Lecture 24 (27th of November 2012) - outline

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4.6 Adiabatic Invariants

We discussed the fundamental of theory of Adiabatic Invariants. If $H(q, p; \lambda(t)$ is a function of some parameter $\lambda(t)$, then energy, E, is not conserved. However, if $\lambda(t)$ changes very slowly with time (we discussed in details the criteria this), then there exist combinations of E and λ, which remains approximately constant. These are called adiabatic invariants (AI). We postulated without proof that AI are angle-actions variables of the system. We illustrated this idea with couple of examples. One of them is given below.

4.6.1 Example: (after Wells-Siklos's paper)

$$H = \frac{p^2}{2m} + \frac{\lambda}{2} q^4 = E. \tag{1}$$

We solve for p:

$$p = \sqrt{m(2E - \lambda q^4)}. \tag{2}$$

We calculate AAV

$$I = \frac{1}{2\pi} \oint p dq = \frac{\sqrt{m}}{2\pi} \oint \sqrt{2E - \lambda q^4} dq = \frac{\sqrt{m}}{\pi} (2E)^{3/4} \lambda^{-1/4} \int_{-1}^{1} \sqrt{1 - x^4} dx. \tag{3}$$

Thus the AI is $I(\frac{E^3}{\lambda})^{1/4}$, i.e. energy varies as $\lambda^{1/3}$ for slowly varying changes in λ.