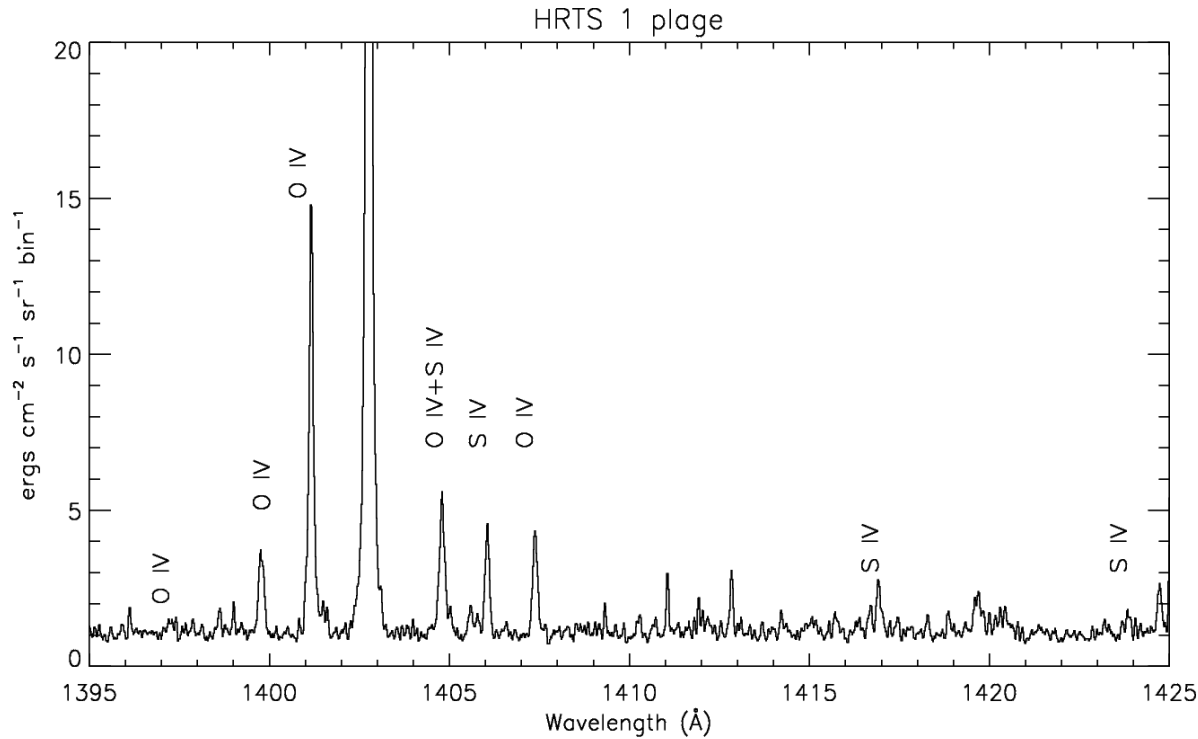
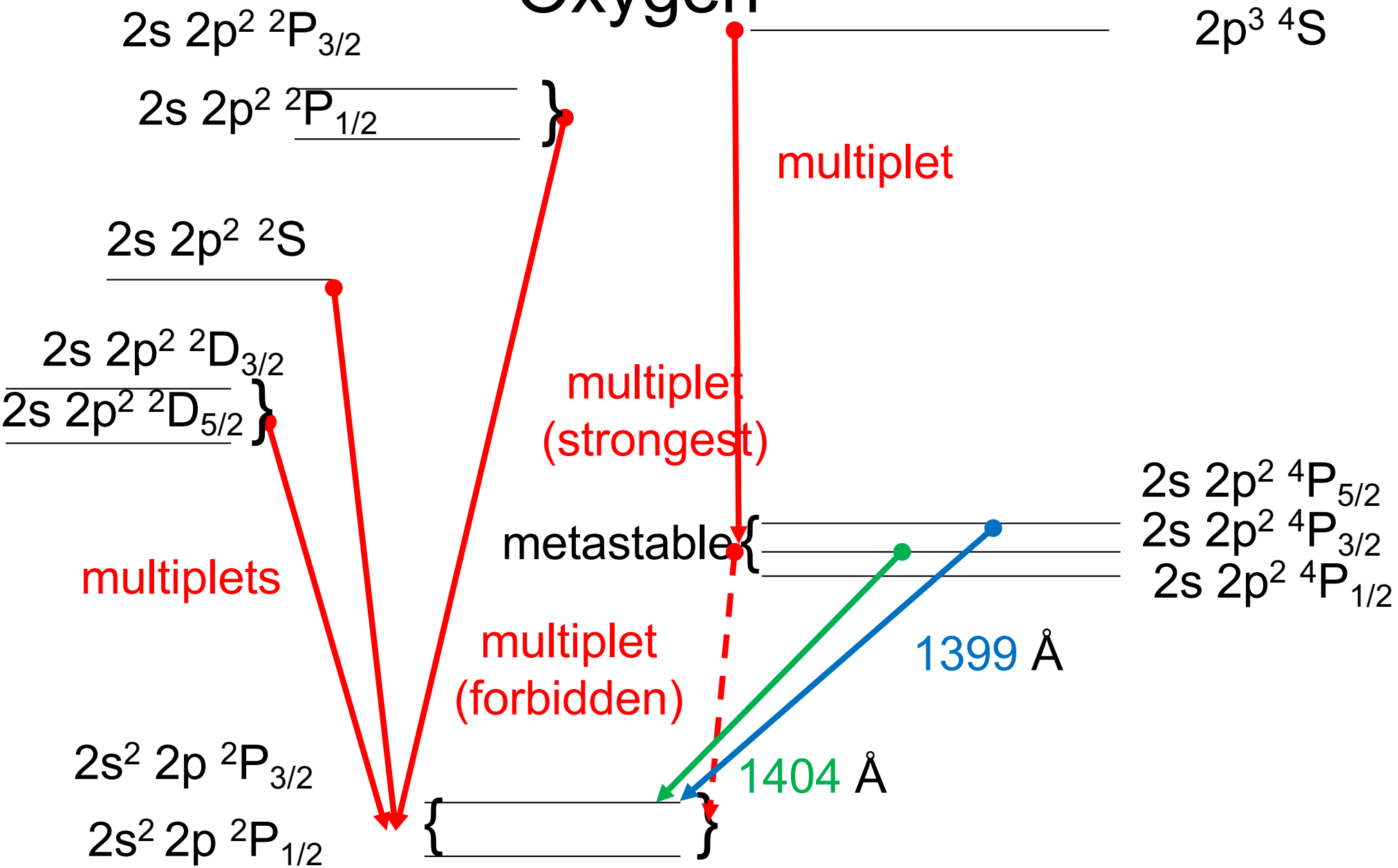


Table 3: O IV transitions commonly used to measure densities.

Transitions	λ (Å)	Instrument	log N _e
$2P_{3/2} - 4P_{5/2}$	1401.16		
$2s^2 2p^2 P_{3/2} - 2s 2p^2 4P_{3/2}$	1404.78 (bl S IV)	Skylab, HRTS, SMM/UVSP SUMER, IRIS	9–12
$2s^2 2p^2 P_{3/2} - 2s 2p^2 4P_{3/2}$	1399.78		

B-like

Oxygen



CHIANTI GUI applications

CHIANTI version 8.0.1 --- ROUTINE: DENS_PLOTTER

Emissivity units: Energy Photons

Proton rates: yes no

Density range, log Ne [cm⁻³]:

Density intervals: 1.0 0.5 0.2 0.1

Temp, log T [K]:

Include photoexcitation? yes no

CHANGE PARAMETERS

NUMERATOR: 629.733
Choose a new line

INTENSITY: SIGMA: PLOT ERROR BARS

629.733 Å 1-5 2s2 1S0 - 2s.2p 1P1

DENOMINATOR: 629.733
Choose a new line

INTENSITY: SIGMA: PLOT ERROR BARS

629.733 Å 1-5 2s2 1S0 - 2s.2p 1P1

Low wavelength limit (Å):

High wavelength limit (Å):

SAVE HARDCOPY QUIT

Send comments to chianti_help@halcyon.nrl.navy.mil

Log Temp [K]: 5.35
Density intervals: 0.2
Photoexc: not included
Protons: included
Units: energy

DERIVED DENSITY [cm⁻³]:
Input line intensities

O V line ratios relative to 629.733 Å

Selected lines: Selected lines All lines

Y-AXIS SCALING: Log Automatic Linear Automatic (ynozero) Manual

Upper limit: Lower limit:

SHOW RATIO VALUES REFRESH PLOT

Input ion (e.g. Fe XII):

Select ratios as a function of: Density Temperature

Min log Ne [cm⁻³]: Max log Ne [cm⁻³]:

Step (log Ne):

Log T [K]:

Add proton rates: Yes

Add photoexcitation: Yes/No

Distance (R/R^{*}): Radiation T [K]:

CHIANTI - O V - lines / 1-5 629.7332

Ratio (ergs)

Log Ne [cm⁻³]

Wavelength range (Å):

No. of lines plotted:

Ergs Phot

Lines plotted: 3-17 192.8011 Å, 4-18 192.9114 Å, 2-17 192.7505 Å, 5-10 774.5192 Å, 1-13 172.1693 Å

Select numerator

Reference: (denominator) 1-5 629.7332 Å

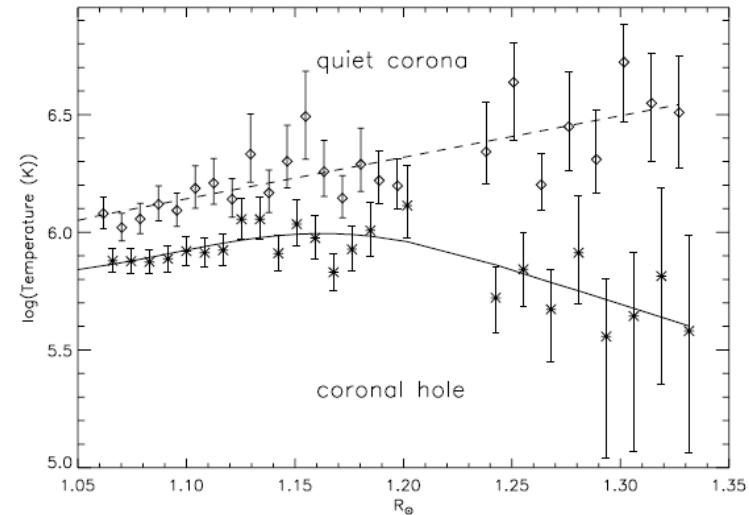
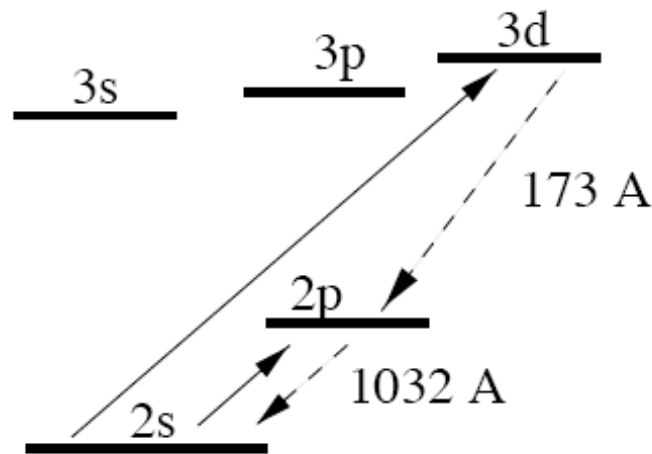
Select new

Save plot as Save emissivities Save ratio

'Direct' temperature diagnostics

For allowed lines, intensities are proportional to the excitation rate.

$$I_{jj} \sim h\nu_{gj} N_e N_g C_{gj}^e \sim h\nu_{gj} N_e N_g \frac{\Upsilon_{gj}}{T_e^{1/2} g_g} \exp\left(-\frac{E_{gj}}{kT_e}\right)$$



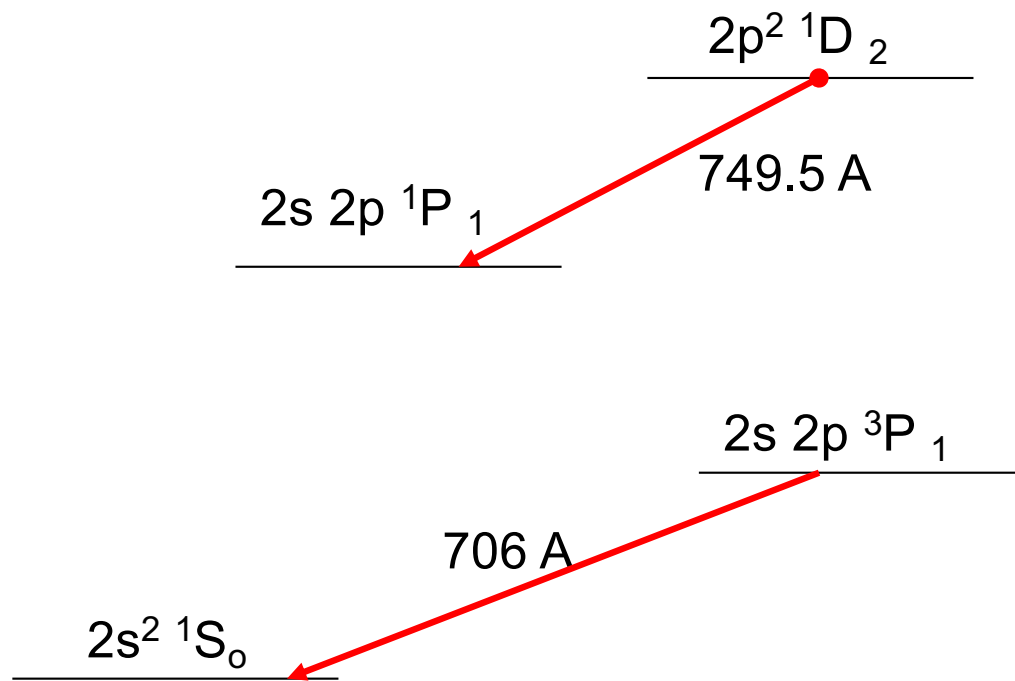
Ratios of lines with very different excitation energies are T-sensitive.

O VI SOHO GIS + SUMER David et al. (1998)

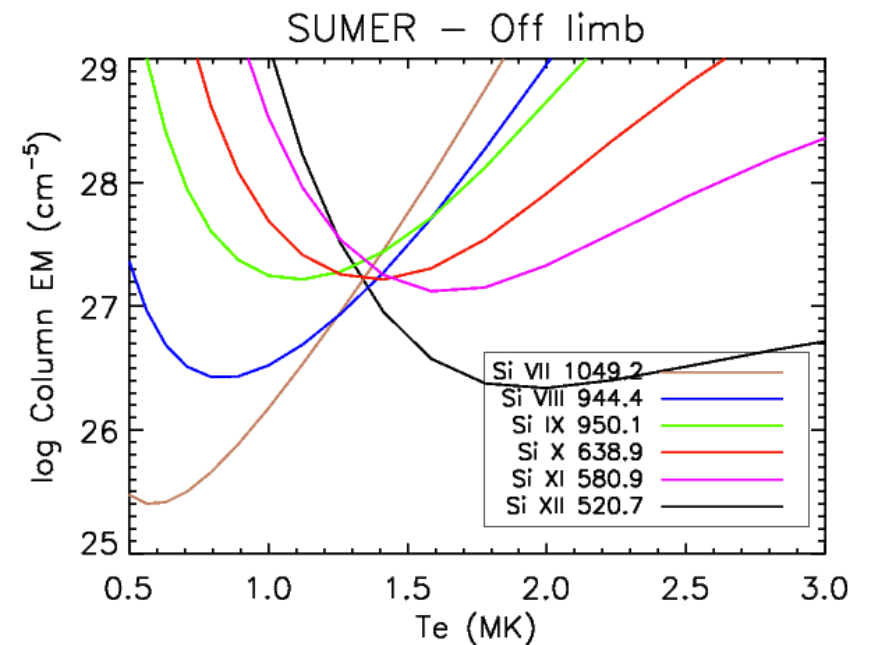
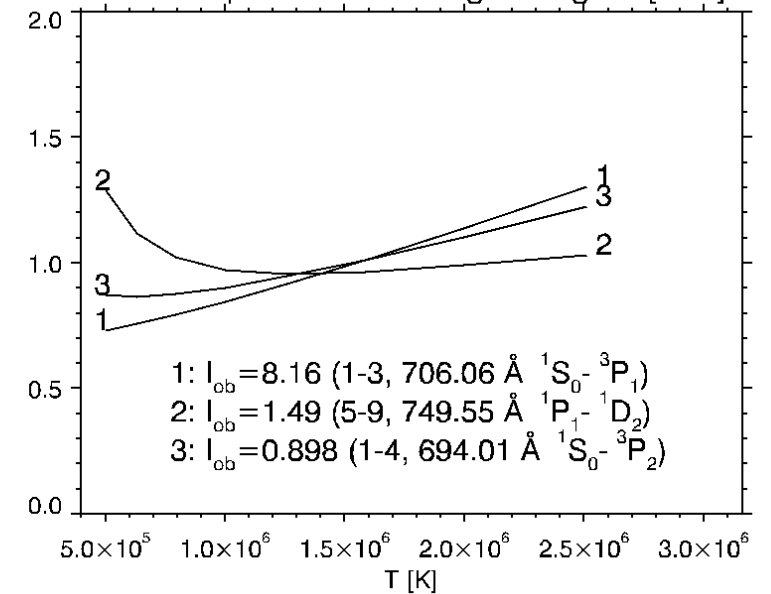
At flare temperatures very good measurements can be obtained from the satellite lines in the X-rays, with different diagnostics.

Measuring Te with Mg IX

One of the best Te diagnostic for 1 MK plasma (Del Zanna et al. 2008).



SOHO SUMER present results Mg IX $\text{Log Ne} [\text{cm}^{-3}] = 8.2$





Break out hands-on session?

Line widths

$$I_\lambda = \frac{I}{\sqrt{2\pi}\sigma} \exp[-(\lambda - \lambda_0)^2 / 2\sigma^2]$$

$$\sigma^2 = \frac{\lambda^2}{2c^2} \left(\frac{2kT}{M_i} + \xi^2 \right) + \sigma_I^2$$

For a Maxwellian distribution

Ion mass

M_i

gaussian instrumental width

σ_I

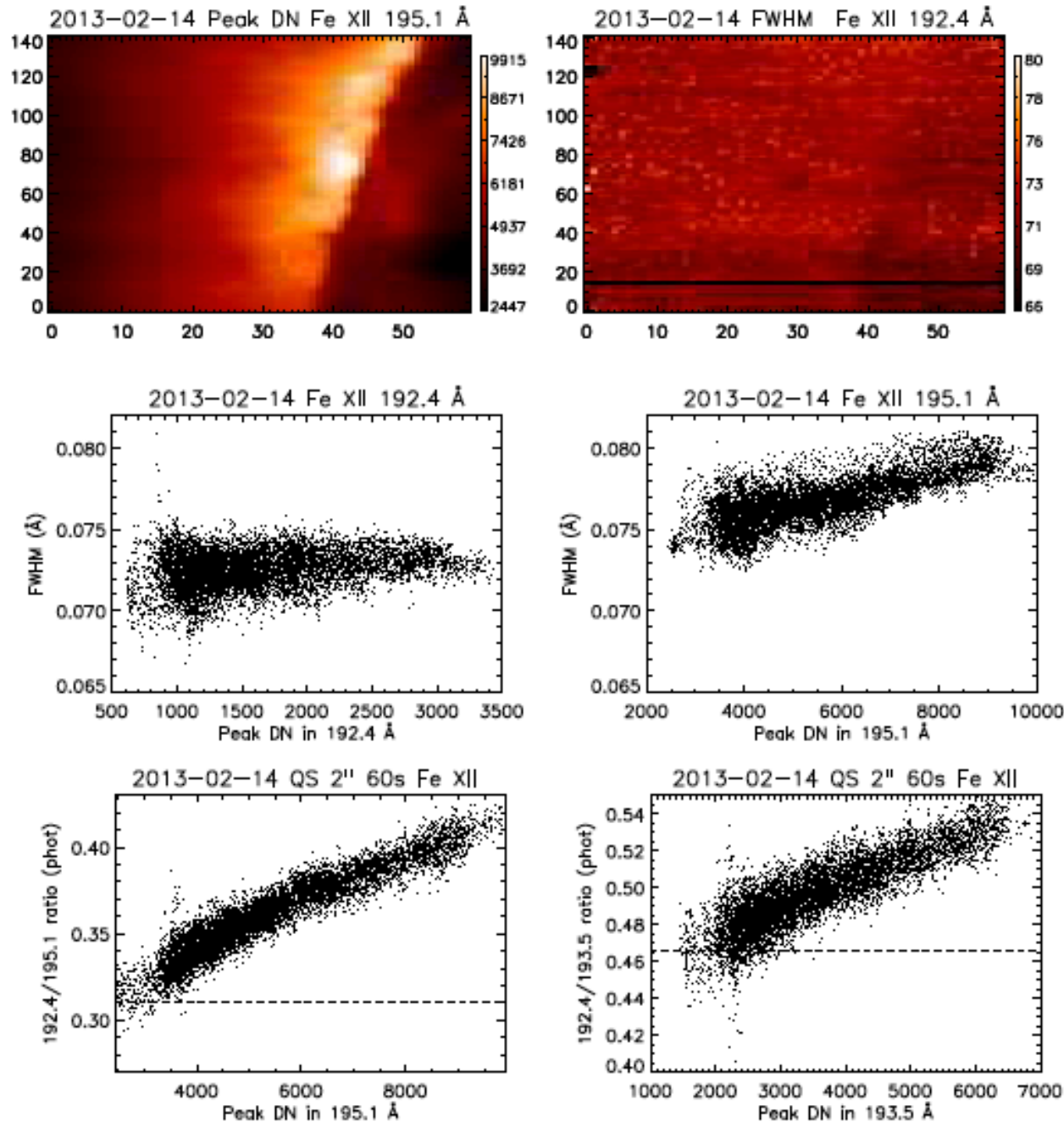
most probable non-thermal velocity

ξ

$$\langle v^2 \rangle^{1/2} = (3/2)^{1/2} \xi$$

A key issue is the ion temperature – not easy to know. Te has hardly been measured. The ionization temperature is normally used.

Opacity in coronal lines



The EIS Fe XII
192, 193.5, 195 Å
Lines should have
the same width and
constant ratios.
They do not in AR
and some off-limb
observations.

Del Zanna+2019

$$\tau_0 = 8.3 \cdot 10^{-21} f_{lu} \frac{\lambda^2}{\Delta\lambda_{FWHM}} N_l \Delta S \quad (4)$$

with λ and $\Delta\lambda_{FWHM}$ expressed in Å. For the 195 Å line, $f_{lu} = 2.97/4$, neglecting the weaker line blending the main line.

The population of the lower level can be written as

$$N_l = \frac{N_l}{N(\text{Fe XII})} \frac{N(\text{Fe XII})}{N(\text{Fe})} Ab(\text{Fe}) \frac{N_H}{N_e} N_e, \quad (5)$$

where $N_l/N(\text{Fe XII}) = 0.9$ is the relative population of the ground state at quiet Sun densities, $N(\text{Fe XII})/N(\text{Fe}) = 0.25$ is the peak relative population of the ion, $Ab(\text{Fe}) = 3.16 \cdot 10^{-5}$ is the Fe photospheric abundance, $N_H/N_e = 0.83$, and N_e is the averaged electron number density. We therefore have $\tau_0 = 6.9 \cdot 10^{-20} N_e \Delta S$ [cm^{-2}] assuming $\Delta\lambda_{FWHM} = 0.02$ Å.

```
plot_populations, 'fe_12', 1.5e6, 20, densities=1e8*[1,3,5]
plot_ioneq, 'fe', ion=[12]
```

for $N_e = 7 \cdot 10^8$ and a path = $1.8 \cdot 10^{10}$ cm (from the EM) we get $\tau = 0.9$ for the 195 Å line.

Assuming that the source function does not vary alos:

$$I_\nu = S_\nu (1 - e^{-\tau_0}) ,$$

while the the line source function S_ν is:

$$S_\nu = \frac{2 h \nu^3}{c^2} \left(\frac{g_u N_l}{g_l N_u} - 1 \right)^{-1} ,$$

for $\tau (195 \text{ \AA}) = 0.9$, $\tau (192 \text{ \AA}) = 0.3$,
the ratio of the source functions is close to 1,
and the 195/192 \AA ratio decreases by $\sim 30\%$,
close to what observed.



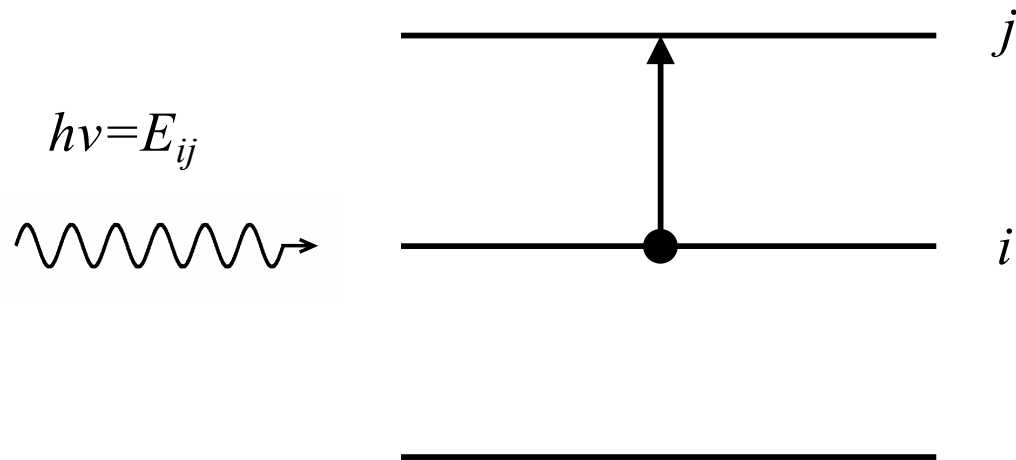
Break out hands-on session?

Photo-excitation (PE)

Photoexcitation and stimulated emission depend on the local radiation field. It is not so important in the low corona and EUV/UV lines.

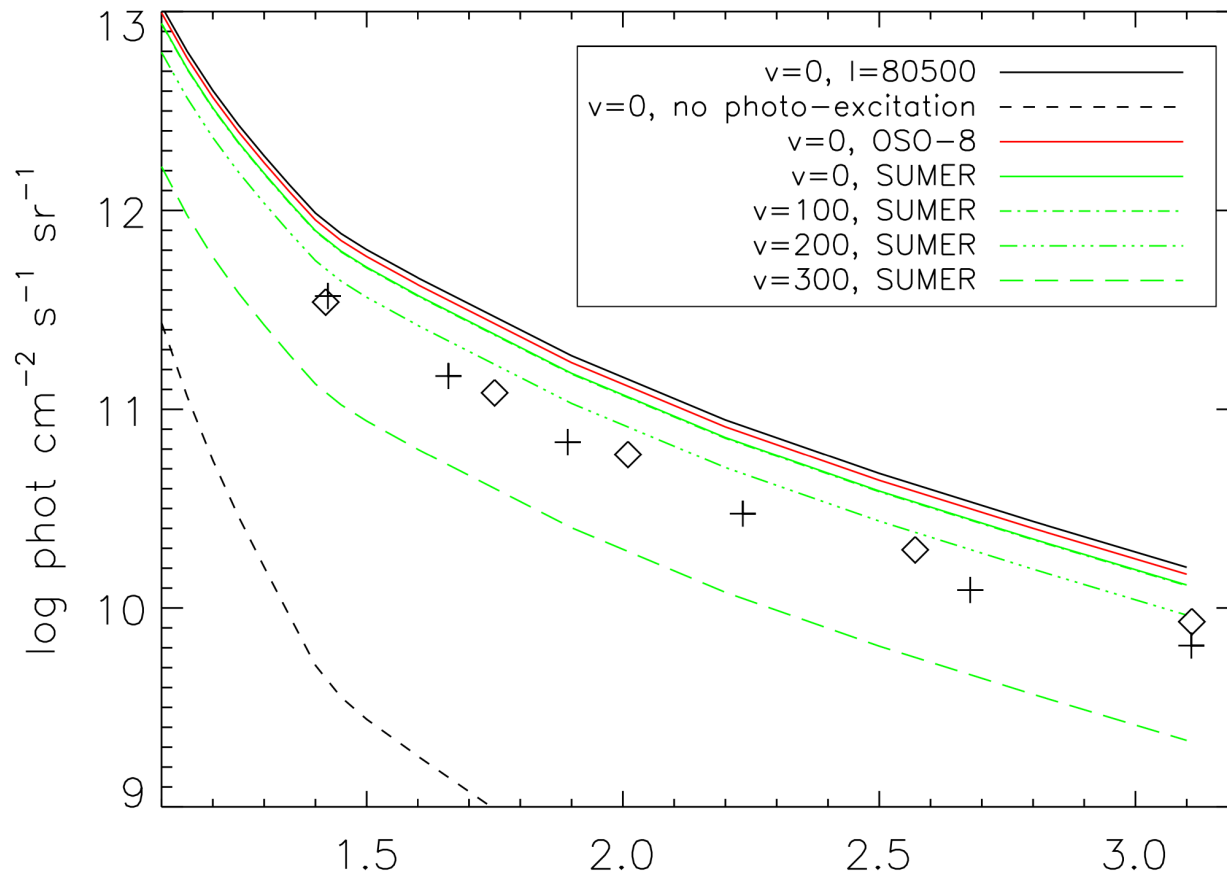
It is very important for the forbidden lines in the corona, when densities are low and little CE is present.

Also, for all cool lines in the corona (esp. H,He), and low charge states in the TR/chromosphere.



PE for H, He in the corona

H I 1216 Å



The abundance of H in the corona is 10^{-7} , still the Ly α is the strongest coronal line!

Related to modelling the outer corona
(Del Zanna+2018)

Neutral He is also strongly affected by PE (Del Zanna+2020)

PE

Assuming a uniform disk intensity:

$$J_\nu = \frac{\Delta \Omega}{4\pi} \bar{I}_\nu = W(r) \bar{I}_\nu \quad (25)$$

where $W(r)$ is the *dilution factor* of the radiation, i.e., the geometrical factor which accounts for the weakening of the radiation field at a distance r from the Sun, and \bar{I}_ν is the averaged disk radiance at the frequency ν .

Assuming spherical symmetry (i.e., the solar photosphere a perfect sphere), and indicating with r the distance from Sun centre R_\odot the solar radius, we have:

$$W(r) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\theta_0} \sin \theta \, d\theta \, d\phi = \frac{1}{2} (1 - \cos \theta_0) = \frac{1}{2} \left(1 - \left[1 - \left(\frac{R_\odot}{r} \right)^2 \right]^{1/2} \right) \quad (26)$$

where θ_0 is the angle sub-tending R_\odot at the distance r , i.e., $\sin \theta_0 = R_\odot/r$.

In terms of the energy density per unit wavelength, U_λ , the photoexcitation rate for a transition $i \rightarrow j$ is:

$$P_{ij} = A_{ji} W(r) \frac{g_j}{g_i} \frac{\lambda^5}{8\pi hc} U_\lambda \quad (27)$$

where A_{ji} is the Einstein coefficient for spontaneous emission from j to i , g_j and g_i are the statistical weights of levels j and i , and $W(r)$ is the radiation dilution factor.

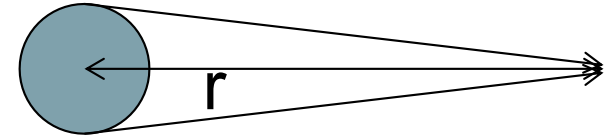
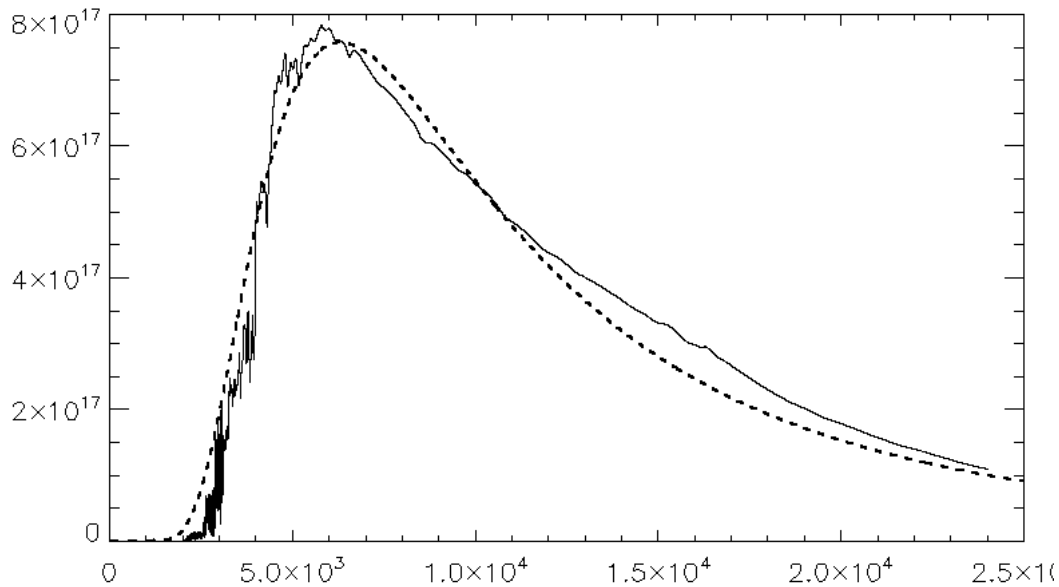


Photo-excitation (PE)



For lines in the vis/IR, $I=B$ (black-body) and the correction to the A-value depends on T_* which is about 6000 K

In CHIANTI, PE is included as a correction to the A-value:

$$A_{ij} = \begin{cases} W(r) A_{ji} \frac{g_j}{g_i} \frac{1}{\exp(\Delta E/kT_*) - 1} & i < j \\ A_{ji} \left[1 + W(r) \frac{1}{\exp(\Delta E/kT_*) - 1} \right] & i > j \end{cases}$$

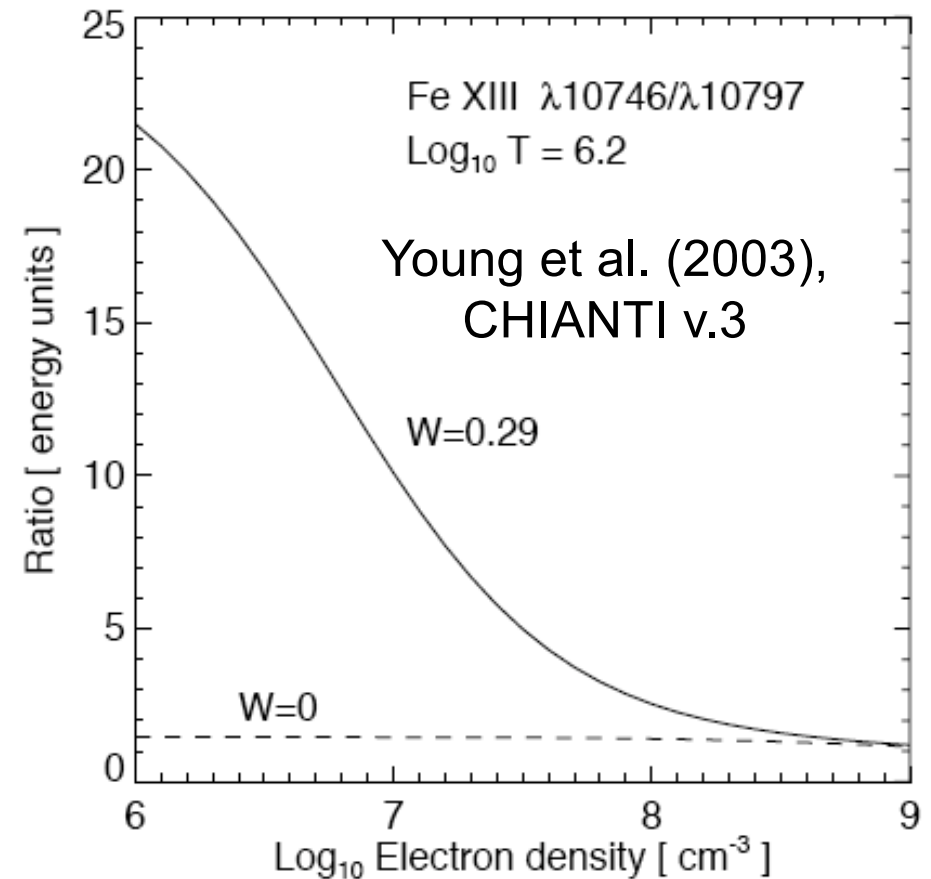
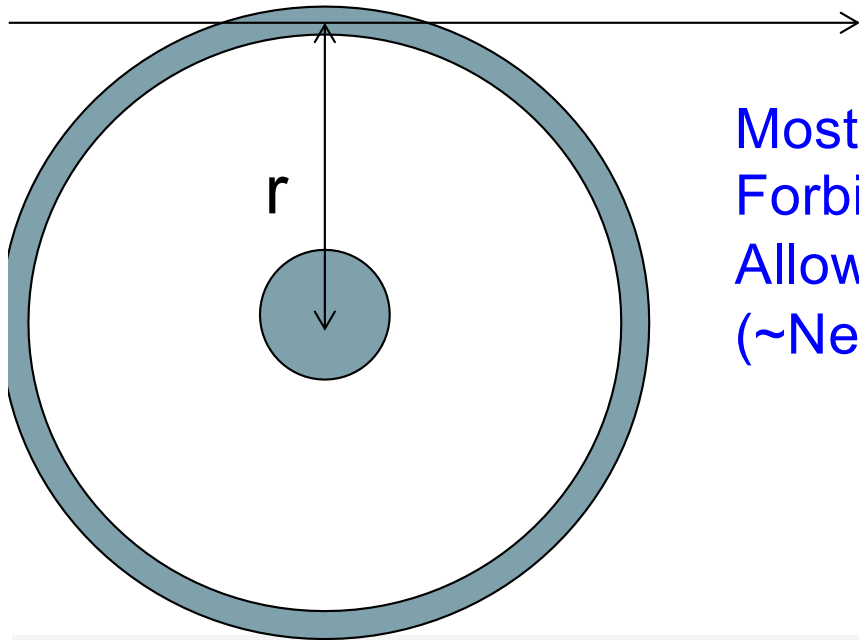
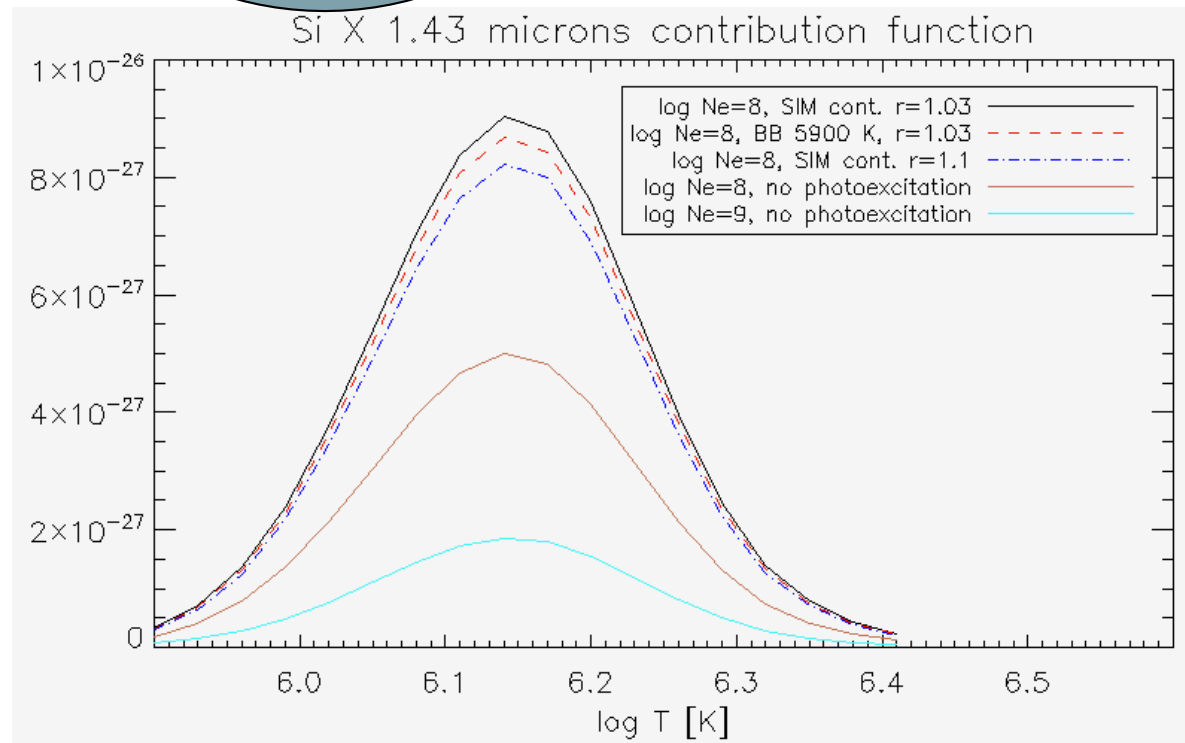


FIG. 3.—Fe XIII $\lambda 10746/\lambda 10797$ ratio plotted as a function of density for two different dilution factors. $W = 0$ corresponds to no radiation field, while $W = 0.29$ corresponds to 0.1 source radii above the source surface ($r_* = 1.1$).



Most allowed (EUV) transitions: $C(T)$; $I \sim Ne^2$
 Forbidden: $C(T, Ne, r, I_{\text{disk}})$
 Allowed, forbidden and scattered I_{disk} emission
 ($\sim Ne$) is not necessarily co-spatial

Photo-excitation from disk
 can be modelled using
 CHIANTI, with user-defined
 disk radiances.



**Unlike allowed transitions,
 a good model of Ne alos
 is fundamental**

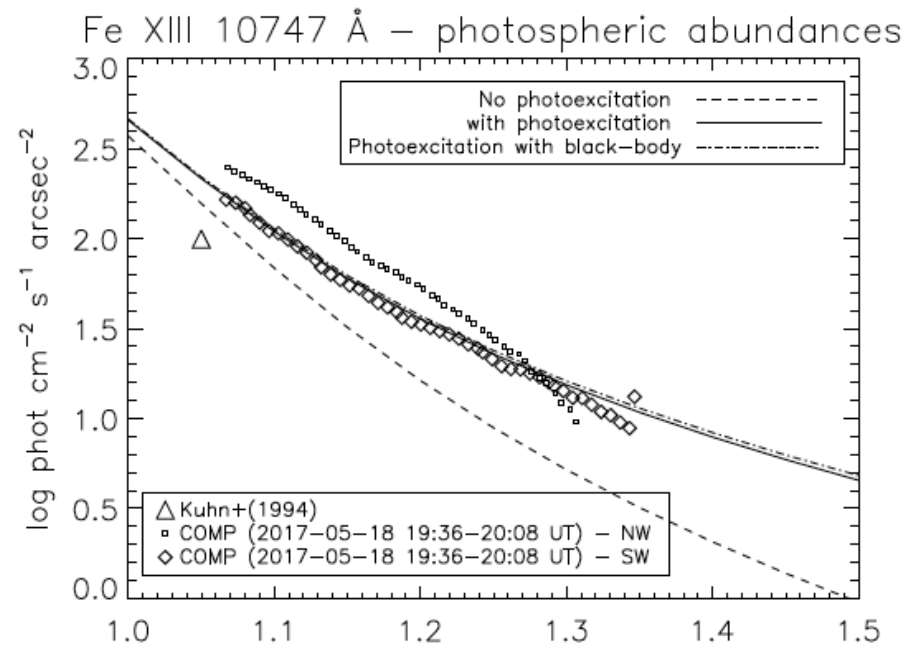
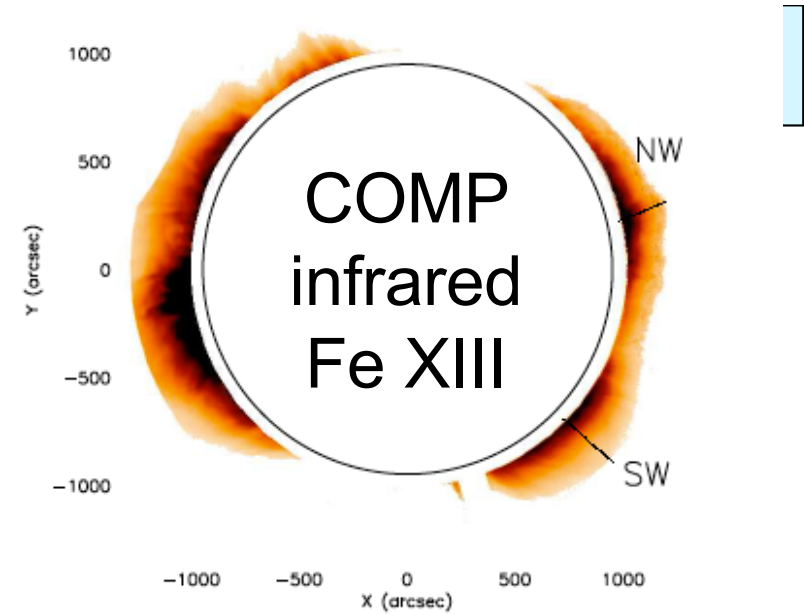
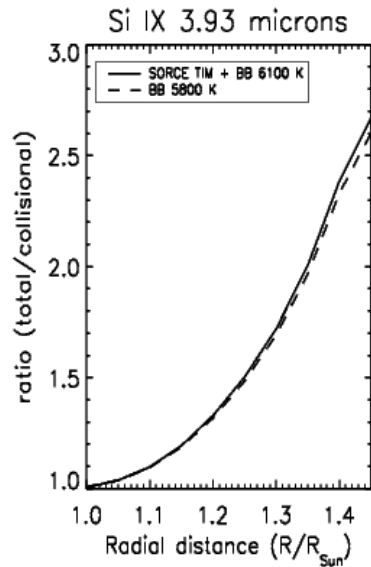
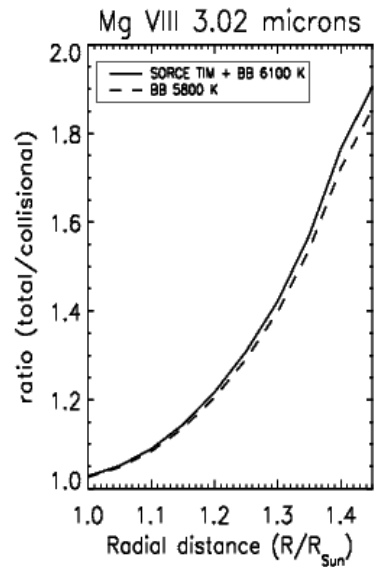
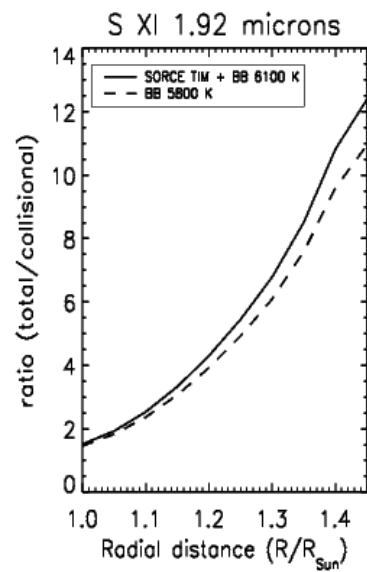
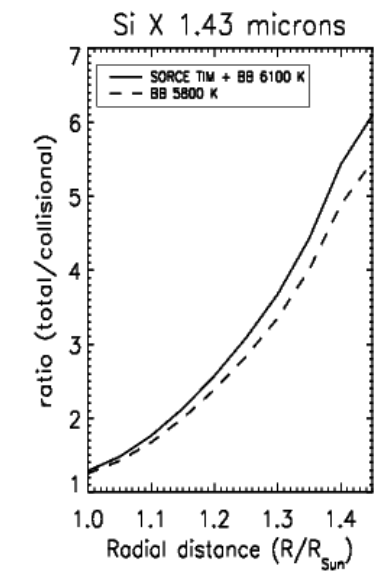


Photo-excitation is pumping the forbidden lines. Each spectral line is sensitive to Ne and disk radiation in a different way.

Del Zanna+2018

PE and coronal lines

Forbidden lines in the visible/IR are great to measure

- Ne via line ratios or actual radiances
- T (ionisation T, but also Te in combination with EUV)
- Chemical abundances
- Non-Thermal effects (line widths, non-thermal electrons)
- Magnetic field (see CoMP Science paper by Yang, Zihao+2020)

However:

- **Atomic data not simple to calculate.** Latest calculations (Del Zanna+ 2012, several A&A papers), made available to CHIANTI v.8 Del Zanna+(2015) showed increases of ~ 2 in the intensities. (cf. [Del Zanna & Mason, 2018, Liv. Rev. Sol. Phys.](#))
- **Visible/IR nearly unexplored !** (see [Del Zanna & DeLuca \(2017\)](#))
- **Modelling the signal is not trivial** (see e.g. [Del Zanna+2018, Dudik+2020](#))
- **Significant atmospheric absorption in the IR (DKIST)**

PE in CHIANTI

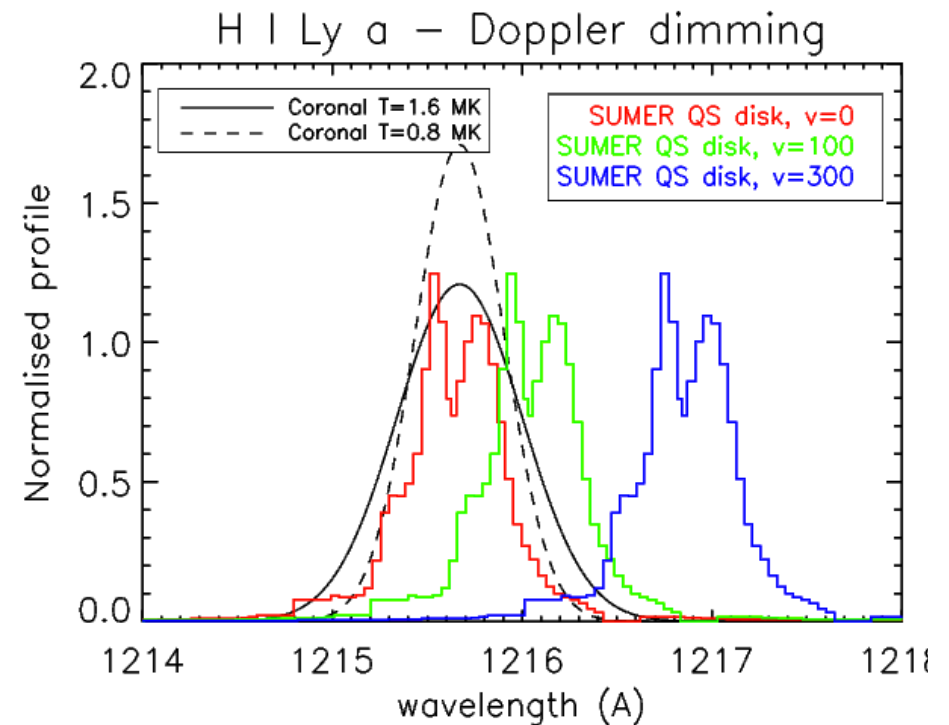
Is included as a correction to the A-value (resonant process) with a dilution W. A black-body is one option. Alternatively an user-defined energy density U file can be passed as a keyword, with RADFUNC= 'my_function,lambda, a, b'. The function must return an array of energy density with the same dimensions of the wavelength array lambda, which is internally defined.

Note: the relation between radiance and energy density is:

$$I_{\lambda} = \frac{c}{4\pi} U_{\lambda}.$$

The optional two parameters can e.g. be the velocity for the Doppler dimming and a temperature.

Doppler dimming is a powerful diagnostic to measure the plane-of-sky solar wind outflow velocity.





Break out hands-on session?

Ionization/Recombination

Processes that depopulate the upper ionization state:

(1): radiative recombination, induced $\sim N_{r+1} N_e R_{r+1}^i$

(2): radiative recombination, spontaneous $\sim N_{r+1} N_e R_{r+1}^r$

is the inverse process of photoionization (cf Badnell 2006)

(3): dielectronic recombination $\sim N_{r+1} N_e R_{r+1}^d$ (Burgess 1964,1965)

Processes that populate the upper ionization state:

(4): collisional ionization (+auto) by direct impact by electrons (inverse of three-body recombination) $\sim N_r N_e S_r^e$

(5): photoionization $\sim N_r R_r^i$

$$R_r^i \sim \int I_\nu d\Omega$$

LTE: detailed balance of (1), (2), (5) --> Saha equation

$$\frac{N_{r+1}}{N_r} = \frac{u_{r+1}}{u_r} e^{-\chi_{r+1}/kT} \frac{2 (2\pi m k T)^{3/2}}{N_e h^3} \quad u_r = \sum_n g_{r,n} \exp(E_{r,n}/kT)$$

Partition function

At low densities plasmas become optically thin and most of radiation escape, therefore processes (1) and (5) are attenuated. The Saha equation does not apply.

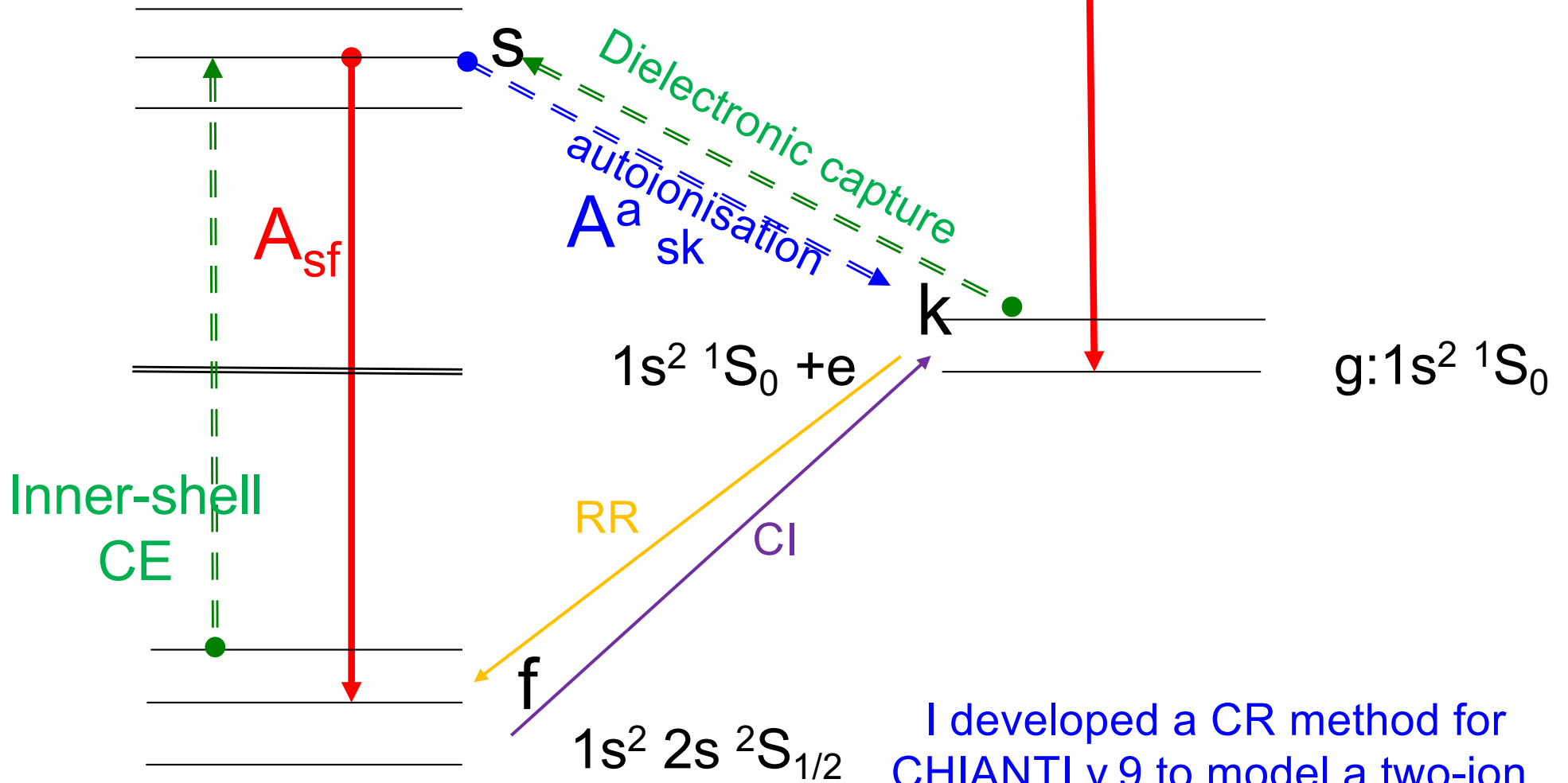
Li-like

He-like

1s 2p +e

u:1s 2p 1P_1

Autoionizing states



I developed a CR method for CHIANTI v.9 to model a two-ion, including autoionizing states

Direct Ionization (DI) by electron impact

$$S^e = \int_{v_0}^{\infty} v\sigma(v)f(v)dv \quad (\text{cm}^{-3}\text{s}^{-1})$$

direct impact coefficients for ionization by collision with electrons (much more efficient than the protons).

Dere (2007) calculated ab-initio direct-ionization (DI) cross-sections **between ground states** and compared them with available experimental data. They are available in CHIANTI.

There are discrepancies among calculations and (sparse) experiments (see also Dufresne & Del Zanna 2019,2020)

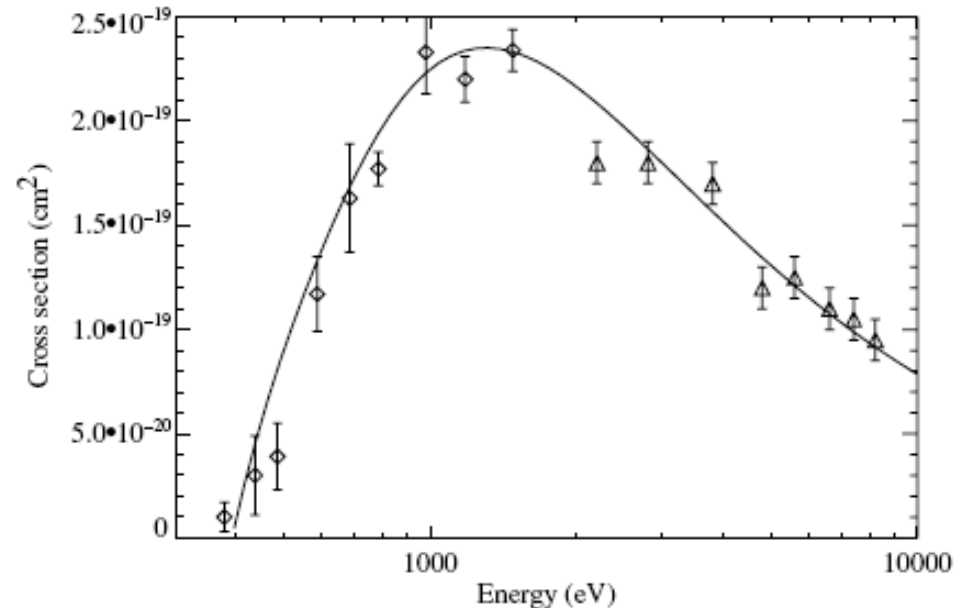


Fig.3. FAC DI cross sections for C V and the measurements of Crandall et al. (1979a) (diamonds) and Donets & Ovsyannikov (1981) (triangles, plotted with an arbitrary 10% experimental error).

CHIANTI:

```
IDL>e= 350+ 10.*indgen(1000)
plot_oi, e,ioniz_cross('c_5',e),chars=2,/xst
```

EA collisional ionisation

- Ionisation via excitation-autoionization (EA) is an additional process important for some ions at higher temperatures.
- Available calculations are sparse. See Dufresne & Del Zanna for examples and references.

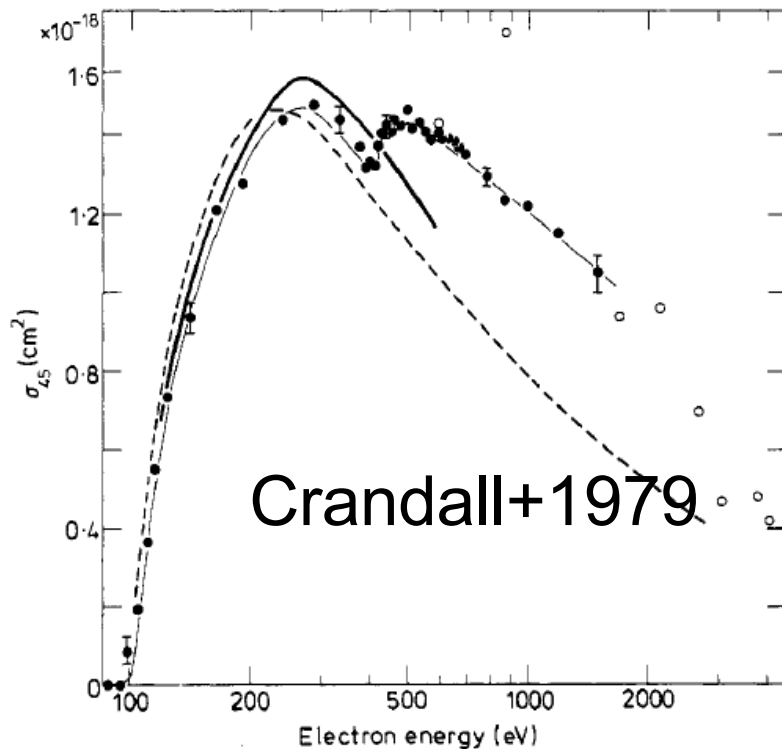


Figure 2. Cross section for electron impact ionisation of N^{4+} . The connected full circles are present data; open circles are data of Donets and Ovsyannikov (1977); broken curve is scaled Coulomb-Born of Golden and Sampson (1977); full bold curve is Coulomb-Born by Moores (1978).

CHIANTI:
IDL>e= 100+ 10.*indgen(1000)
plot_oi, e,ioniz_cross('n_5',e),chars=2,/xst

3-Body recombination

Three body recombination is the inverse process of collisional ionization. The rate coefficient for the three body recombination C_{ji}^{3B} can be obtained by applying the principle of detailed balance:

$$N_e N_i(Z^{+r}) C_{ij}^I = N_e N_j(Z^{+r+1}) C_{ji}^{3B} \quad ,$$

which leads to

$$C_{ji}^{3B} = C_{ij}^I = \frac{g_i N_e}{g_j 2} \left(\frac{h^2}{2 \pi m k T_e} \right)^{3/2} e^{-(E_i - E_f)/k T_e}$$

for Maxwellian electrons

Radiative recombination (RR)

- It is a recombination by a free electron. The photon-induced is not usually relevant. Rates are normally obtained from the PI cross-sections via detailed balance. Level-resolved (initial and final) cross-sections are needed. Very accurate values are available for H, He and several ions.
- Level-resolved rates for all ions obtained from simplified PI (background distorted-wave) are available at the UK APAP network.
- CHIANTI has the total of these rates, between ground states. Since v.9 CHIANTI introduced some level-resolved rates in two-ion models (Del Zanna, see Appendix of Dere+2019).

Dielectronic recombination (DR)

When a free electron is captured into an autoionization state of the recombining ion while a bound electron is excited. The ion is in a doubly-excited unstable state. It can then autoionize (releasing a free electron) or produce a radiative transition into a bound state (a satellite line) producing a recombined ion.

Shown by Burgess (1964,1965) to be a very important effect for the solar corona at high temperatures, typically 10 times more effective than RR

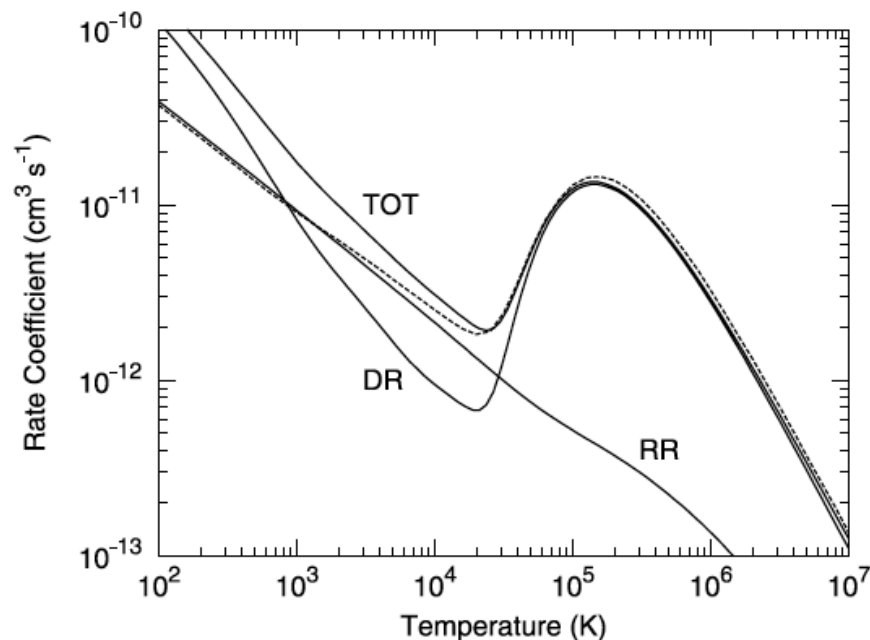


FIG. 9.—Total ground state rate coefficients for O²⁺. *Solid curves*, RR (present), DR (Zatsarinny et al. 2004), and total (DR + RR) from AUTOSTRUCTURE; *dashed curve*, total (unified DR + RR) from *R*-matrix (Nahar 1999).

Badnell (2006)

CHIANTI:

```
IDL> t= 100+ 10.*lindgen(1e6)
plot_oo, t, recomb_rate('o_3',t,$
    /radiative),chars=2,/xst
```

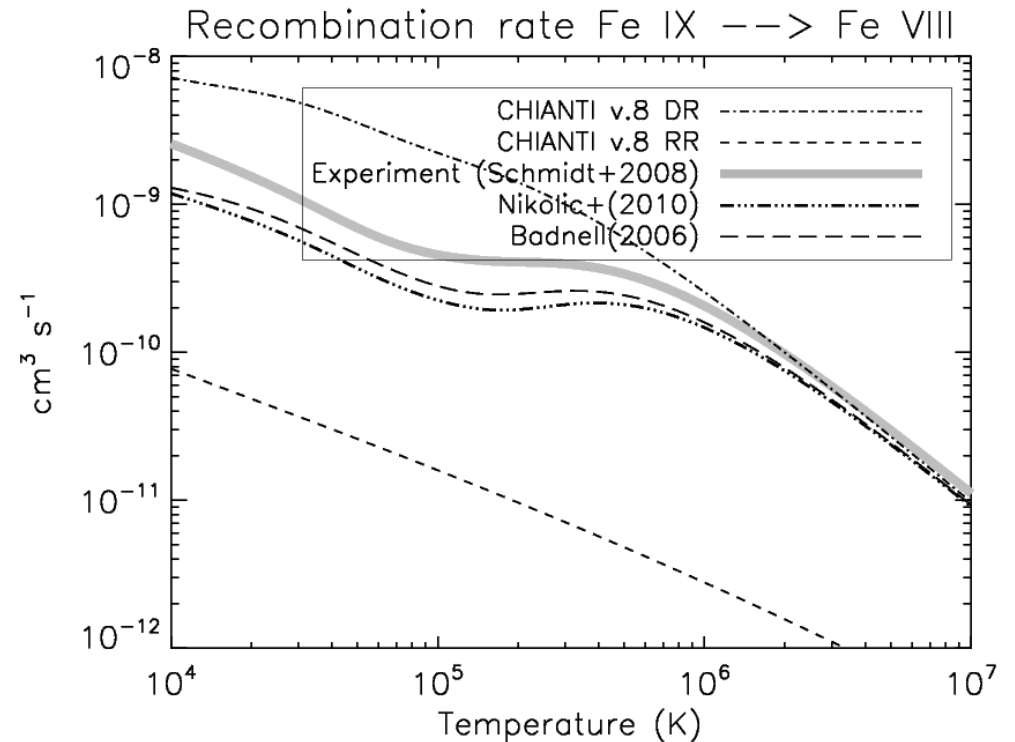
```
oplot, t, recomb_rate('o_3',t, /diel),line=2
```

DR

Level-resolved (final states) DR rates from ground and metastable states are available for many ions at the UK APAP network, calculated as part of the DR project (Badnell+1998)

CHIANTI mostly has these DR rates, but totals and from ground states only

There is a series of papers with experimental results



Density dependence on DR

High electron densities affect the DR rate coefficients in a complex way.

Discussed by
Burgess & Summers (1969)
(see also Summers 1972, 1974).

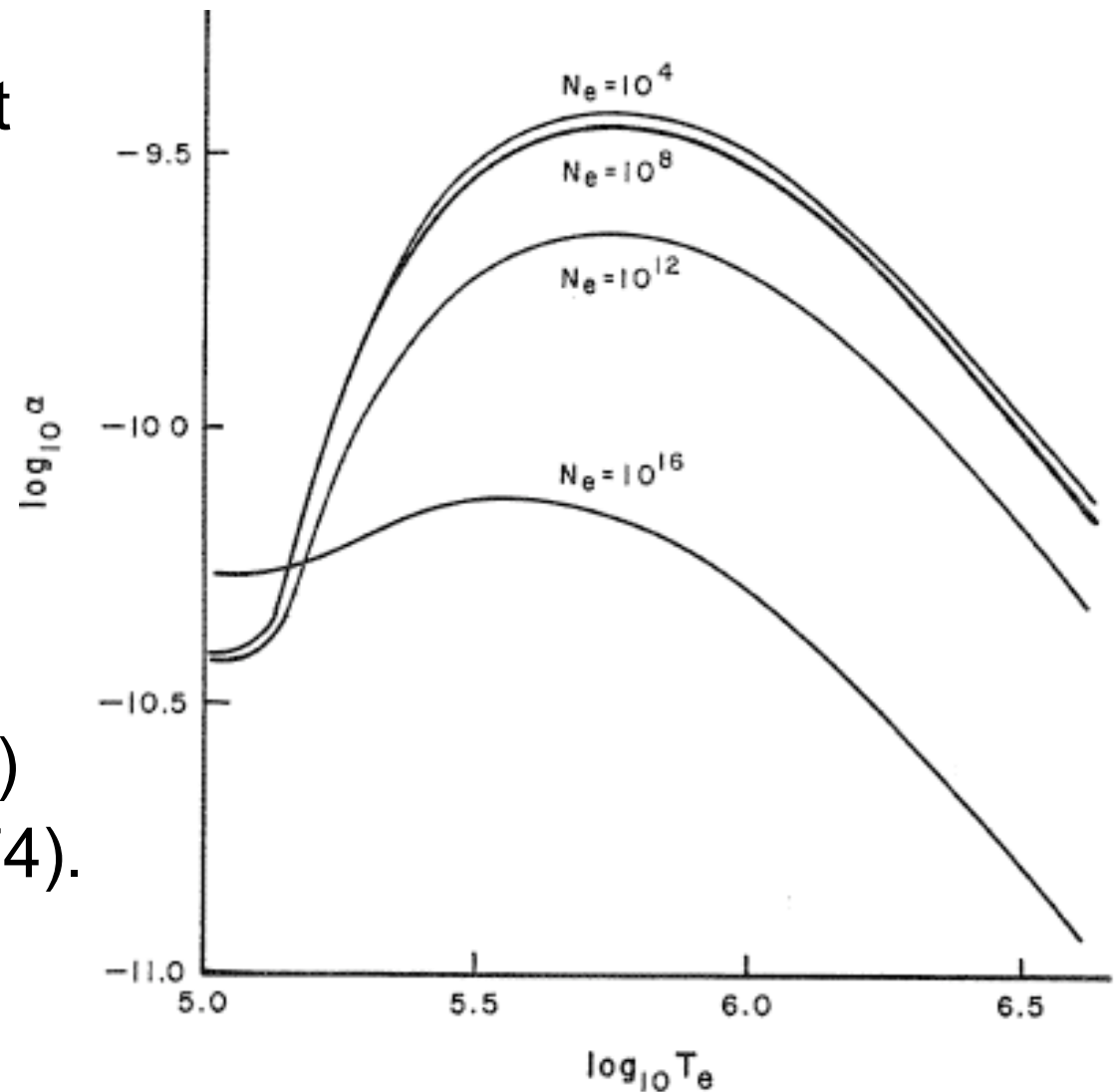


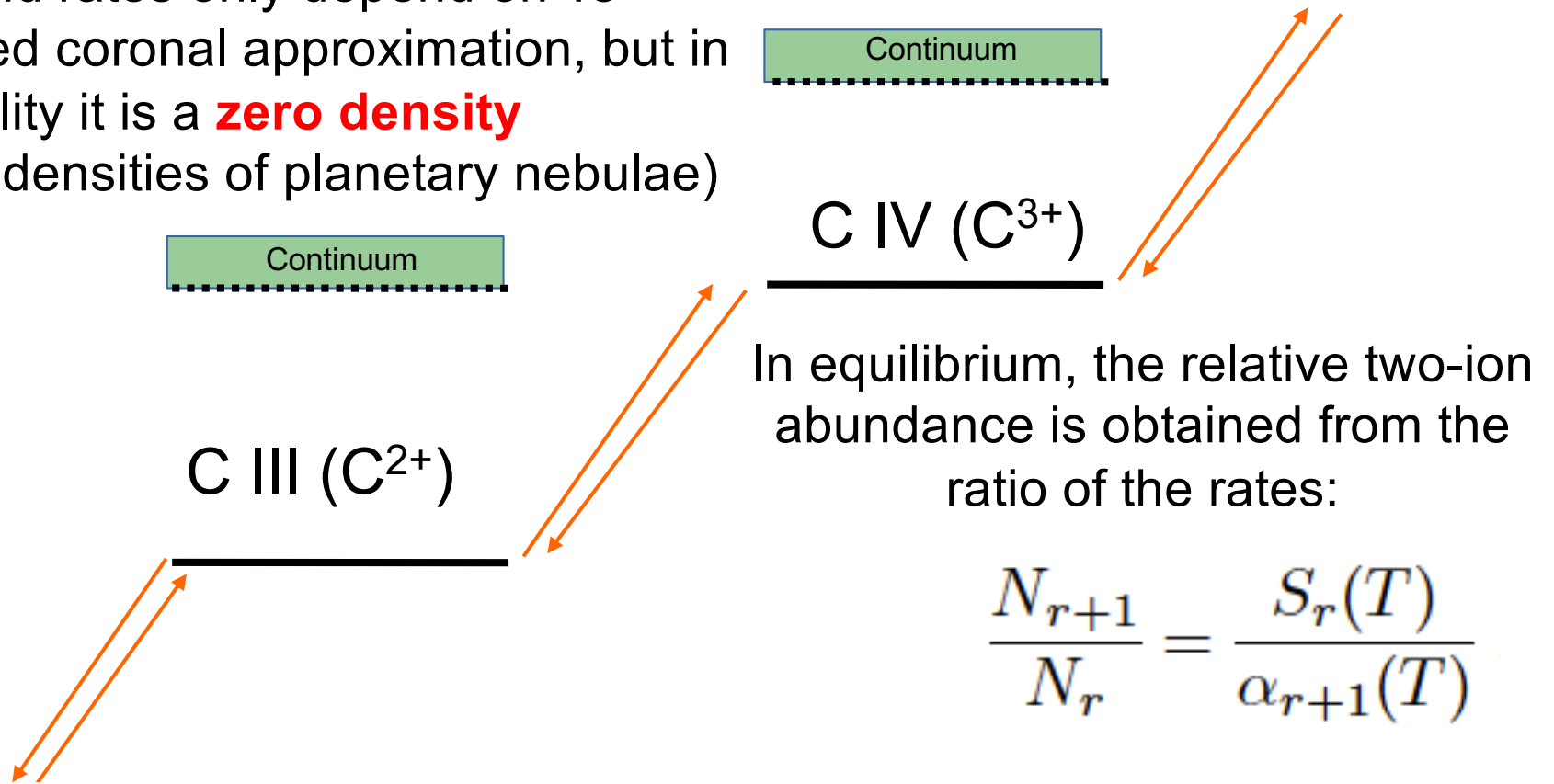
FIG. 7.—Fe⁺⁸ + e recombination coefficient

Ionisation equilibrium at zero density

$$\frac{1}{N_e} \frac{dN_r}{dt} = N_{r-1} S_{r-1} - N_r (S_r + \alpha_r) + N_{r+1} \alpha_{r+1}$$

Each ion is considered to be in the ground state and rates only depend on T_e

Usually called coronal approximation, but in reality it is a **zero density** (lower than densities of planetary nebulae)

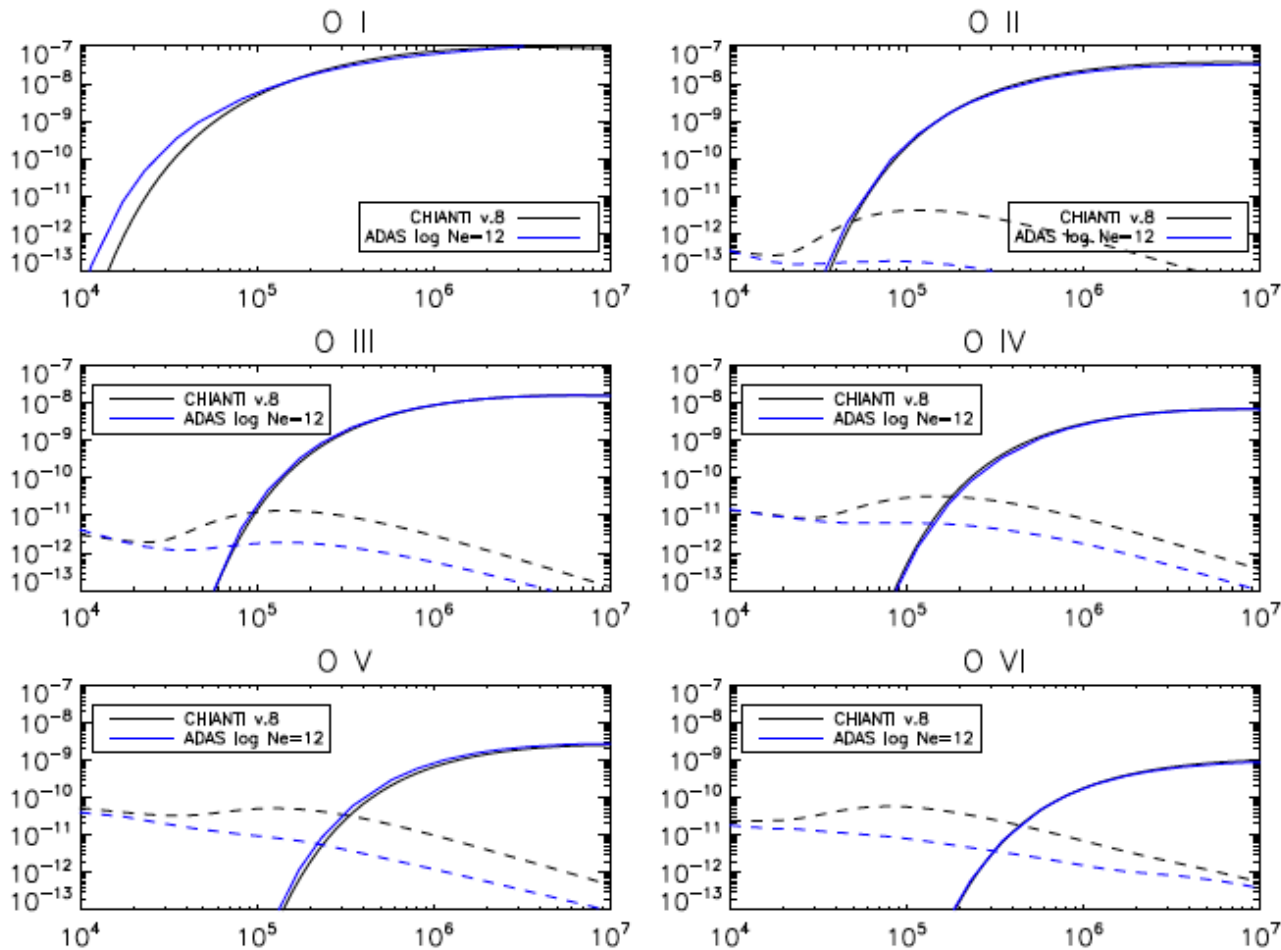


In equilibrium, the relative two-ion abundance is obtained from the ratio of the rates:

$$\frac{N_{r+1}}{N_r} = \frac{S_r(T)}{\alpha_{r+1}(T)}$$

the timescales for N_r to ionise are $1/(N_e S_r^e)$, while to recombine $1/(N_e \alpha_r)$

CI - DR

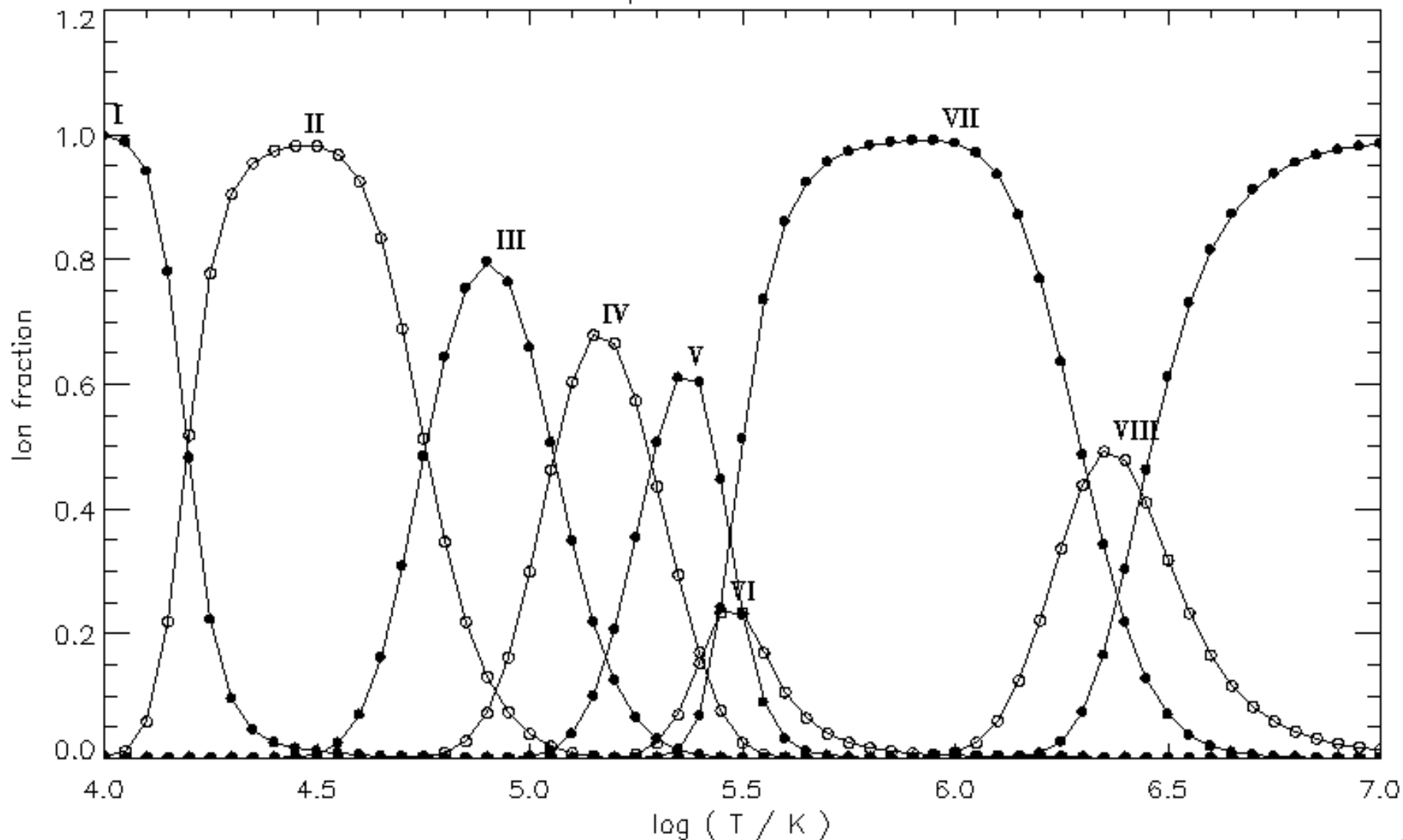


There are CHIANTI routines to plot these rates.
Note the Ne-dependence of the DR rates (ADAS)

```
t=10^[5.+0.2*indgen(10)]  
plot_io, chars=2 ,alog10(t), ioniz_rate('o_4',t)
```


Zero density

Ionization equilibrium for O – chianti



CHIANTI: IDL> plot_ioneq,'o'

ionisation T - DKIST

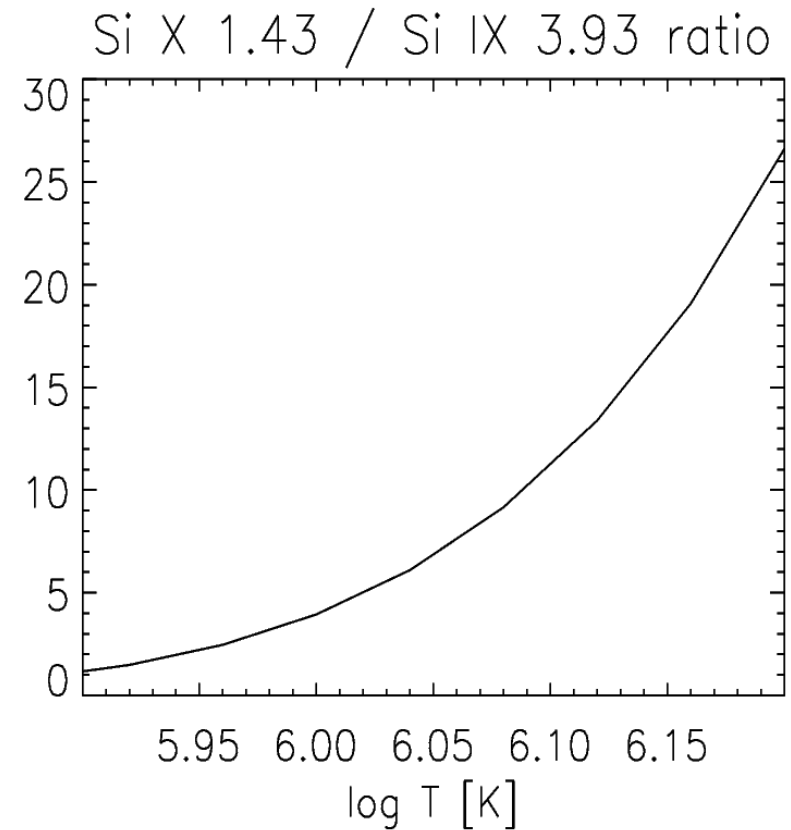
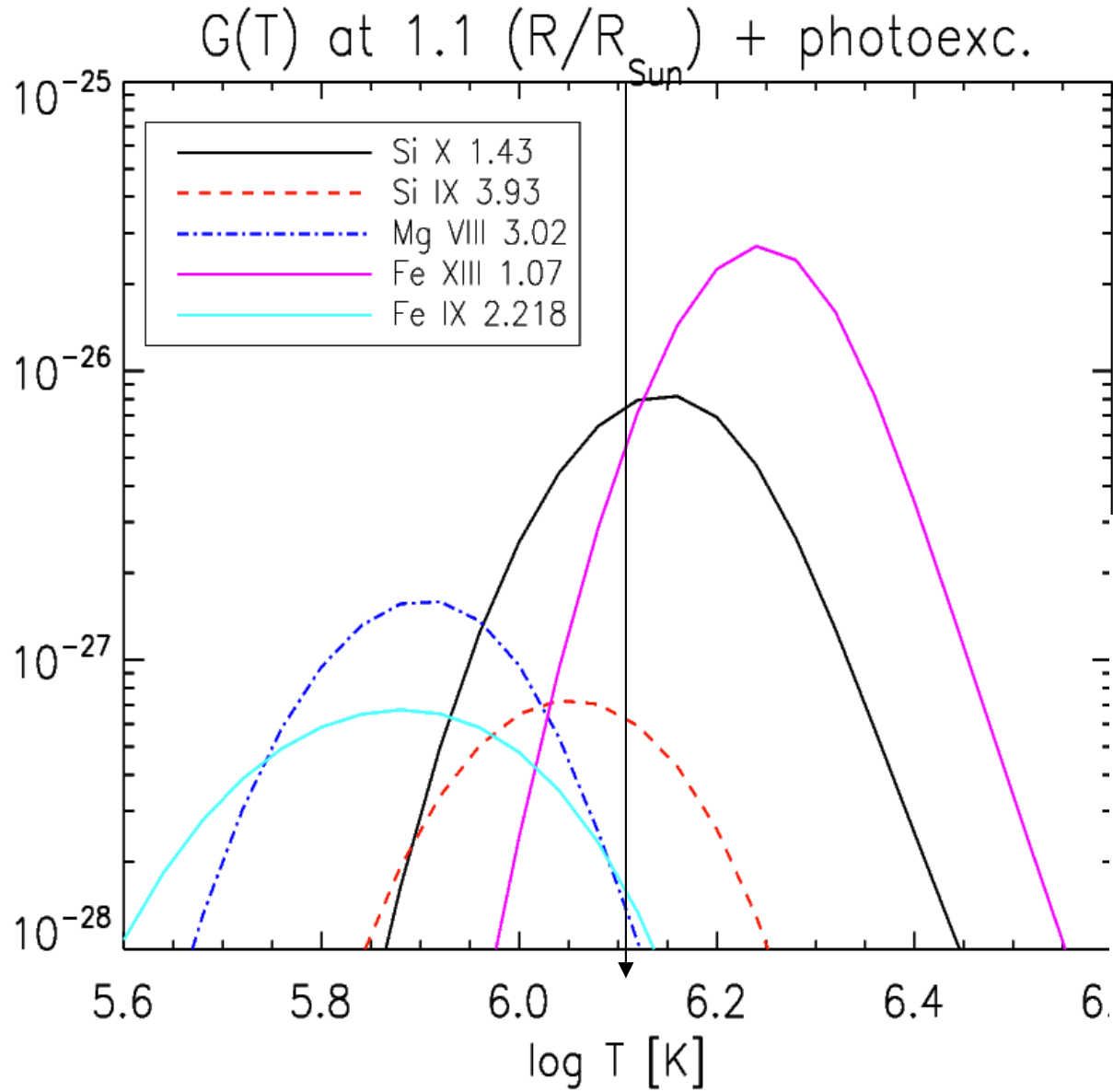
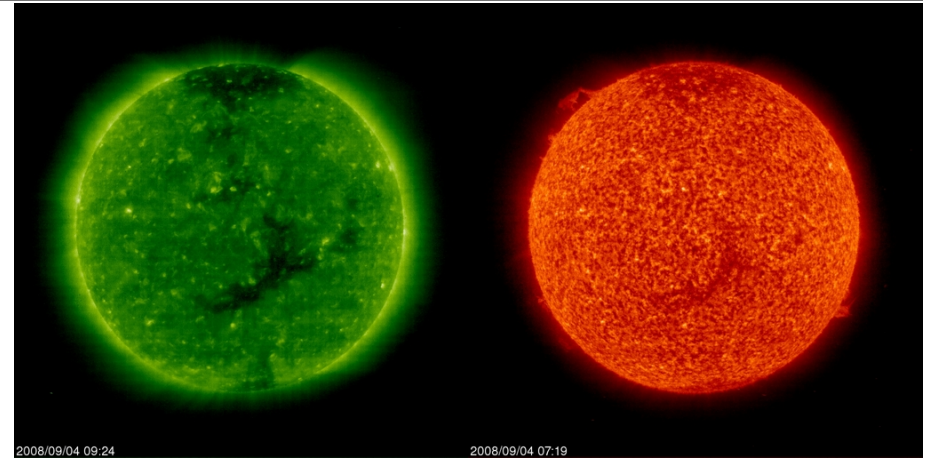
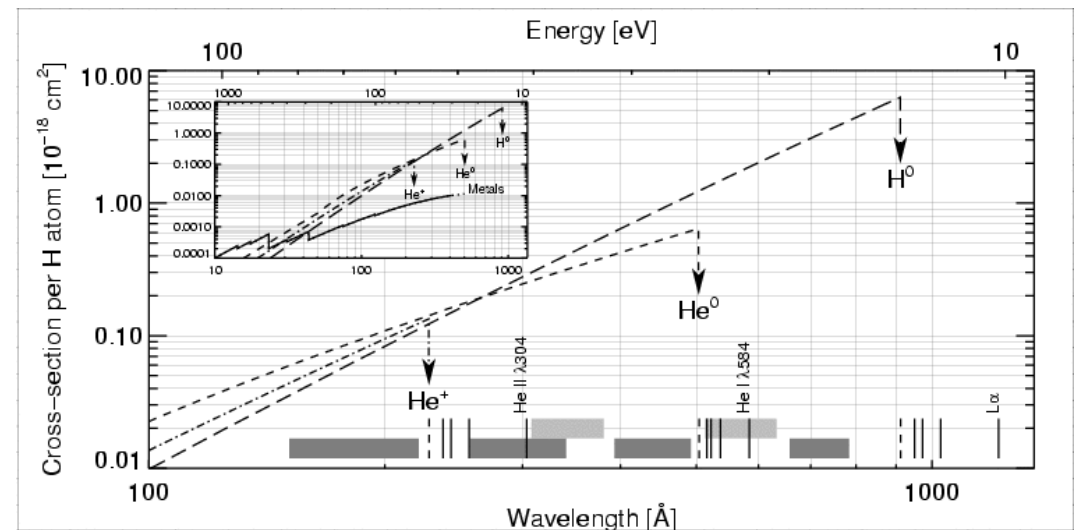
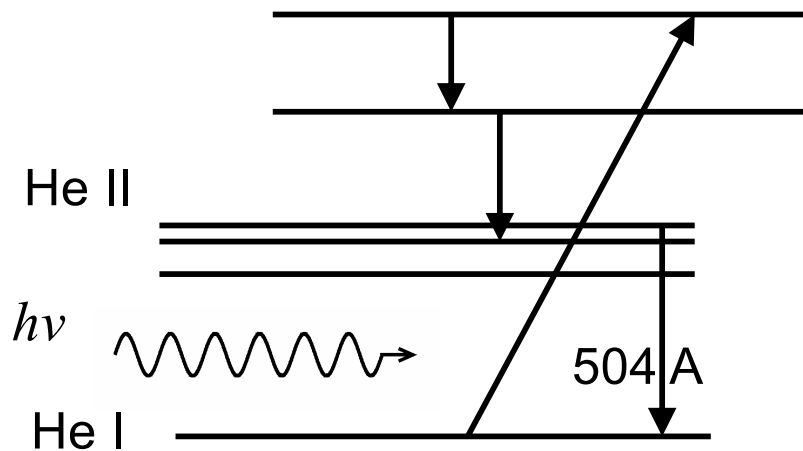


Photo-Ionisation (PI)

Photoionization is normally negligible in the lower solar corona. But it is important for example for prominences, the low transition region and the outer corona.



Important for H, He which recombine after being ionized by the coronal radiation.



Plot created on Thu Nov 15 18:48:22 2001 by cross_sections.pro

PI

$$\alpha_{ic}^{\text{PI}} = 4\pi \int_{\nu_0}^{\infty} \frac{\sigma_{ic}^{(\text{bf})}(\nu)}{h\nu} J_{\nu} d\nu$$

$$J_{\nu} = \frac{\Delta \Omega}{4\pi} \bar{I}_{\nu} = W(r) \bar{I}_{\nu} ,$$

$$W = \frac{1}{2} \left[1 - \left(1 - \frac{1}{r^2} \right)^{1/2} \right]$$

$$r = \frac{R}{R_{\star}}$$

Data are available from the Opacity Project (Seaton+), the UK APAP Network and the literature.

Only cross-sections from ground states are available in CHIANTI.

PI from populated excited states can be very important, see e.g. Dufresne, Del Zanna+ (2019,2020,2021) and for He Del Zanna+2020

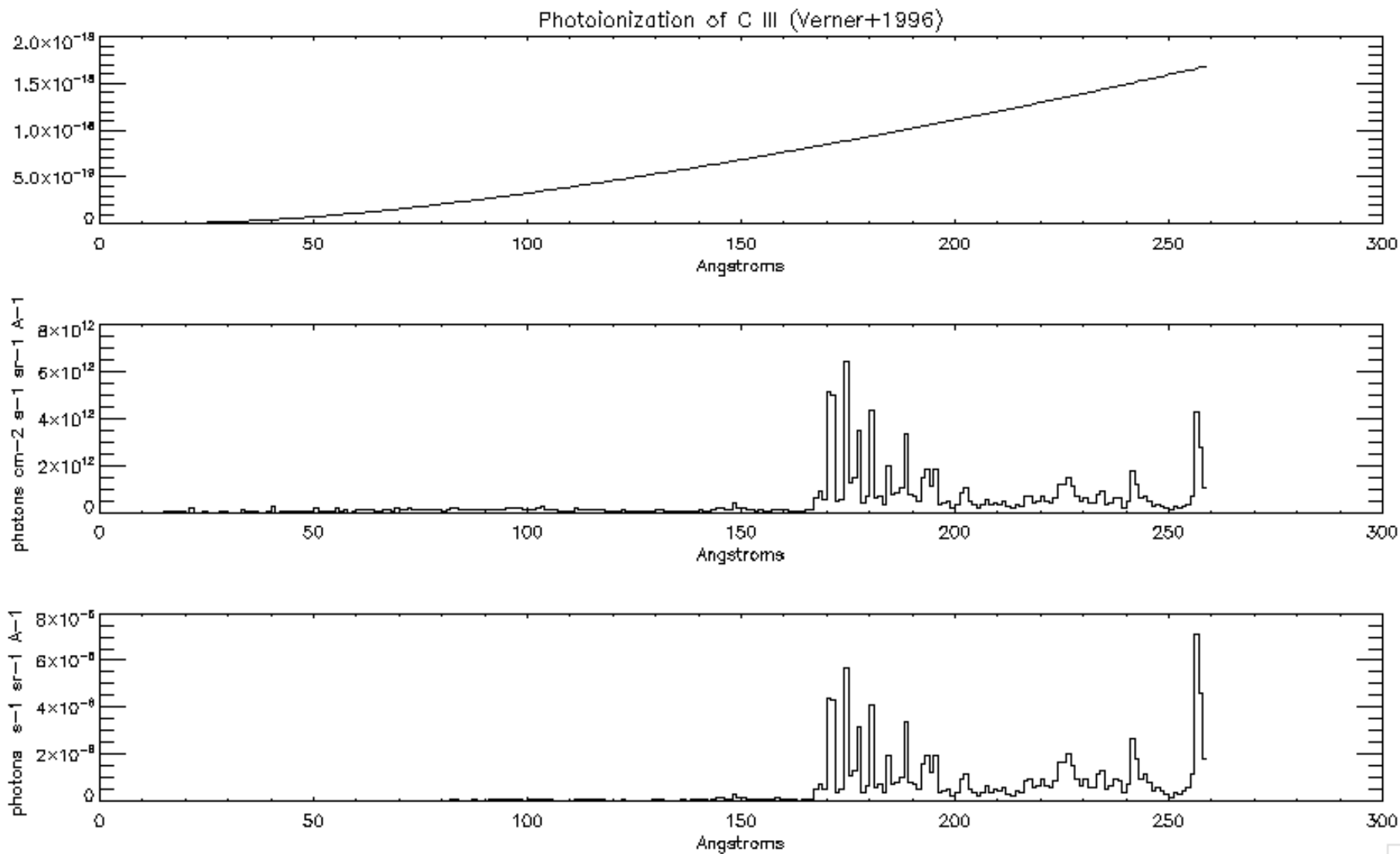
C III PI for QS

lam=1.+lindgen(300)

plot, lam, verner_xs(6,3, lam), chars=2, ytit='10!U-18!N cm!U-2!N', \$

xtit='Wavelength (Angstroms)

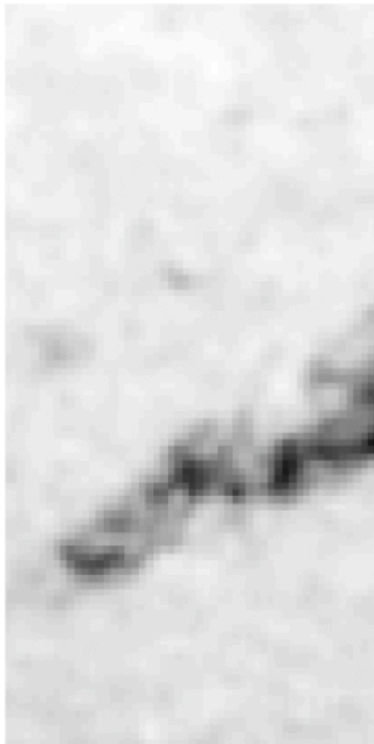
The spectrum close to the PI edge is important.



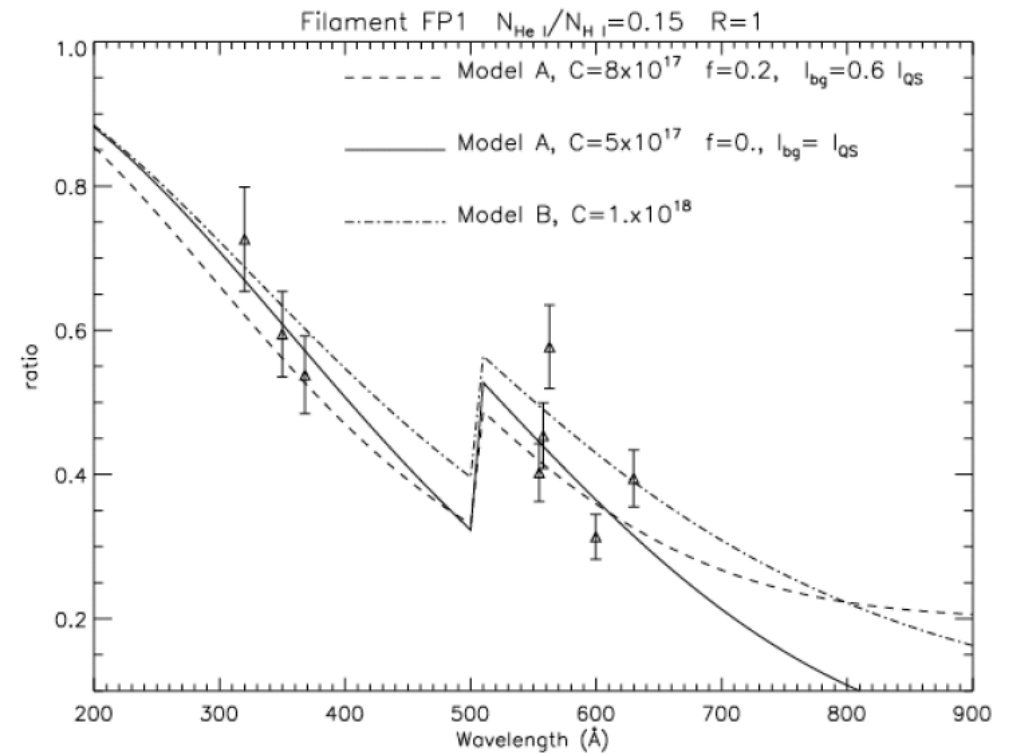
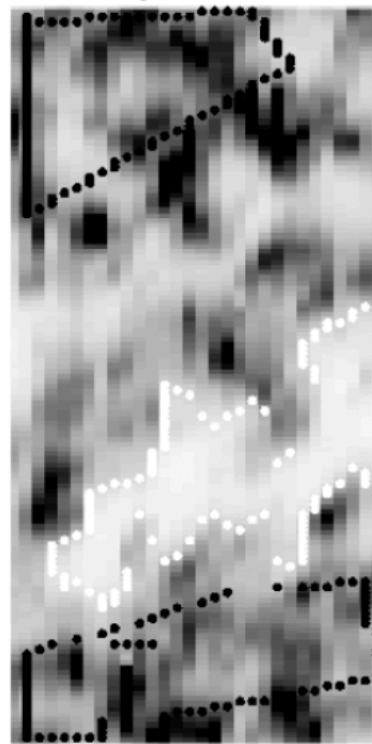
PI

PI is also important when H,He absorbs background emission as in filaments, spicules, etc.

Meudon Observatory 06:40:00 UT

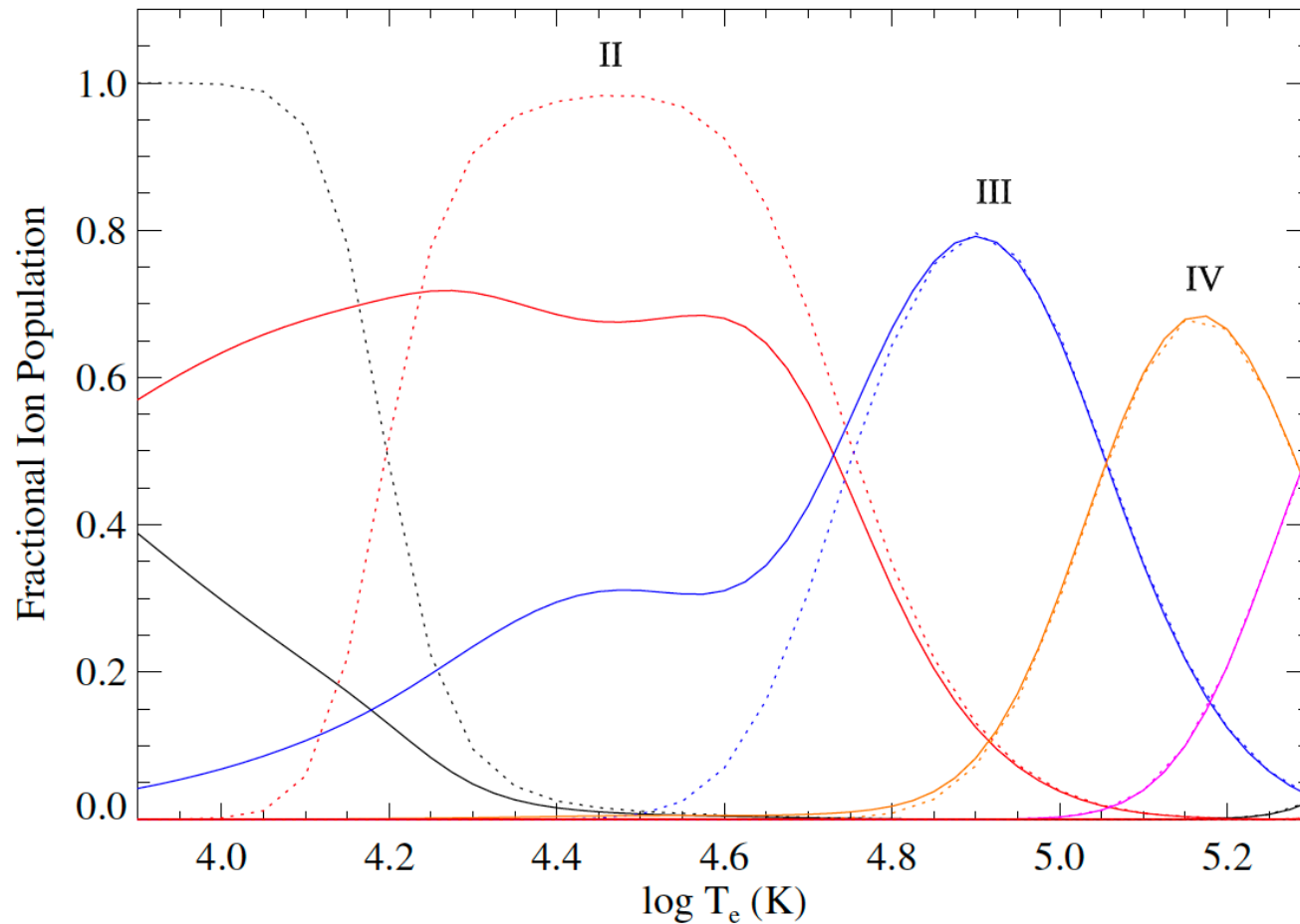


O V 629.7 Å log T = 5.4 07:00:56.938 UT



Del Zanna+2004

PI for O

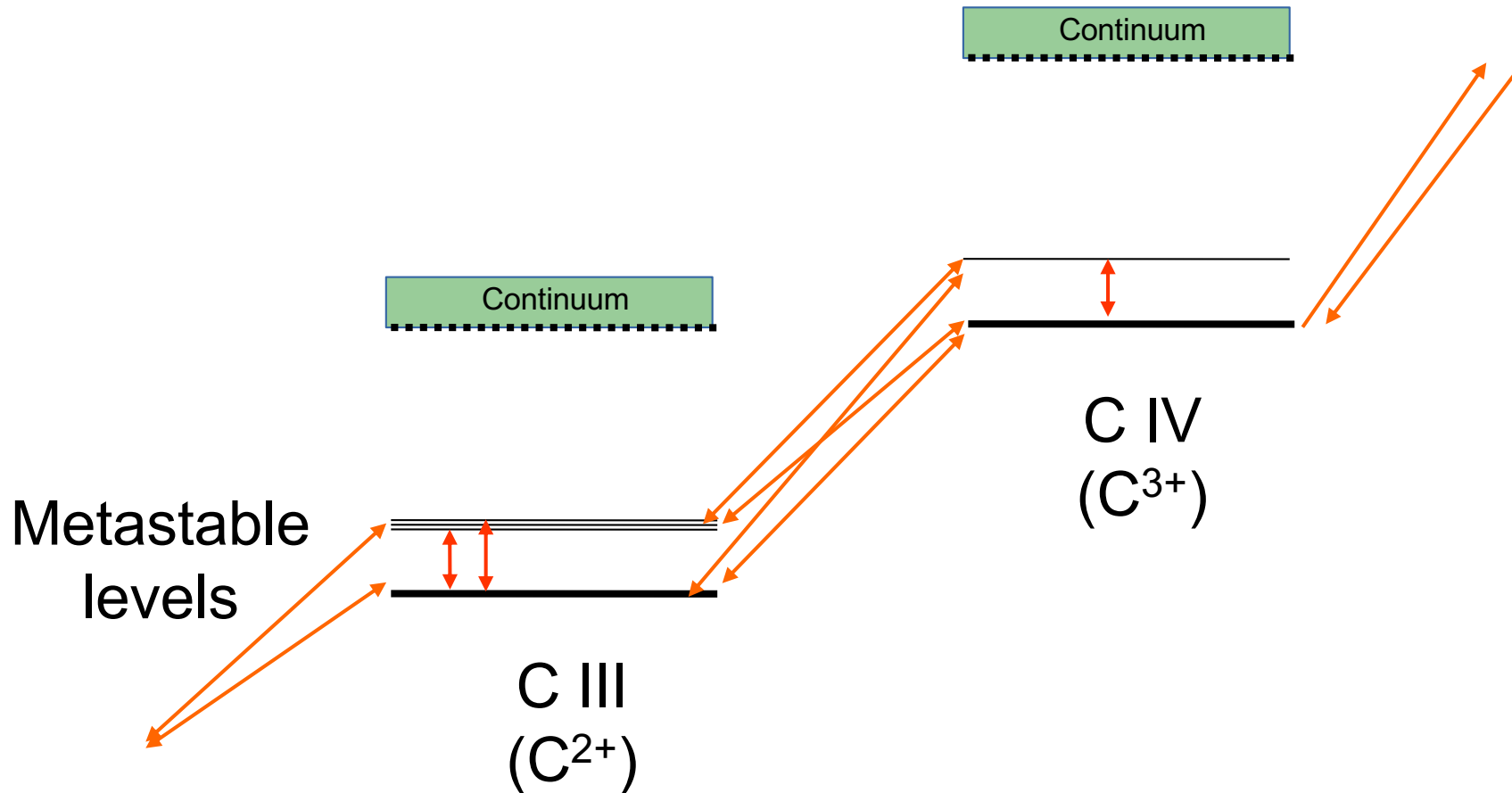


Dufresne Del Zanna
Badnell (2020)

Figure 3. Coronal approximation for oxygen: solid line - this work including photo-ionisation only, dotted - CHANTI v.9. Ions are highlighted by Roman numerals and different colours.

CR Modelling: Level-Resolved

Two main effects: 1) suppression of DR and 2) CI from metastable levels. A new approach – building a level-resolved matrix with all the main levels for all the ions, see Dufresne & Del Zanna (2019,2020) with some approximations. Full CRMs are being built.



Combined CI and DR effects - C

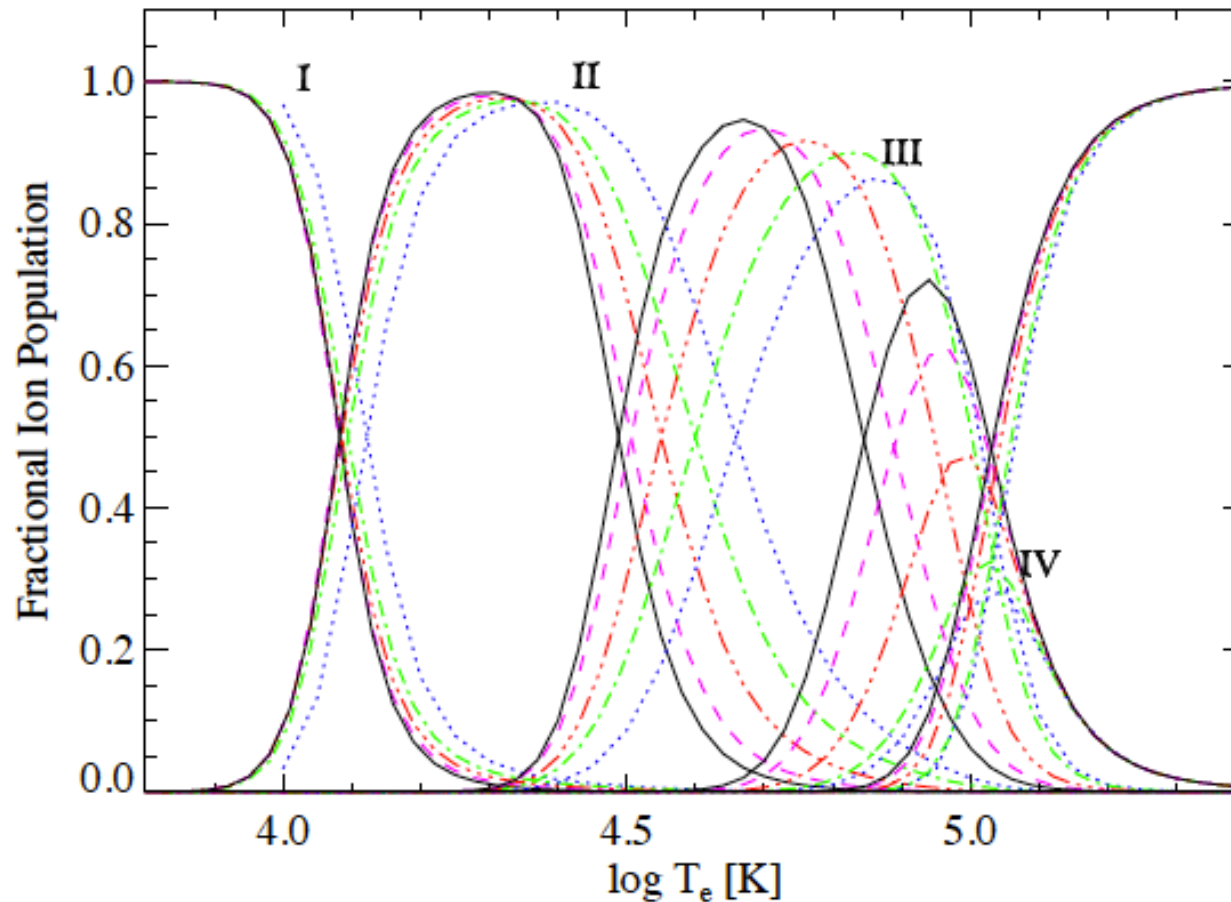


Fig. 12. Combined effect of density on level-resolved, electron impact ionisation and DR suppression in the CR model; blue dotted line - CHIANTI v.8, green dash-dotted - this work at 10^4 cm^{-3} density, red dash-dot-dotted - 10^8 cm^{-3} , purple dashed - 10^{10} cm^{-3} , black solid - 10^{12} cm^{-3} .

Dufresne &
Del Zanna

Combined CI and DR effects - O

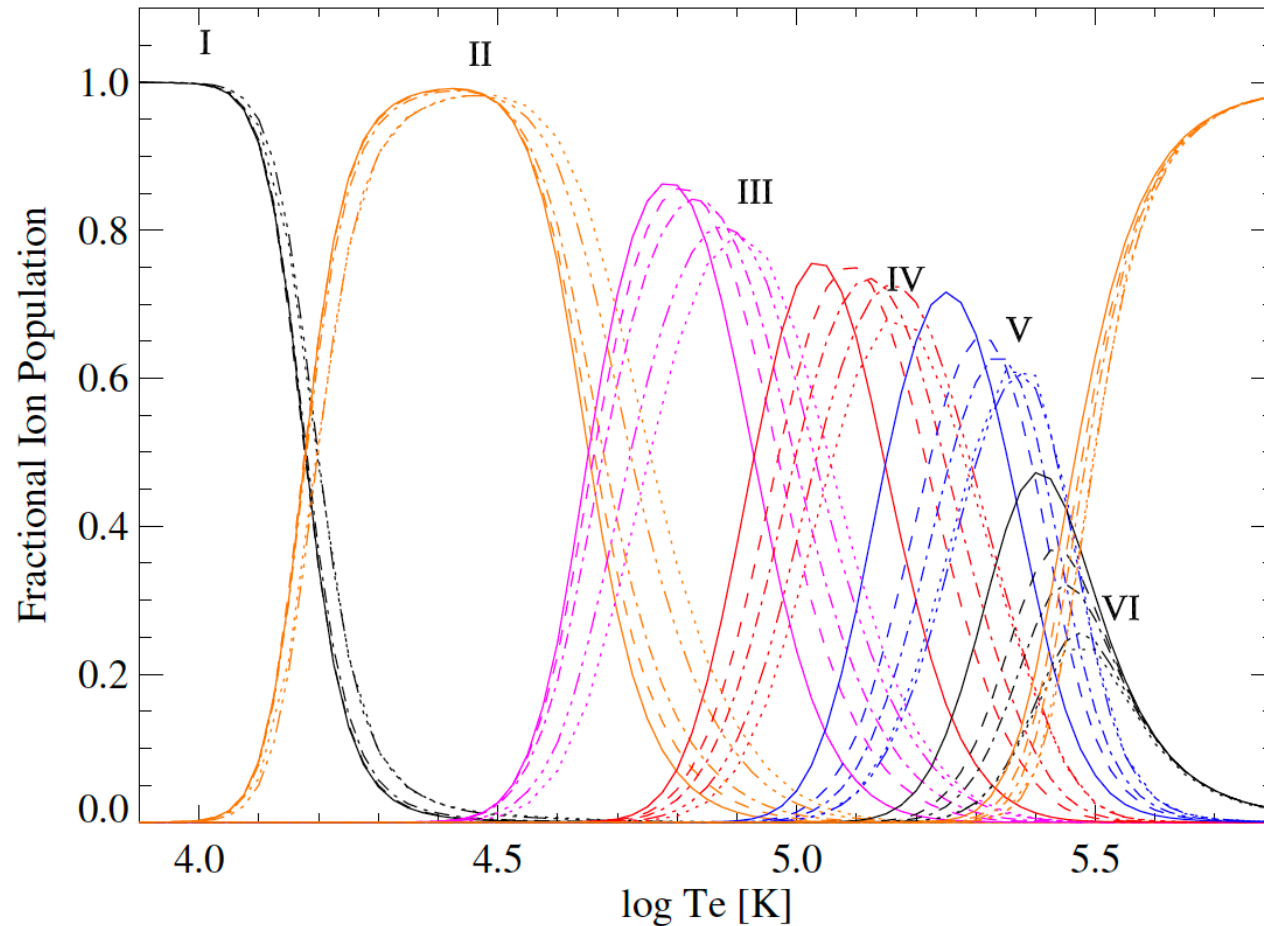


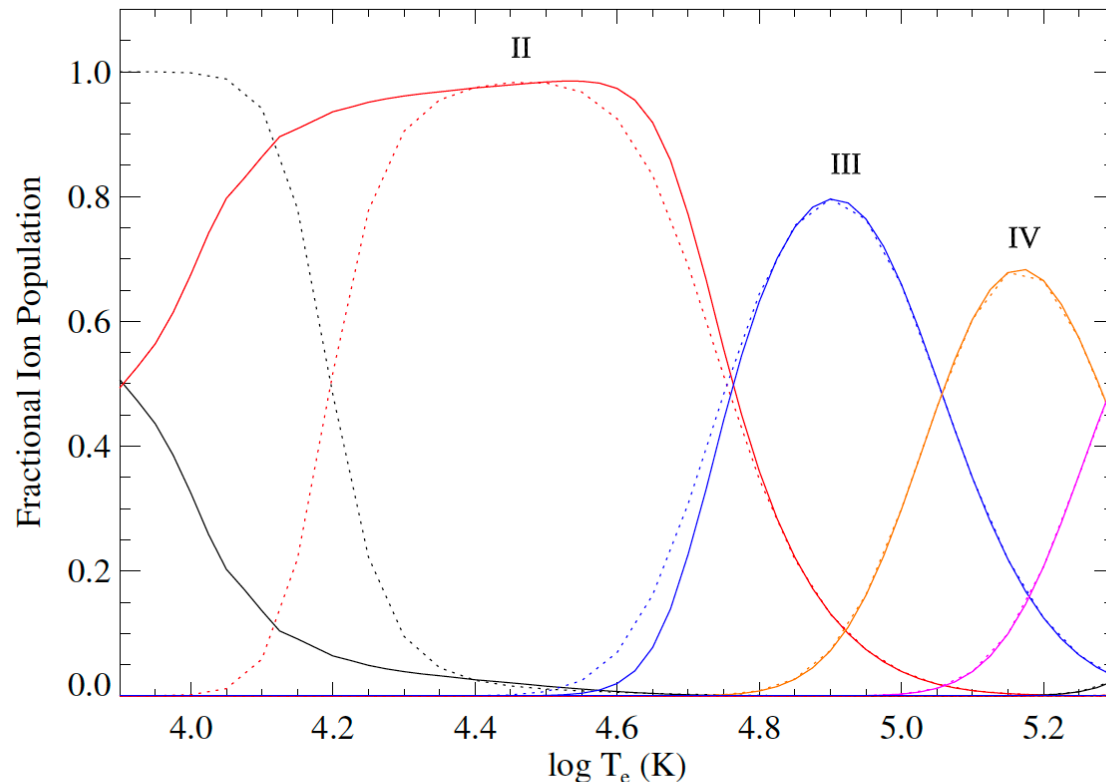
Figure 9. Final ionisation equilibrium of oxygen at various densities; dotted line - CHIANTI, dash-dot-dotted - this work at 10^4 cm^{-3} density, dash-dotted - 10^8 cm^{-3} , dashed - 10^{10} cm^{-3} , solid - 10^{12} cm^{-3} . Individual charge states are highlighted by Roman numerals and different colours.

Dufresne
Del Zanna,
Badnell

Charge tranfer (CT)

Very important for low charge states of some elements. Mostly due to interaction (both ways) with neutral H,He, which themselves are difficult to model (see issues with He in the corona in Del Zanna+2020).

Availability of CT rates is sparse.



O results for quiet Sun

Dufresne, Del Zanna
Badnell 2021

Figure 4. Coronal approximation of oxygen: solid line - this work including charge transfer only, dotted - CHANTI v.9. Ions are highlighted by Roman numerals and different colours.

Ne-dependent CI, DR, + PI, CT

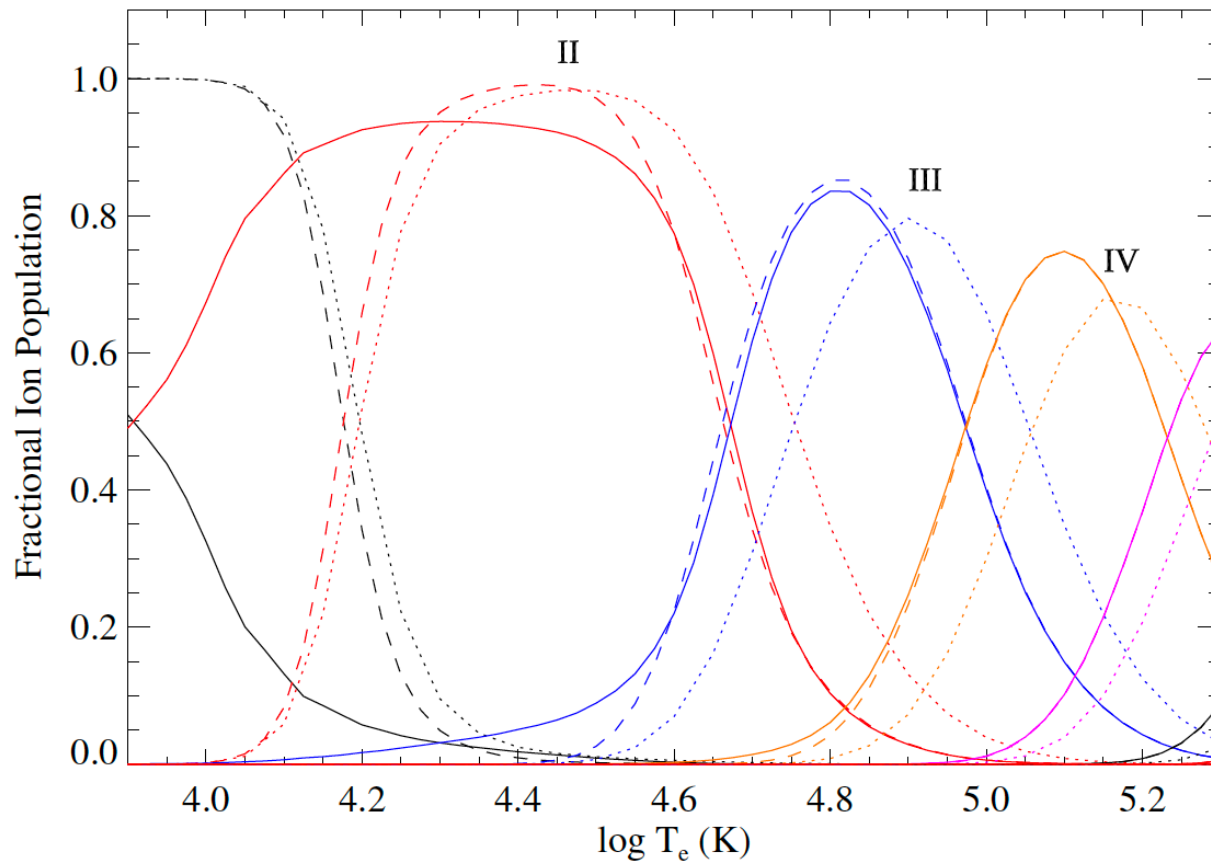


Figure 5. Ionisation equilibrium of oxygen: solid line - full model, dashed - Dufresne et al. electron collisional model, dotted - CHIANTI v.9. Ions are highlighted by Roman numerals and different colours.

O results for quiet Sun

Dufresne, Del Zanna
Badnell 2021

Ionisation (time dependent)

Time-dependent ionization can substantially affect ion populations. Important in the solar wind (low Ne) and in active regions (whenever the heating timescales are short).

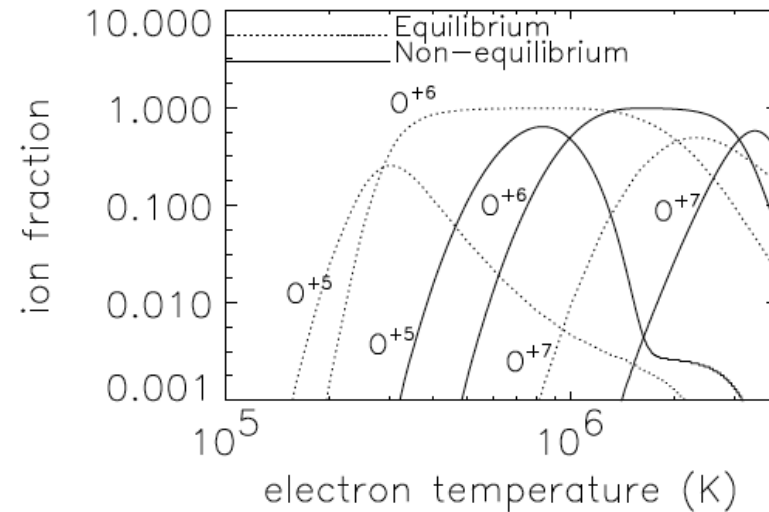


FIG. 5.—Oxygen ion fractions as a function of temperature for the equilibrium case (*dotted lines*) and when a constant flow speed of 30 km s^{-1} is assumed (*solid lines*).

Esser+1998

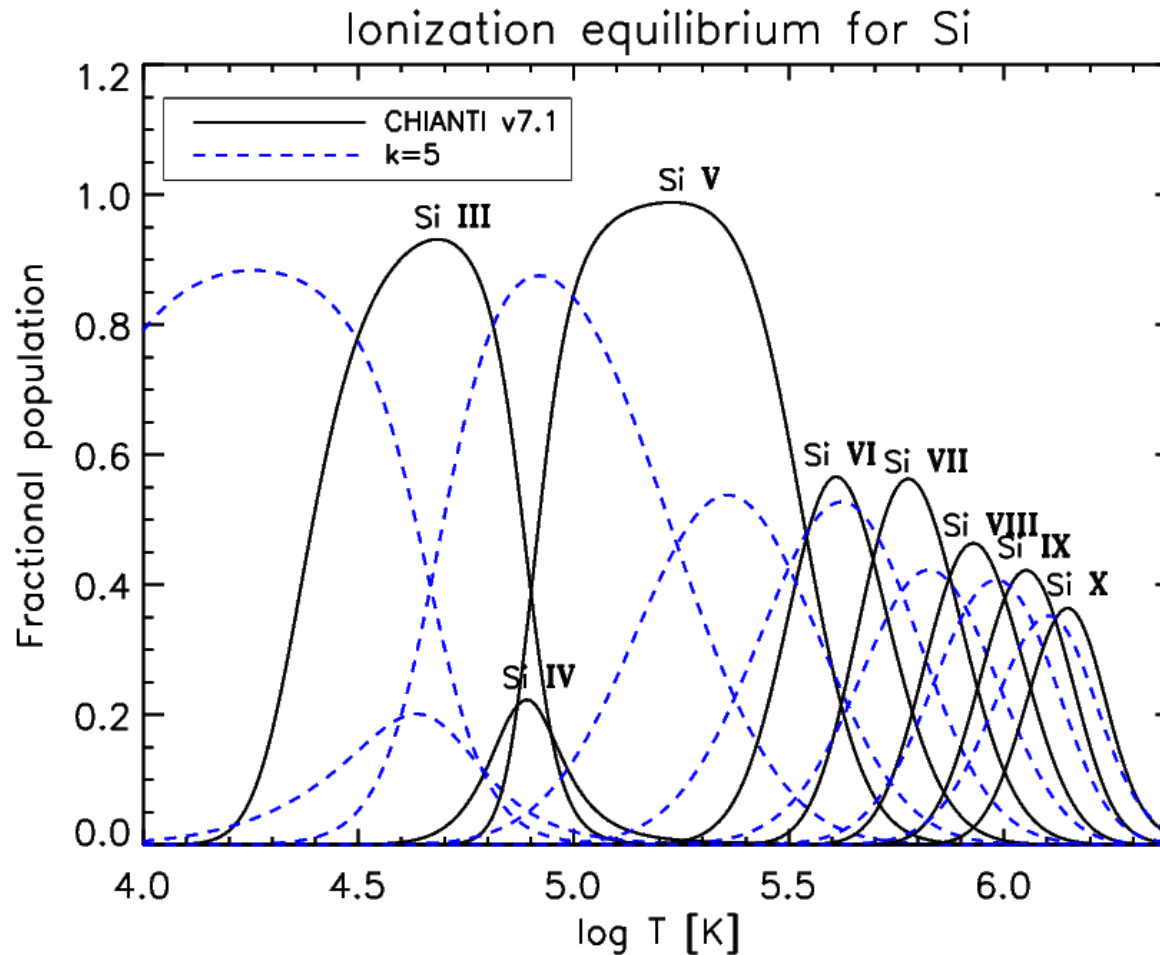
Effects on spectral lines can be studied e.g. with the HYDRAD code (see examples in Bradshaw, Del Zanna & Mason (2003):

The radiative losses are calculated by solving the time-dependent ionization, including the advective term (the rate coefficients I and R are functions of temperature)

$$\frac{\partial}{\partial t}(N_r) + \frac{\partial}{\partial s}(N_r v) = N_e(N_{r-1}S_{r-1}^e - N_r(S_r^e + R_r) + N_{r+1}R_{r+1}) \quad N(X) = \sum_r N_r$$

Ionization equilibrium with NMED

A fully self-consistent model is very complex and is not available. Approximations used within the KAPPA package show the effects for strong NMED:



DEM / EM

$$I(\lambda_{ji}) = \int_h N_e N_H A(X) G(N_e, T, \lambda_{j,i}) dh$$

If there is a continuous distribution of densities and temperatures along the line of sight, then we can define the column Differential Emission Measure (DEM) and total emission measure EM:

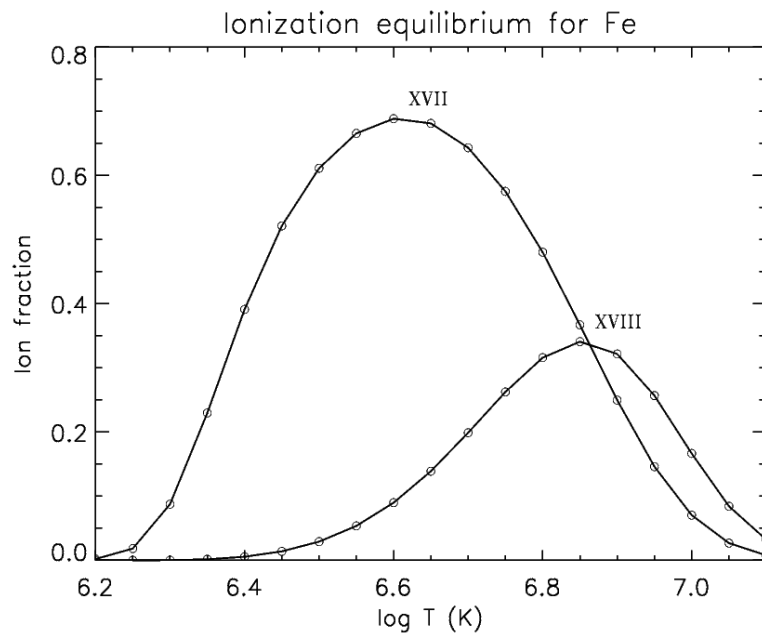
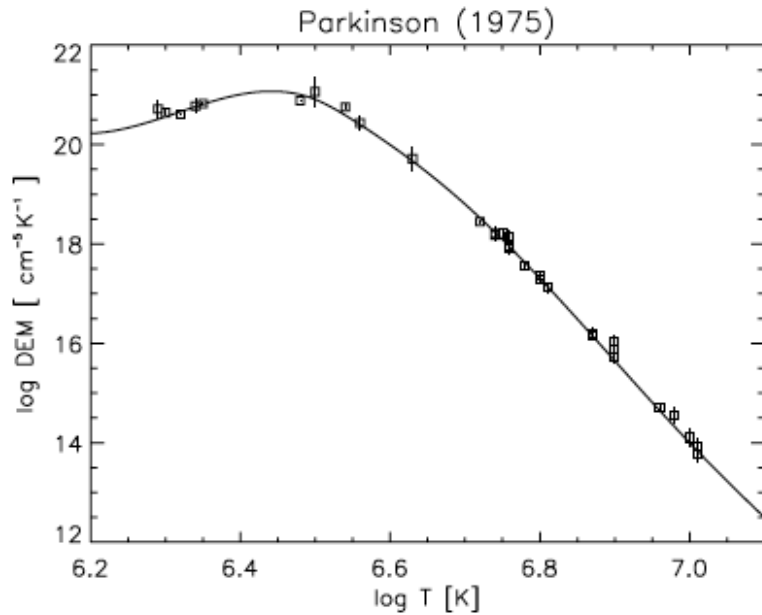
$$\begin{aligned} \text{DEM}(T) &\equiv N_e N_H \frac{dh}{dT} \quad [\text{cm}^{-5} \text{K}^{-1}] \\ \text{EM} &\equiv \int_h N_e N_H dh = \int_T \text{DEM}(T) dT \quad [\text{cm}^{-5}] \end{aligned}$$

$$I(\lambda_{ij}) = A(X) \int_T G(T) \text{DEM}(T) dT$$

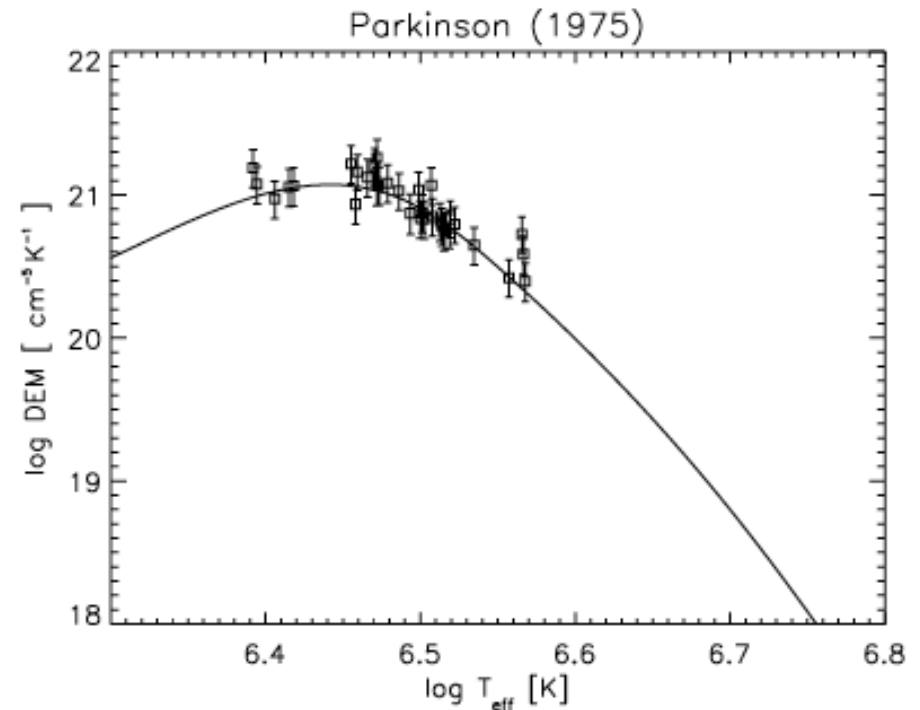
The DEM(T) gives an indication of the amount of plasma along the line of sight that is emitting the radiation observed and has a temperature between T and T+dT. It is also used for measuring chemical abundances. Note that in the literature many different definitions of DEM, EM, and approximations can be found, for example:

$$\begin{aligned} \text{DEM}(T) &= N_e^2 (dT/dh)^{-1} \\ \text{EM} &= \int N_e^2 dh = \int \text{DEM}(T) dT \end{aligned}$$

Effective Te (Del Zanna & Mason 2014)



λ_{obs}	I_{obs}	T_{max}	T_{eff}	R	Ion
13.55	0.163	6.98	6.47	0.59	Ne IX
13.70	0.334	6.56	6.47	0.92	Ne IX
13.82	0.174	6.80	6.51	1.01	Fe XVII
14.21	0.098	6.90	6.57	0.85	Fe XVIII
					Fe XVIII
16.78	1.9	6.74	6.50	1.15	Fe XVII



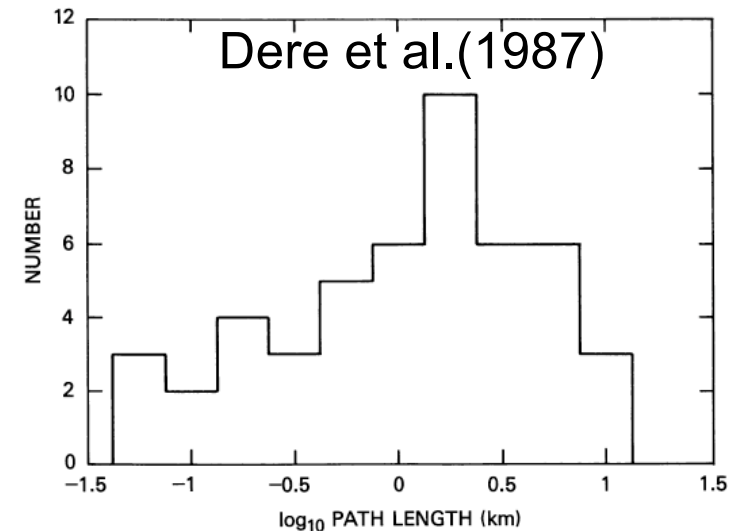
$$T_{\text{eff}} = \int G(T) \text{DEM}(T) T \, dT / \int G(T) \text{DEM}(T) \, dT$$

Spectroscopic filling factors

If one assumes an homogeneous slab of plasma of thickness dh , once elemental abundances are known, and a line intensity measured, one can obtain an average $\langle EM \rangle = \langle N_e^2 \rangle dh$ from which the path length dh can be estimated once N_e is measured from e.g. a line ratio. Dere et al.(1987) used HRTS transition region C IV intensities and O IV densities from a line ratio to obtain path lengths of 0.1-10 km, much smaller than the observed sizes of the spicular structures (2400 km).

In other words, average $\langle N_e^2 \rangle$ obtained from the measured EM and estimated dh are much smaller than expected from the averaged densities obtained from line ratios.

$$f = \frac{\langle N_e^2 \rangle}{\langle N_e \rangle^2} = \frac{\langle EM \rangle}{\langle N_e \rangle^2 \Delta h}$$



For coronal loops is close to 1 (SOHO/CDS: Del Zanna 2003, Del Zanna & Mason 2003; Hinode/EIS: Tripathi et al. 2009)

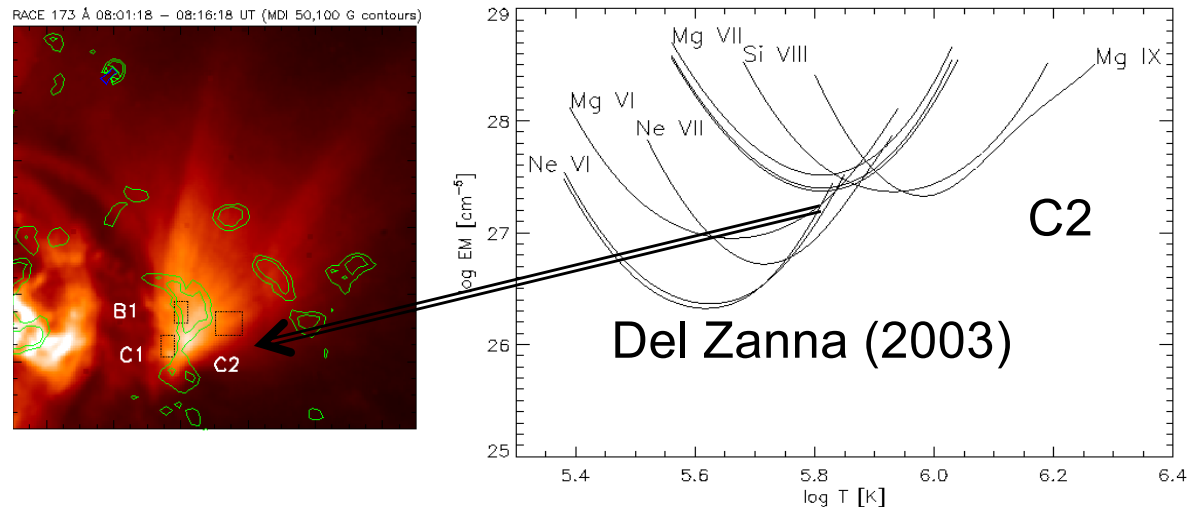
EM loci

$$I_{ji} = \int f Ab(Y) G_{ji}(T_e) N_e^2 dh \cong$$

$$Ab(Y) \int G_{ji}(T_e) dT < \int N_e^2 dh / dT >$$

$$EM = \int N_e(h)^2 dh \quad \frac{I_{obs}}{Ab(Y) G_{ji}(T_e)}$$

Del Zanna (2003) re-introduced the EM loci method (Strong 1978) to AR loops. **Isothermal !**



For each line and temperature T_i the value $I_{ob} / G(T_i)$ represents an upper limit to the value of the emission measure at that temperature, assuming that all the observed emission I_{ob} is produced by an isothermal plasma at the temperature T_i .

Anomalous ions

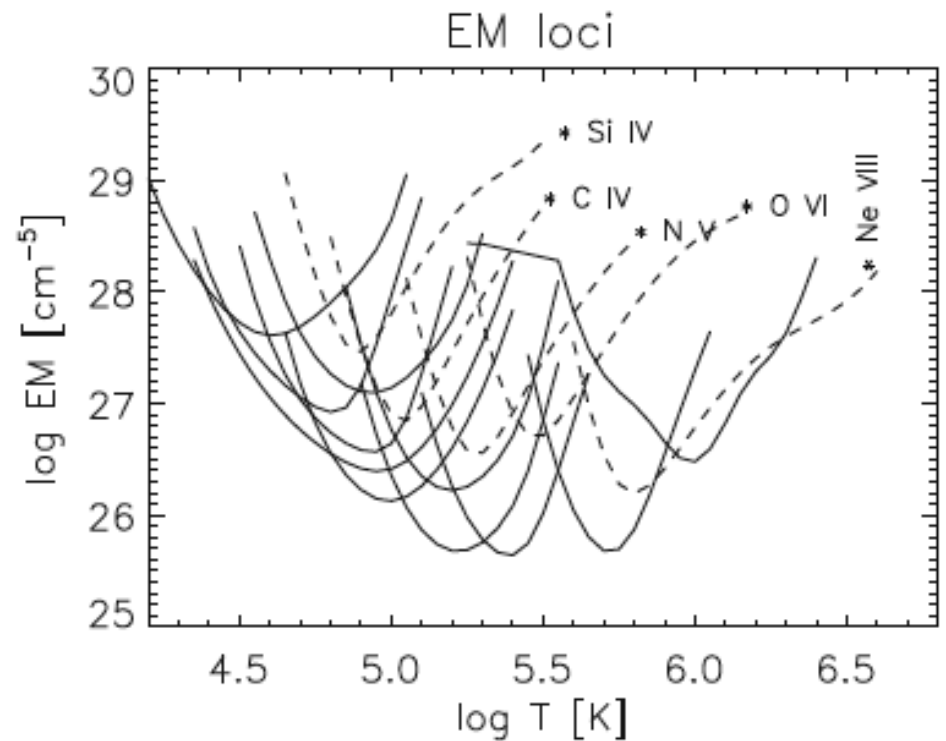
Lines from Li-like and Na-like ions are stronger (factors of 2-5) than predicted. These lines are the strongest in the UV !

Si IV and C IV

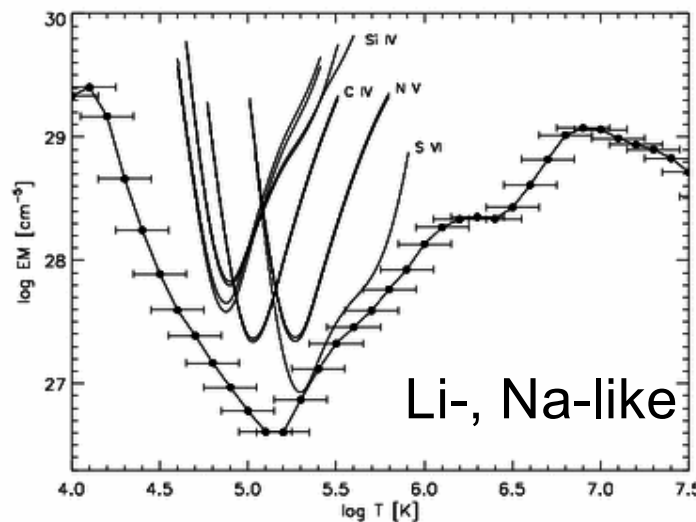
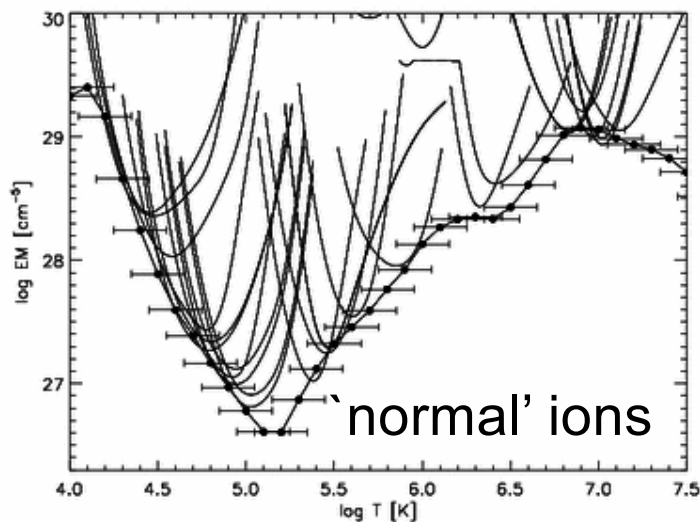
$$I_{ji} = \int Ab(X) G_{ji}(T_e) N_e^2 dh$$

$$EM = \int N_e(h)^2 dh$$

$$EMLoci = \frac{I_{obs}}{Ab(X) G_{ji}(T_e)}$$



Sun-as-a-Star (Judge+1995, Del Zanna& Mason)



Del Zanna+(2002): first to show that the problem is present also in UV stellar observations

Chemical abundances

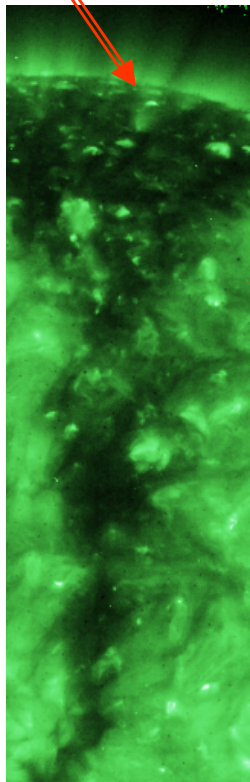
- EM Loci or EM, DEM curves have been used since 1963 to measure the relative abundances in the corona, which are different than photospheric. There is a correlation with the FIP, indicating that the process occurs in the chromosphere (hard to model !), see one of the possible explanations in Laming, Sol. Phys. Living Review.
- Abundances relative to H can be measured using the continuum.
- For a review of key results see our Living Review, as many in the literature are incorrect for various reasons, one related to how the plasma is distributed in temperature.

Chemical abundances in CH plumes

The Widing and Feldman (1989) approximation imposes a continuous distribution of the values DEM_L plotted at the temperature of maximum ion abundance.

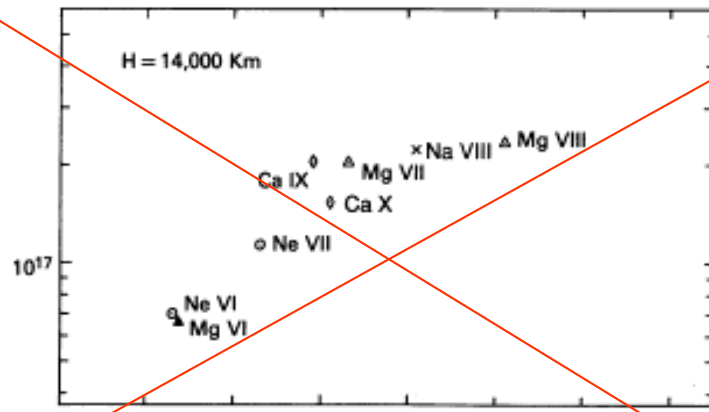
Incorrect abundances (by a factor of 10) can be obtained when plasma is isothermal as in CH plumes.

Plumes in coronal holes



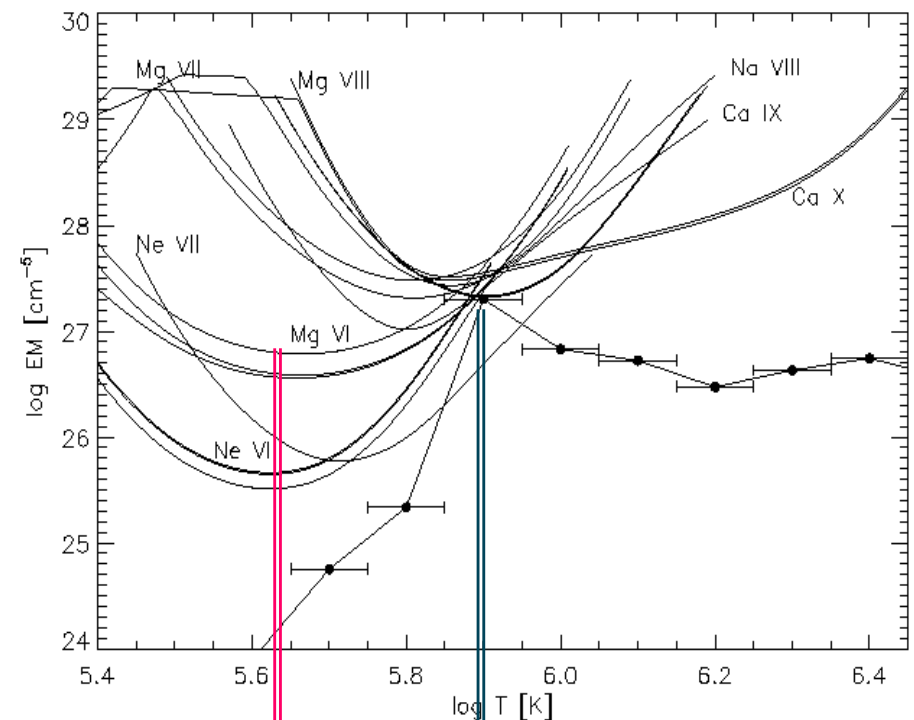
$$DEM_L \equiv \frac{I_{ob}}{Ab(X) \int_T C(T) dT}$$

FIP=10



Widing & Feldman (1992)

FIP=0

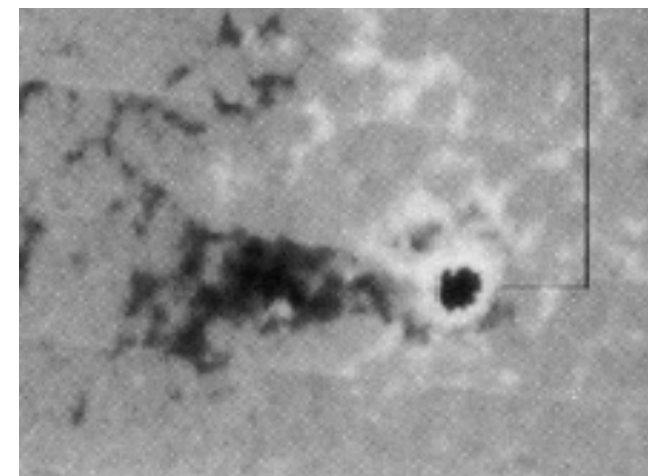
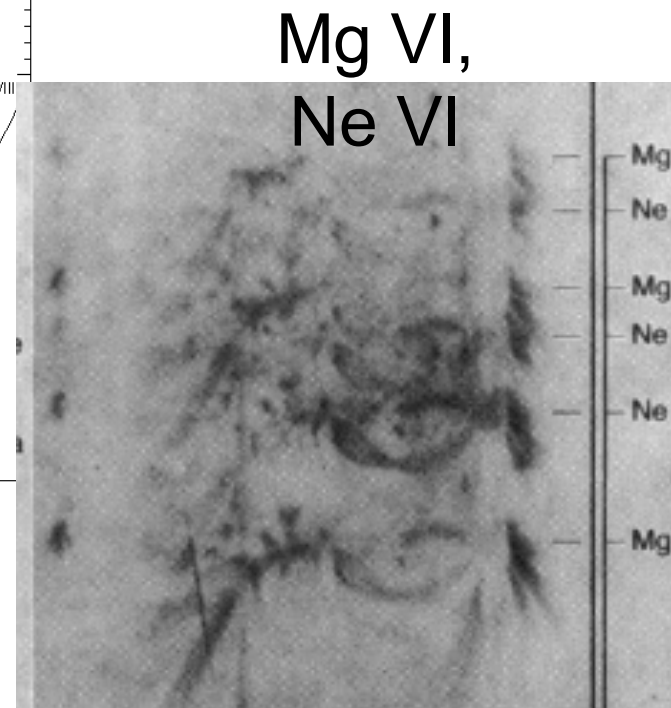
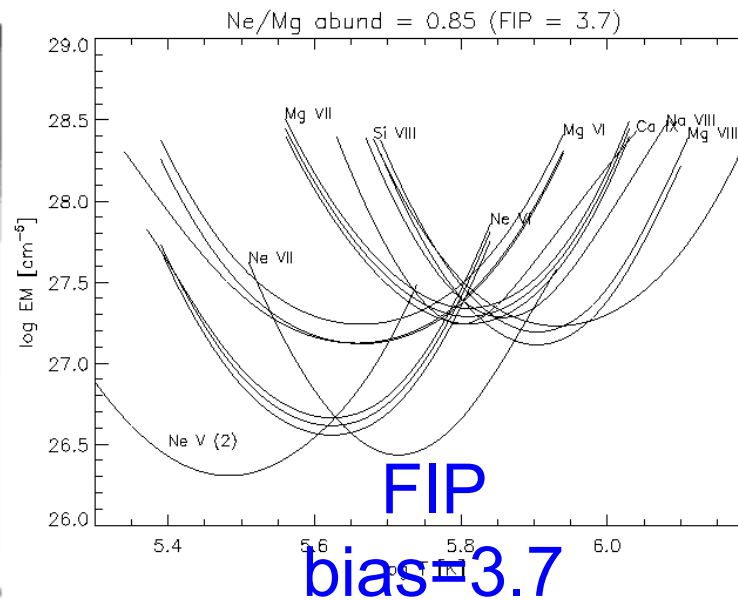
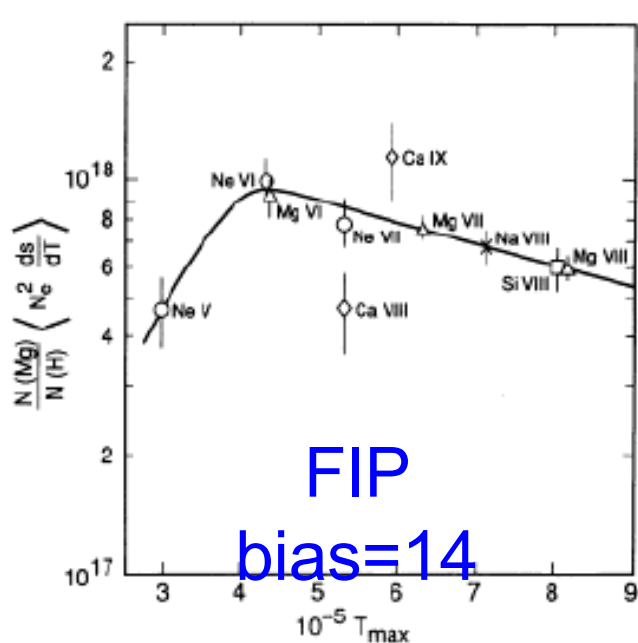


Te of maximum
emissivity in
ionization
equilibrium

Del Zanna et al. (2003)

Actual
Te

FIP BIAS in AR loops (Skylab)



The EM Loci curves are consistent with an FIP bias present but 4 times lower !
(Del Zanna 2003)

Continuum

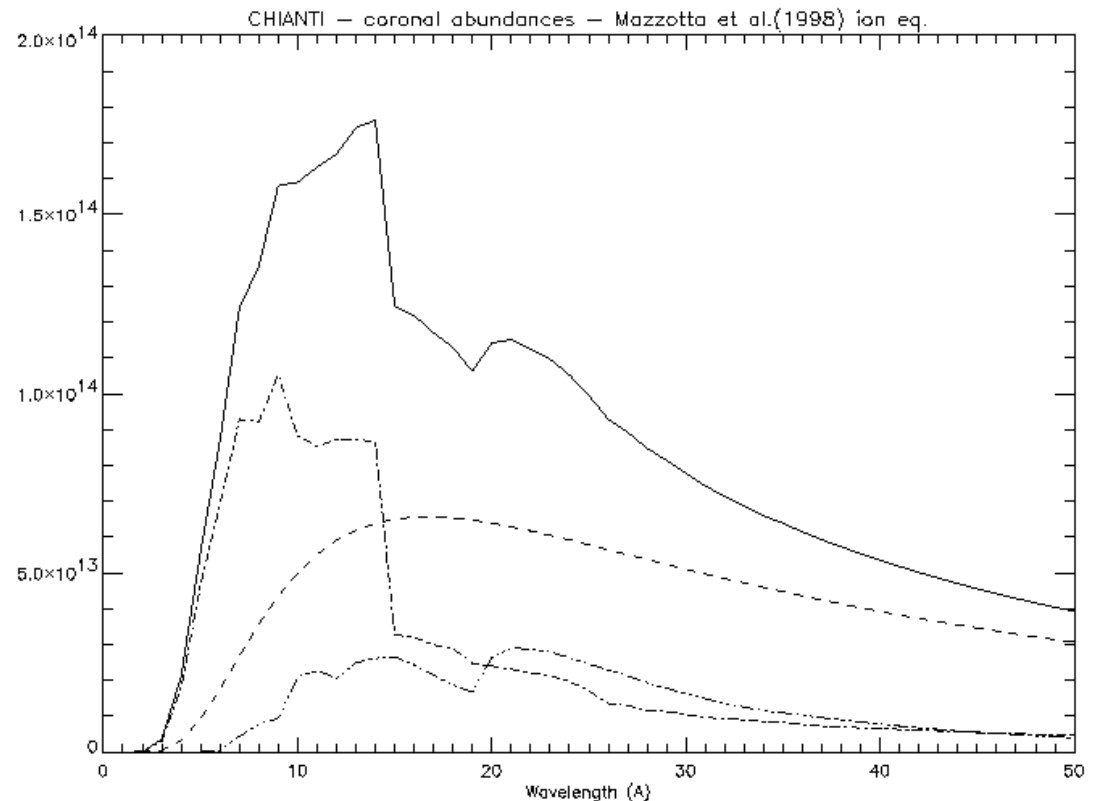
Free-free (f-f) produce a continuum of photons with energies $E=0-1/2 mv^2$)

Free-bound (f-b) produces photons at energies $1/2 mv^2 - E_f$

with E_f the final energy of the atomic state. F-b depends on the chemical abundances.

Continuum is present at all wavelengths

Note that the expressions found in the literature for the f-f and f-b normally assume Maxwellian electron distributions.



END

Thank you