Lecture 14 (3rd of November 2012) - outline

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3.3 Euler's Equations

We derived Euler's Equations for the components of angular velocity $\boldsymbol{\omega}$ in body frame

$$\begin{aligned} I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 &= \tau_1^{\text{ext}}, \\ I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 &= \tau_2^{\text{ext}}, \\ I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 &= \tau_3^{\text{ext}}, \end{aligned} \tag{1}$$

where τ_a^{ext} are components of external torque.

3.4 Free Tops

Free tops are bodies which are not subject to torques, i.e. free rotating bodies.
We discussed the following examples of free tops.

3.4.1 The Symmetric Top

When $I_1 = I_2 = I_3$, such as a sphere, the motions is simple - $\boldsymbol{\omega}$ is conserved and the body spins around a fixed axis with constant angular frequency. In this case $\mathbf{L}$ is parallel to $\boldsymbol{\omega}$.

When $I_1 = I_2 \neq I_3$, e.g. in case of the Symmetric Ellipsoid, the motion is more interesting. Using Euler's Equations (1) we showed that ω_3 component of is conserved, while the vector $\boldsymbol{\omega}$ is precessed in $\mathbf{e}_1, \mathbf{e}_2$-plane with angular frequency

$$\Omega = \omega_3 \frac{I_3 - I_1}{I_1}.$$

In the space frame the motion looks like a wobble. It is a combination of two precessions: one of $\boldsymbol{\omega}$ about $\mathbf{e}_3$ and the other of $\mathbf{e}_3$ about $\mathbf{L}$ ($\tilde{\mathbf{e}}_3$). Notice that $\boldsymbol{\omega}$ and $\mathbf{L}$ are not parallel!