THE

Secret Life

OF SCATTERING AMPLITUDES
A scattering process is a description of what happens when stuff bumps into other stuff.

See what comes out

Choose what to send in

Messy collision happens here
Rutherford discovered the atomic nucleus by scattering \( \alpha \)-particles off thin gold foil.

"It was quite the most incredible event that has ever happened in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."
Most of what we know about the Standard Model - our best theory of particle physics - has been gleaned from scattering experiments.
You draw all diagrams with the same external lines; Feynman then tells you the amplitude.

We teach these rules to our Part III students.

- the reason they work was explained by Freeman Dyson in the late 1940s.
Result of a Feynman diagram calculation for 2 particles producing a third in a QFT (YM):

\[ k_1 \cdot k_4 \varepsilon_2 \cdot k_1 \varepsilon_1 \cdot \varepsilon_3 \varepsilon_4 \cdot \varepsilon_5 \]

+ seventeen more pages!
$$\frac{\langle ij \rangle^4 \delta^4 (\sum p_i)}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$
Why is the answer so simple?

\[
\frac{\langle ij \rangle^4 \delta^4(\sum p_i)}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}
\]
Twistor Theory
Twistors were developed in the 1970s by Penrose, intended as a new framework for physics. He wished to promote causal relationships between events above the events themselves.

Thanks to Atiyah, Hitchin, Ward & many others they quickly found application in *mathematics*, but physics was slow to catch on...
Space-time

Twistor space

Point in space-time

Separation is light-like

Sphere in twistor space

Two spheres intersect
Physical objects are encoded in *global* properties of objects in twistor space.

Physics is not described by point-like objects on twistor space: no “particles”
Replacing many Feynman diagrams with one single object is common in string theory, but at a high price:

- 10 dimensions
- Infinitely many new types of particle
In late 2003, Witten realized that a string theory *in twistor space* was just what was needed.

Only 3+1 dimensions

Only particles we’ve seen
Figure 5: The polygon is specified at the AdS boundary by the positions of the cusps $x_i$. These positions are related to an ordered sequence of momenta $k_i$ by $k_i = x_i - x_{i-1}$. The two-dimensional minimal surface stretches in the AdS bulk and ends on the polygonal contour at the boundary.

2. The classical sigma model and Hitchin equations

The classical AdS$_5$ sigma model is integrable. This can be shown by exhibiting a one parameter family of flat connections. For our problem, it will be convenient to choose this one parameter family in a special way which will simplify its asymptotic behavior on the worldsheet. In fact, to make this choice we will make use of the Virasoro constraints of the theory. This has been explained in detail in previous papers [8, 9, 2x, 2y]. Instead of repeating the whole discussion, we will present a slightly more abstract and algebraic version here.

2.1 General integrable theories and Hitchin equations

Let us assume that we have a coset space $G/H$. Let us assume that the Lie algebra $G$ has a $\mathbb{Z}_2$ symmetry that ensures integrability. In other words, imagine that the Lie algebra has the decomposition $G = H \ltimes K$ so that $H$ is left invariant under the action of the $\mathbb{Z}_2$ generator while elements in $K$ are sent to minus themselves. We then write the $G$ invariant currents $J = g^{-1} dg$. This is a flat current $dJ_s J \wedge J^s = 0$. We can decompose $J$ in terms of its components along $H$ and $K$ as $J = g^{-1} dg = H_s K$. When we gauge the sigma model, we add a gauge field along $H$ and we can do local $H$ gauge transformations. The equations of motion of the system can be written in terms of...