Lectures 15, 16 and 17 (6th, 8th and 10th of November 2012) - outline

Comments and questions should be sent to Berry Groisman, bg268@

3.4 Free Tops (continued)

3.4.2 General Asymmetric Top: Poinsot Construction and Solution in Term of Elliptic Integrals

First, we analysed a simplest case of rotation of a top about one of its principal axes. We discussed stability of such rotations and we have shown that rotation is stable about the axis with the largest of smallest I.

Next, we discussed Poinsot construction, named after French mathematician Louis Poinsot. This is a nice geometrical representation of the motion of $\boldsymbol{\omega}$ or $\mathbf{L}$ in the body frame. We started with writing the two constants of motion in this case, namely

$$T = \frac{I_1\omega_1^2}{2} + \frac{I_2\omega_2^2}{2} + \frac{I_3\omega_3^2}{2},$$
$$\mathbf{L^2} = I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2.$$

They can be rewritten as two ellipsoids with axes ω_a ($\boldsymbol{\omega}$-space representation). The motion of the vector $\boldsymbol{\omega}$ lies on the intersection of the two ellipsoids (the first is known as *inertia ellipsoid*).

Alternatively, we can rewrite them in terms of components L_a ($\mathbf{L}$-space representation). In this case the first is an ellipsoid and the second is a sphere with radius L. The motion of the vector $\mathbf{L}$ lies on the intersection of the ellipsoid and the sphere. We discussed in detail this latter possibility.

Solution in Term of Elliptic Integrals was not discussed. It is available as a handout.

3.5 Euler's Angles

We introduced an explicit parametrization of the configuration space, i.e. the matrix R_{ab}, with the help of three angles ϕ, θ and ψ, known as *Euler's angles.* We derived the components of $\boldsymbol{\omega}$ in the body frame in terms of these angles.

3.6 Heavy Symmetric Top

In cases where gravity exerts torques while acting through centre of mass we say that tops are *heavy* tops. We have analysed the case of a heavy symmetric top, while comparing it with the analysis of the spherical pendulum. We have discussed three different types of behaviour and derived the condition for uniform precession.