Example slides from a few Prosper-based seminar presentations

...with lots of graphics

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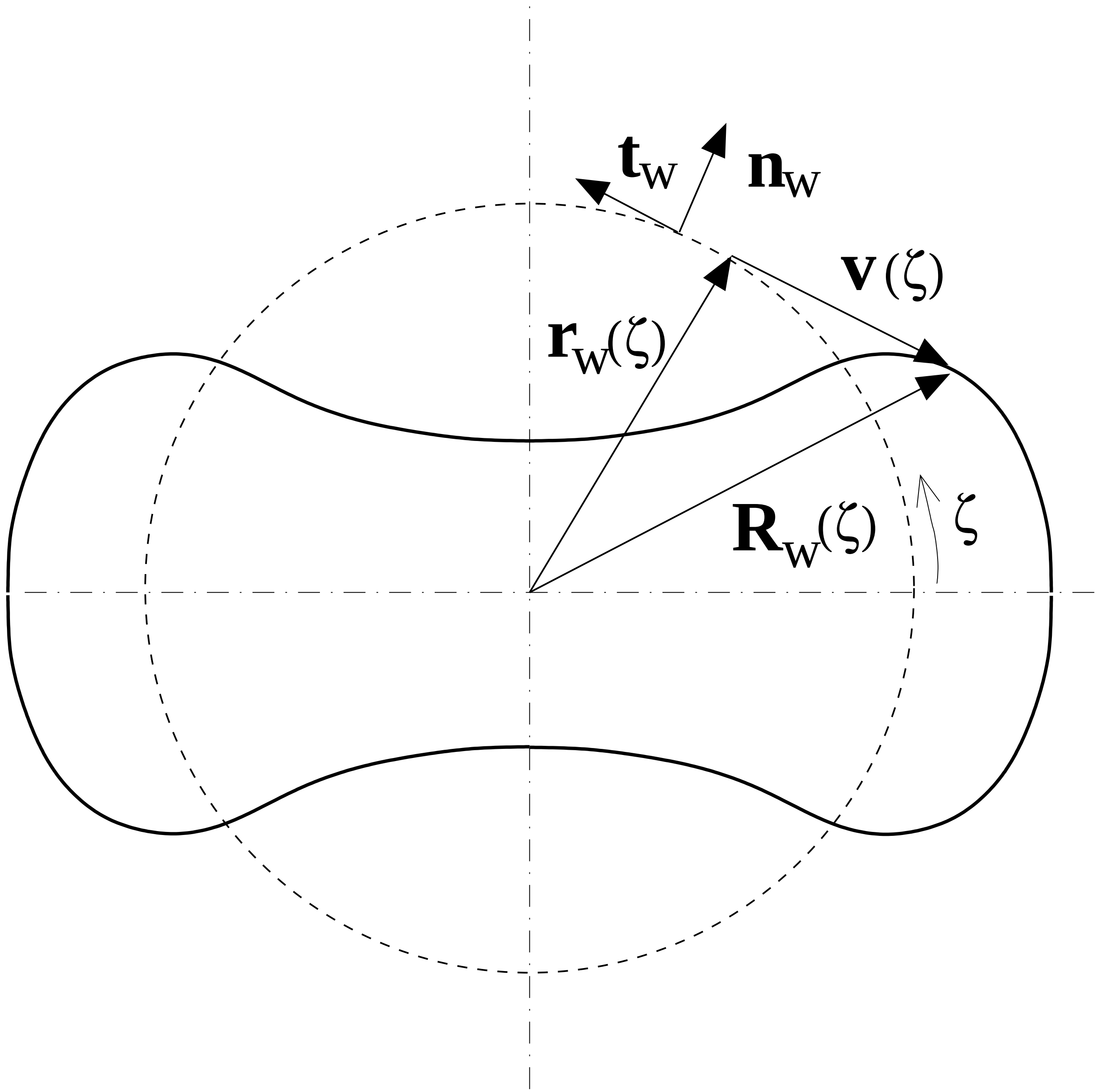
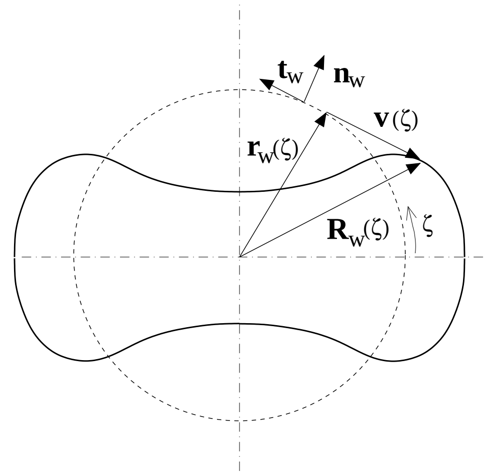
Lagrangian wall mechanics



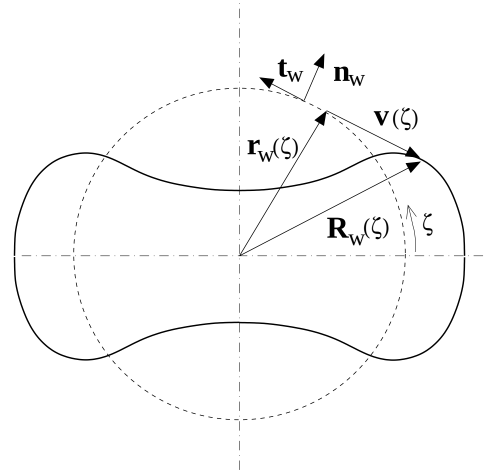
Lagrangian wall mechanics



- Principle of virtual displacements for a linearly elastic ring of undeformed radius R_0 and thickness h, subject to load $\mathbf{f}$:

$$\int_0^{2\pi} \left[\gamma\, \delta\gamma + \frac{1}{12}\left(\frac{h}{R_0}\right)^2 \kappa\, \delta\kappa - \frac{1}{12}\left(\frac{h}{R_0}\right)^3 \left(\left(\frac{R_0}{h}\right)\mathbf{f} - \lambda_T^2 \frac{\partial^2 \mathbf{R}_w}{\partial t^2}\right) \cdot \delta\mathbf{R}_w\right] d\zeta = 0.$$

Lagrangian wall mechanics



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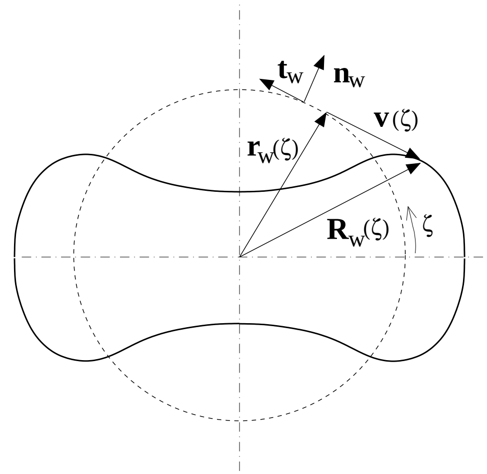
- γ and κ represent the ring's mid-plane strain and change of curvature, respectively.

Lagrangian wall mechanics



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- Load non-dimensionalised by bending stiffness K, i.e.

$$\mathbf{f}^* = K\, \mathbf{f} \qquad \text{where} \qquad K = \frac{E}{12(1-\nu^2)} \left(\frac{h}{R_0} \right)^3.$$

Fluids: Weak form of equations

Momentum:

$$\iiint \left[-\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \psi^{(F)} \, \mathrm{d}V = 0$$

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[...integrate by parts.]

Fluids: Weak form of equations

Momentum:

$$\iiint \left[p \frac{\partial \psi^{(F)}}{\partial x_i} - \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial \psi^{(F)}}{\partial x_j} \right] \mathrm{d}V$$

$$+ \iint \left[-p n_i + \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) n_j \right] \psi^{(F)} \, \mathrm{d}S = 0$$

[...split the surface integral into $\mathrm{d}S = \mathrm{d}S_f + \mathrm{d}S_{\backslash S_f}$]

Fluids: Weak form of equations

Momentum:

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[Note: $\psi^{(F)} = 0$ on $S_{\backslash S_f}$]

Fluids: Weak form of equations

Momentum:

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$$+ \iint \left[-p n_i + \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) n_j \right] \psi^{(F)} \, \mathrm{d}S_f = 0$$

[...use the traction boundary condition on the free surface S_f.]

Fluids: Weak form of equations

Momentum:

$$\iiint \left[p \frac{\partial \psi^{(F)}}{\partial x_i} - \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial \psi^{(F)}}{\partial x_j} \right] \, \mathrm{d}V$$

$$- \iint \left[p_b + \frac{1}{\mathrm{Ca}} \kappa \right] \psi^{(F)} n_i \, \mathrm{d}S_f = 0$$

[...apply Weatherburn's surface divergence theorem to the surface integral.]

Fluids: Weak form of equations

Momentum:

$$\iiint \left[p \frac{\partial \psi^{(F)}}{\partial x_i} - \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial \psi^{(F)}}{\partial x_j} \right] \, \mathrm{d}V$$

$$+ \frac{1}{\mathrm{Ca}} \iint \frac{1}{g} \left[\boldsymbol{g}_1 \right]_i \left(g_{22} \frac{\partial \psi^{(F)}}{\partial \zeta_1} - g_{12} \frac{\partial \psi^{(F)}}{\partial \zeta_2} \right) + \frac{1}{g} \left[\boldsymbol{g}_2 \right]_i \left(g_{11} \frac{\partial \psi^{(F)}}{\partial \zeta_2} - g_{12} \frac{\partial \psi^{(F)}}{\partial \zeta_1} \right) \, \mathrm{d}S_f$$

$$- \iint p_b \psi^{(F)} n_i \, \mathrm{d}S_f - \frac{1}{\mathrm{Ca}} \oint \psi^{(F)} m_i \, \mathrm{d}s = 0$$

Fluids: Weak form of equations

Momentum:

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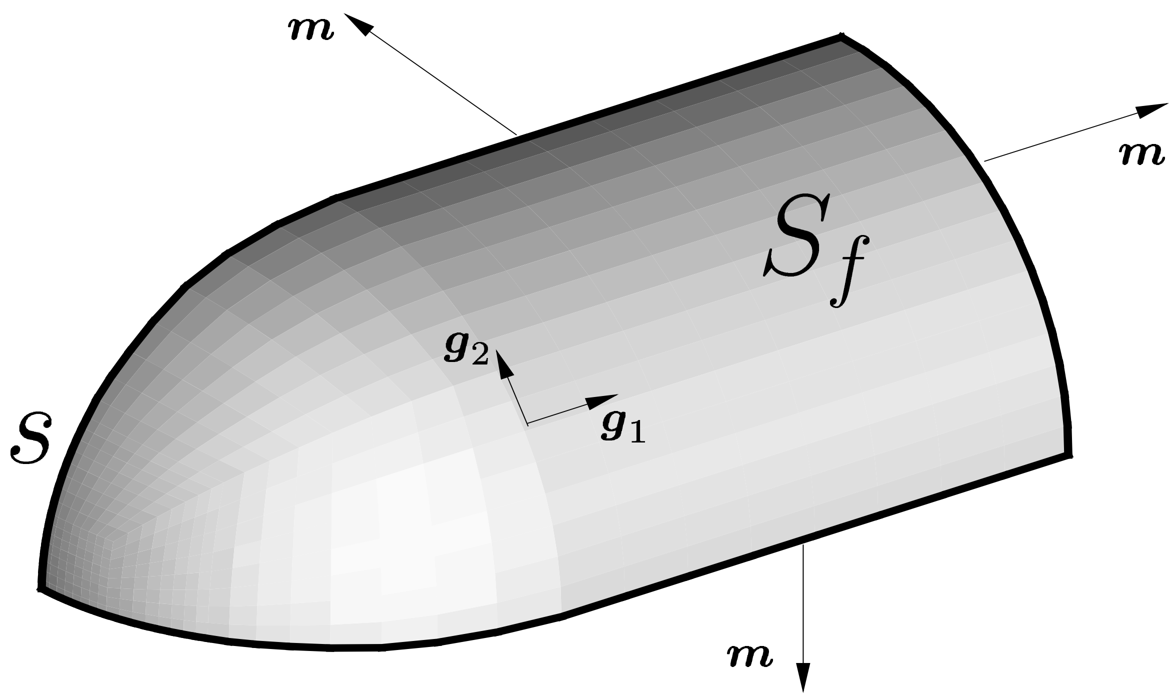

Fluids: Weak form of equations

Momentum:

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$$- \iint p_b \psi^{(F)} n_i \, \mathrm{d}S_f - \frac{1}{\mathrm{Ca}} \oint \psi^{(F)} m_i \, \mathrm{d}s = 0$$

Conservation of mass:

$$\iiint \frac{\partial u_i}{\partial x_i} \psi^{(P)} \, \mathrm{d}V = 0$$

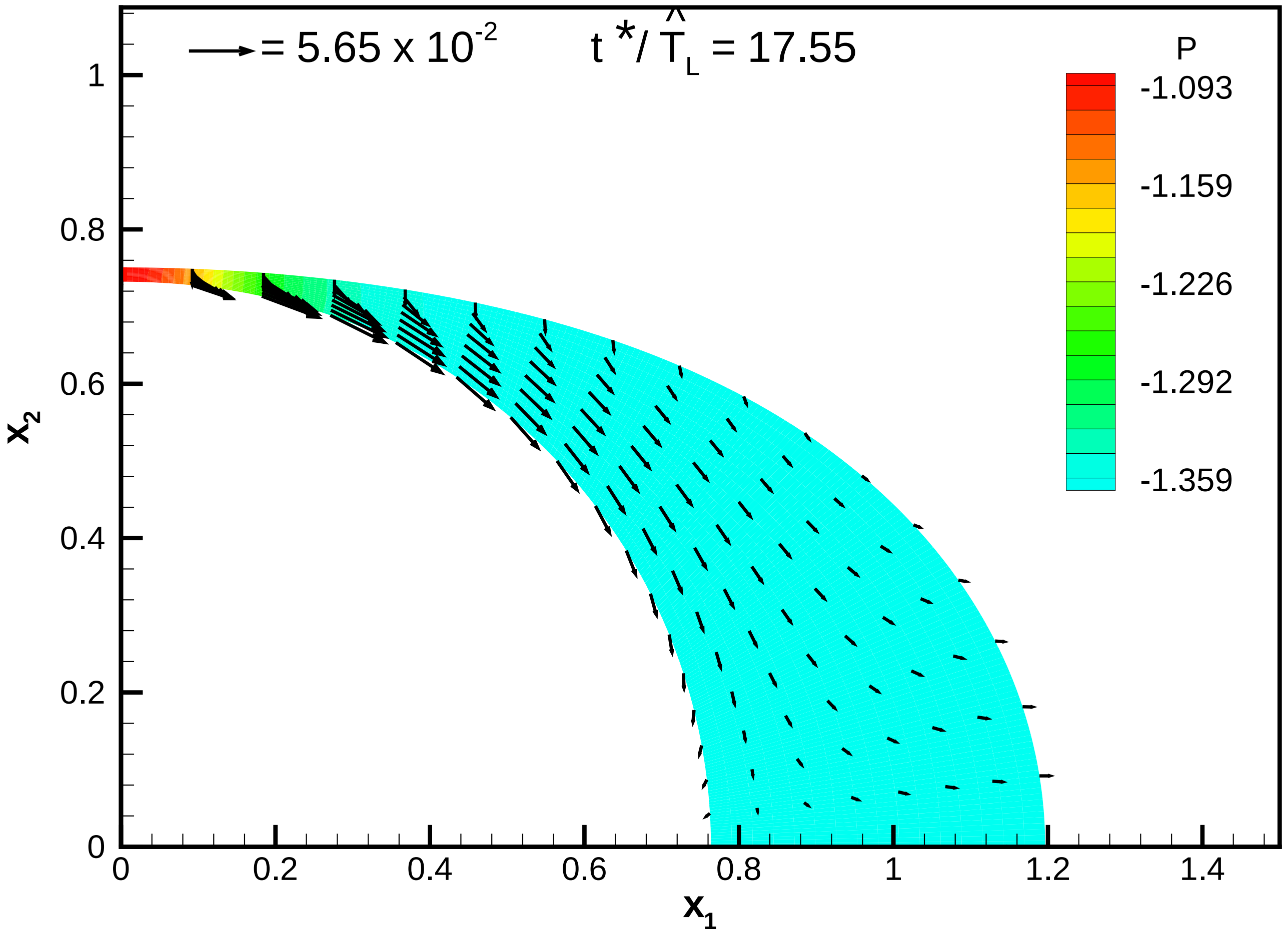
Non-penetration on free surface:

$$\iint u_i n_i \psi^{(H)} \, \mathrm{d}S_f = 0$$

- Discretise using Taylor–Hood elements
- Solve matrix equations directly by frontal method (HSL 2000)

Results (displayed as a pseudo-animation)

Surface-tension-driven collapse of a liquid-lined elastic ring:



- Fully coupled discretisation of the free-surface Navier-Stokes equations and the equations of large-displacement shell theory.
- Solution by the Newton-Raphson method.

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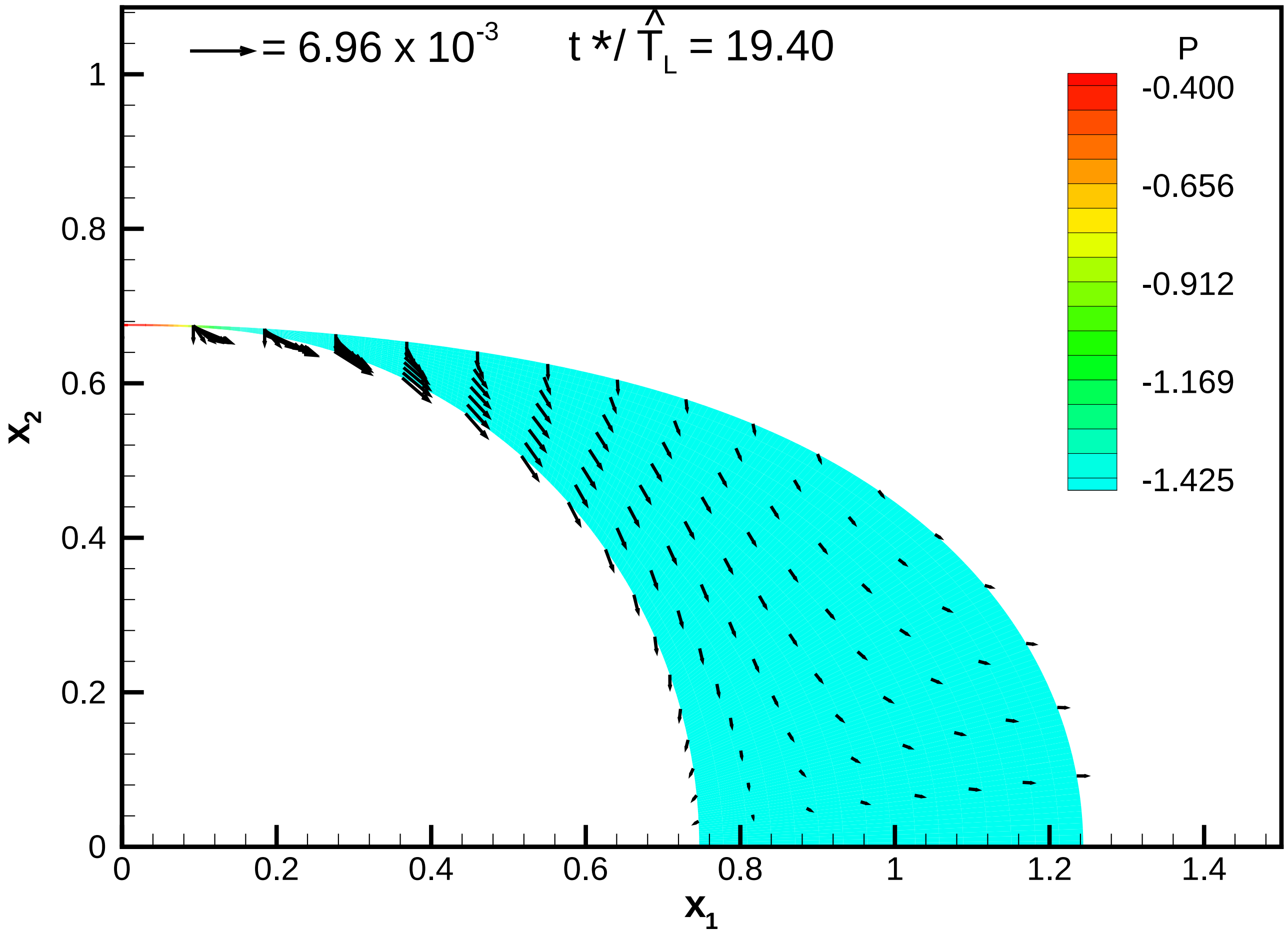

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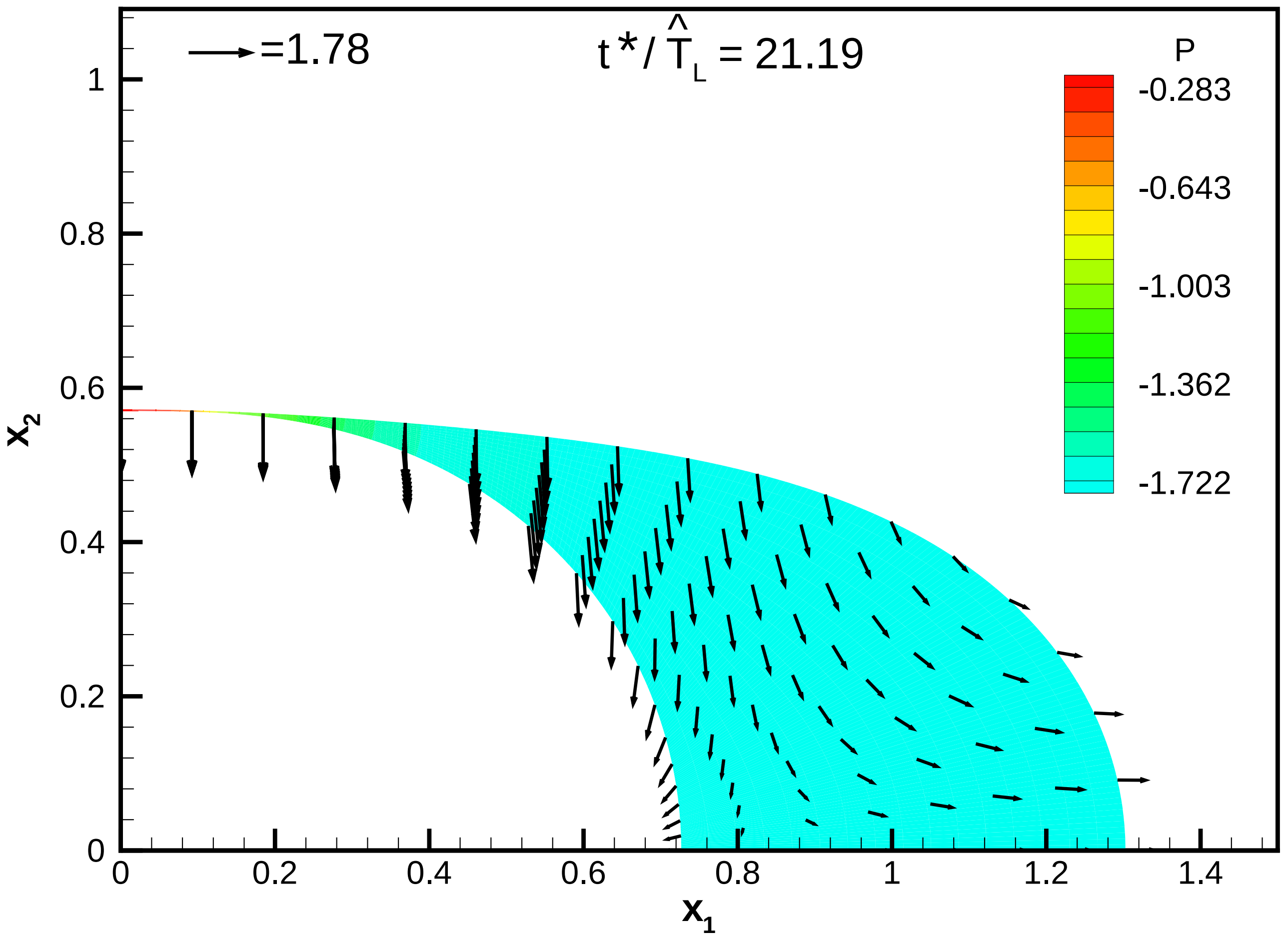

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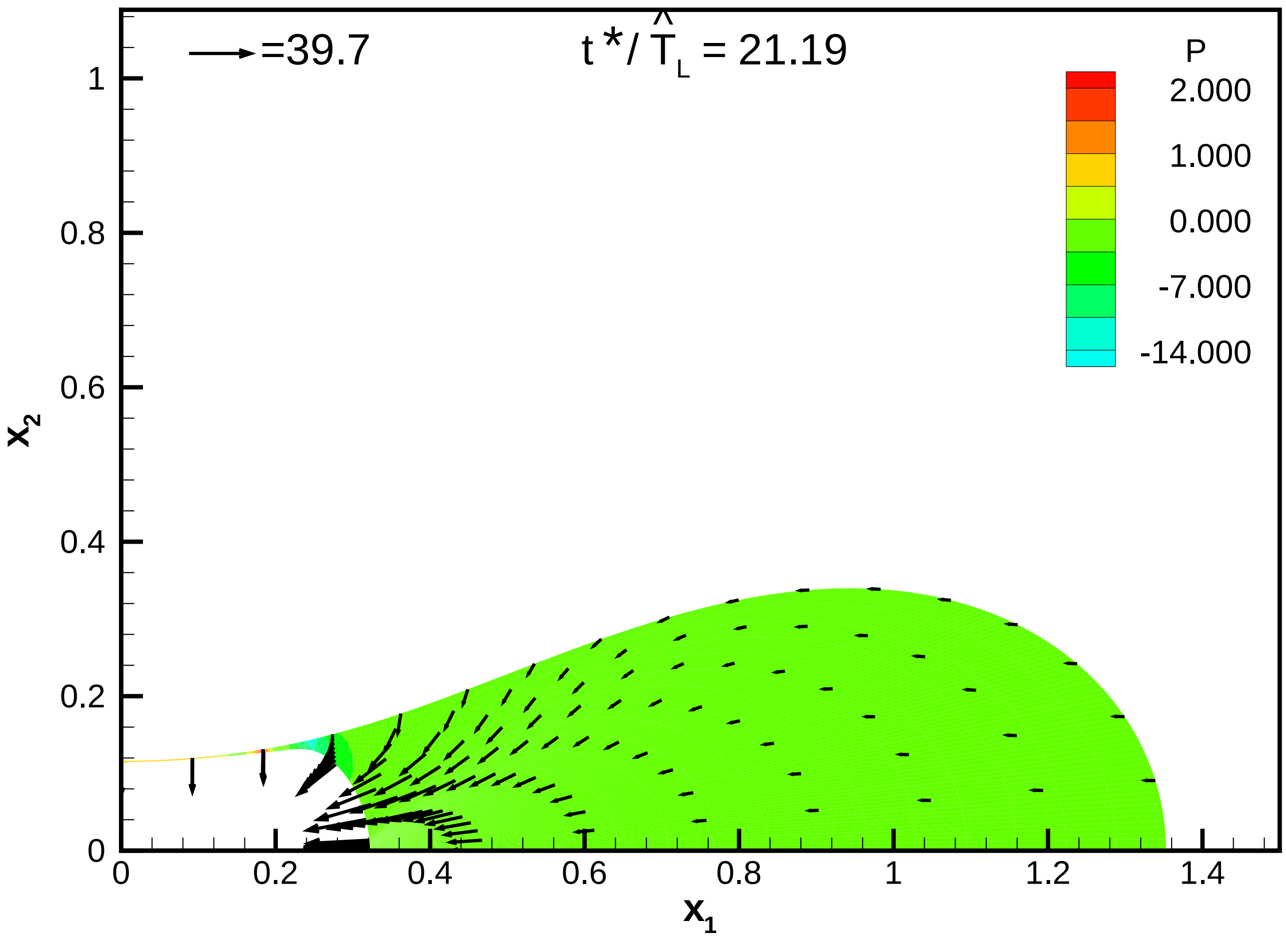

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Results (displayed as real animation)

Self-excited oscillations during finite Reynolds number flow in a collapsible channel.

Start external animation

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