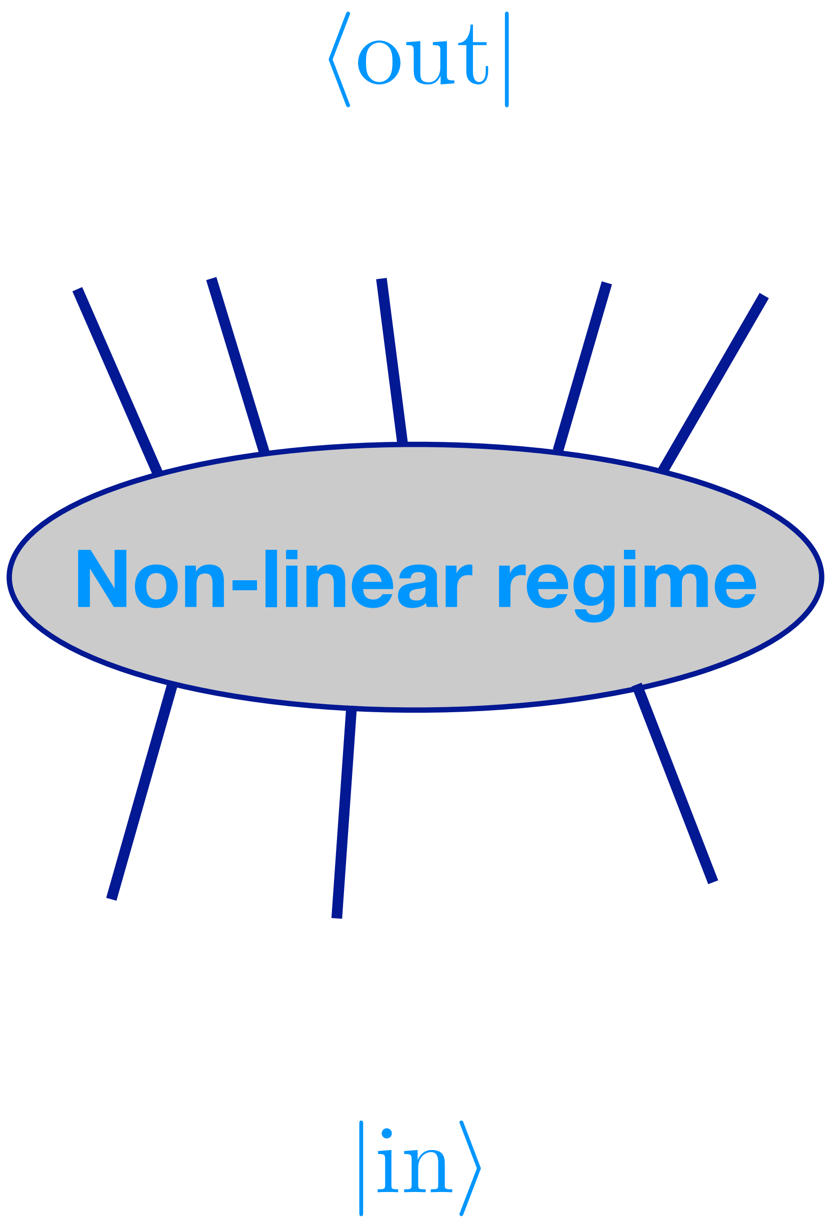
Ambitwistors and the Scattering Equations

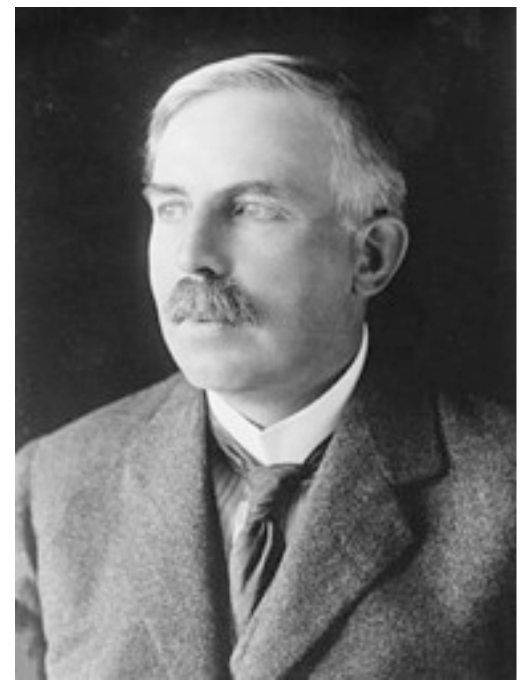
based on work with L. Mason and with T. Adamo & E. Casali

David Skinner DAMTP, Cambridge 18th February 2015

Scattering processes have led the way in understanding Nature at the subatomic level



True both for experiments...



$\Rightarrow$

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True both for experiments...

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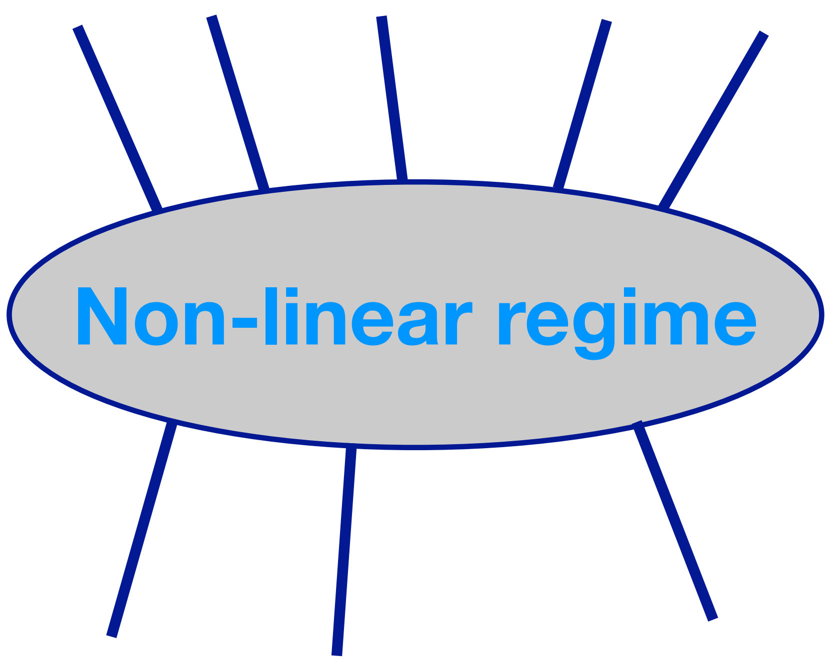
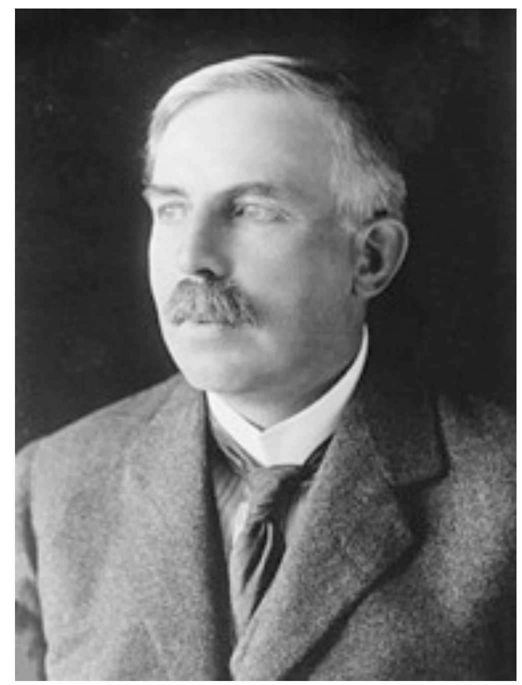


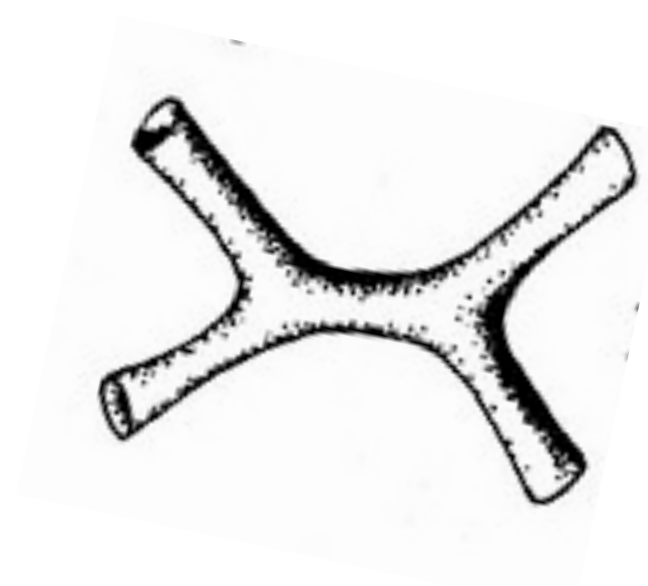


$\Rightarrow$

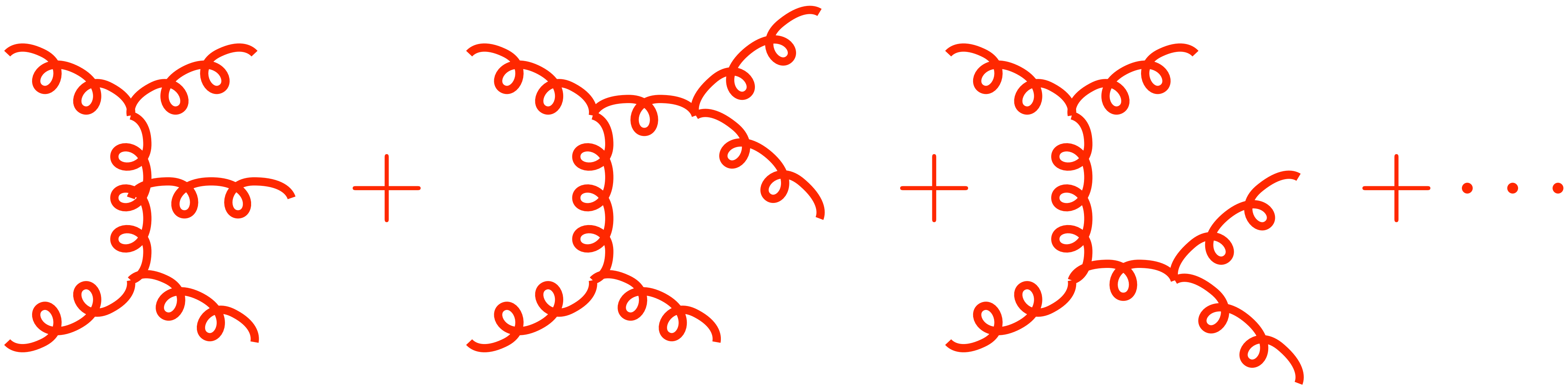
$| \mathrm{in} \rangle$

... and for theory

$$\frac{\Gamma(-1-s)\,\Gamma(-1-t)}{\Gamma(-2-s-t)} \Rightarrow$$



In YM and gravity - theories we care most about - Feynman diagrams rapidly become very messy

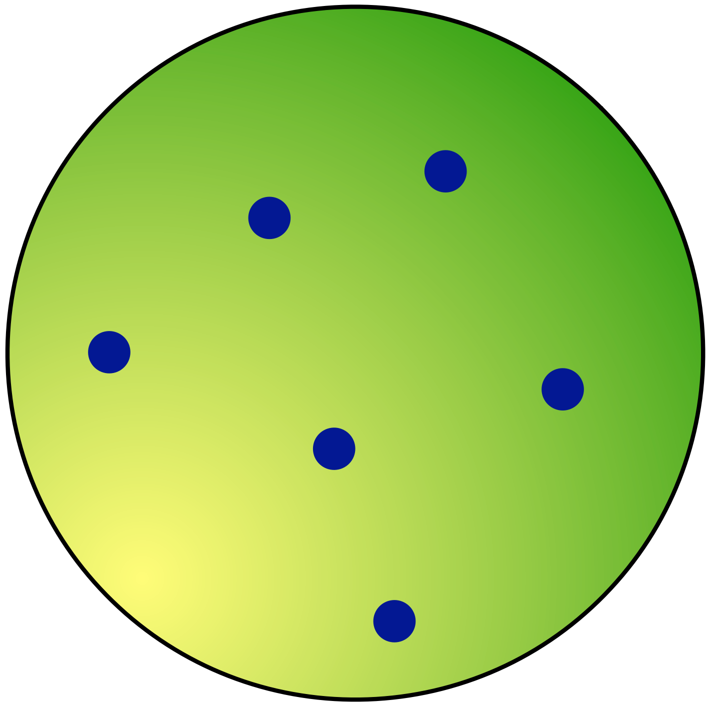


- Even individual vertices and propagators are complicated expressions involving many terms

In recent years, work by many people has shown that the *amplitudes themselves* have many fascinating structures that are completely obscured by their representation in terms of Feynman diagrams

Cachazo, He & Yuan have proposed several remarkable formulae for tree-level amplitudes in massless theories

External states are associated to points $z_i \in \mathbb{CP}^1$



Up to $\mathrm{SL}(2,\mathbb{C})$, these points are then fixed in terms of external data by the *scattering equations*

$$\sum_{j\neq i} \frac{k_i \cdot k_j}{z_i - z_j} = 0 \quad \text{for all } i \in \{1, \ldots, n\}$$

$$\mathcal{M} = \sum_{\text{solns}} \frac{\mathrm{Obj}(\epsilon_i, k_i, z_i, \ldots)}{\mathrm{Jac}}$$

'Obj' depends on the theory

'Jac' is Jacobian of sct. eqs.

For example, in gravity & Yang-Mills define

$$\Psi = \left[\begin{array}{cccc:cccc} 0 & \frac{k_1\cdot k_2}{z_{12}} & \cdots & \frac{k_1\cdot k_n}{z_{1n}} & C_{11} & \frac{k_1\cdot \epsilon_2}{z_{12}} & \cdots & \frac{k_1\cdot \epsilon_n}{z_{1n}} \\ \frac{k_2\cdot k_1}{z_{21}} & 0 & & & \frac{k_2\cdot \epsilon_1}{z_{21}} & \ddots & & \\ \vdots & & \ddots & & \vdots & & \ddots & \\ \frac{k_n\cdot k_1}{z_{n1}} & \cdots & & 0 & \frac{k_n\cdot \epsilon_1}{z_{n1}} & \cdots & & C_{nn} \\ \hdashline -C_{11} & \frac{\epsilon_1\cdot k_2}{z_{12}} & \cdots & \frac{\epsilon_1\cdot k_n}{z_{1n}} & 0 & \frac{\epsilon_1\cdot \epsilon_2}{z_{12}} & \cdots & \frac{\epsilon_1\cdot \epsilon_n}{\epsilon_{1n}} \\ \frac{\epsilon_2\cdot k_1}{z_{21}} & \ddots & & & \frac{\epsilon_2\cdot \epsilon_1}{z_{21}} & 0 & & \\ \vdots & & \ddots & & \vdots & & \ddots & \\ \frac{\epsilon_n\cdot k_1}{z_{n1}} & \cdots & & -C_{nn} & \frac{\epsilon_n\cdot \epsilon_1}{z_{n1}} & \cdots & & 0 \end{array}\right]$$

$$\mathcal{M}_{\text{grav}} = \sum_{\text{sols}} \frac{\text{Pfaff}'\Psi \ \text{Pfaff}'\tilde{\Psi}}{\text{Jac}}$$

$$\mathcal{M}_{\text{YM}} = \sum_{\text{sols}} \frac{\text{Pfaff}'\Psi}{\text{Jac}} \left[\frac{\text{tr}(T_1\cdots T_n)}{z_{12}z_{23}\cdots z_{n1}} + \cdots\right]$$

“Gravity = Gauge x Gauge”

These expressions have many weird & wonderful properties:

- each contribution algebraic but non-rational
- manifest permutation invariance
- gauge / diffeo invariant term-by-term
- transparent behaviour in soft limits term-by-term
- same scattering equations as dominate high energy, fixed angle string scattering[Gross-Mende]
- expressions also exist for Einstein-Yang-Mills, NLSM, DBI, scalar ϕ^3, ...

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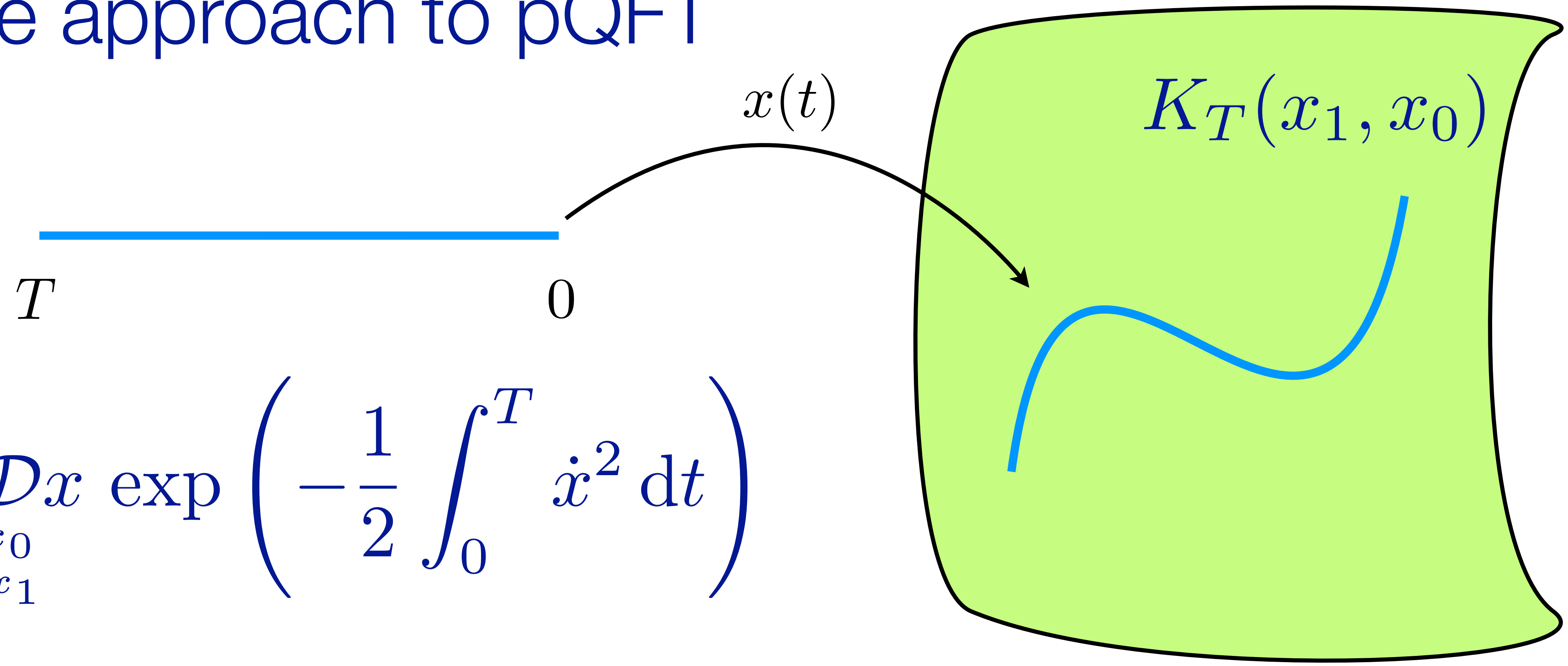
What lies behind such magic formulae? What do they teach us about these theories?

Chiral strings on the space of light rays

Worldline approach to pQFT



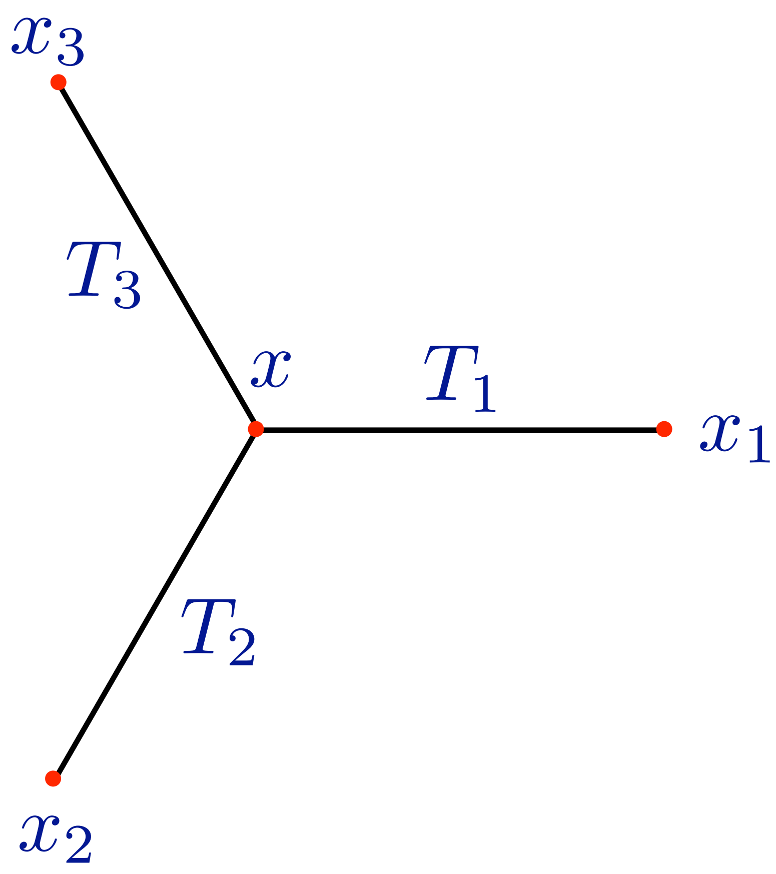
$$\int_{\substack{x(0)=x_0 \\ x(T)=x_1}} \mathcal{D}x \, \exp\left(-\frac{1}{2}\int_0^T \dot{x}^2 \, \mathrm{d}t\right)$$

The basic path integral gives the heat kernel, while canonical quantization leads to the Klein-Gordon eq

Integrating over the Schwinger parameter (length) T leads to the propagator $D(x_1, x_0) = \int \frac{\mathrm{d}^d p}{(2\pi)^d} \frac{\mathrm{e}^{\mathrm{i}p\cdot(x_1 - x_0)}}{p^2}$

Replacing the worldline by a graph Γ gives the basic structure of massless field theory

eg



$$= \int \mathrm{d}^3 T \int \mathcal{D}x \, \mathrm{e}^{-S_\Gamma[x]}$$

$$= \int \mathrm{d}^d x \prod_{i=1}^{3} \frac{\mathrm{d}^d p_i}{(2\pi)^d} \frac{\mathrm{e}^{\mathrm{i} p_i \cdot (x_i - x)}}{p_i^2}$$

$$= \langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle_{\phi^3}^{\mathrm{tree}}$$

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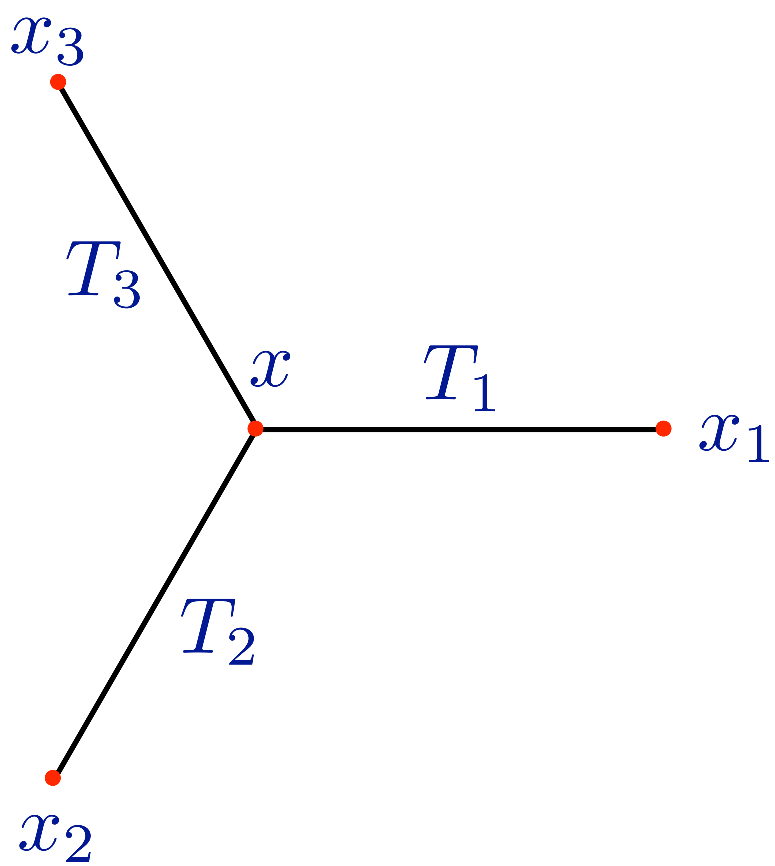

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$$= \langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle_{\phi^3}^{\text{tree}}$$

- different pQFTs obtained from more elaborate worldline actions / different allowed graph topologies
- integral over lengths is really an integral over the moduli space $\mathrm{Met}(\Gamma)/\mathrm{Diff}(\Gamma)$

The basic structure of pQFT is thus provided by

$$\sum_{\Gamma} \int_{\mathcal{M}_\Gamma} \mathrm{d}^{|E|}\mu \int \mathcal{D}x\,\mathcal{D}p \; \exp\left(\mathrm{i} \int_{\Gamma} p\,\mathrm{d}x - \frac{1}{2} e p^2 \right)$$

in first-order form, where e is an einbein. Similarly, the basic structure of pST is the genus expansion

$$\sum_{g} \int_{\mathcal{M}_g} \mathrm{d}^{6g-6}\mu \int \mathcal{D}x\,\mathcal{D}p \; \exp\left(\mathrm{i} \int_{\Sigma_g} p\,\mathrm{d}x - \frac{1}{2} p \wedge *p \right)$$

where p is now a 1-form on the worldsheet.

- various pSTs (type II, heterotic, type I, ...) obtained by allowing more elaborate worldsheet actions or different worldsheet topologies

The CHY formulae come from an intermediate theory, based on the purely chiral worldsheet action

$$S = \int_\Sigma P_\mu \bar{\partial} X^\mu - \frac{1}{2} e P_\mu P^\mu$$

where $P_\mu \in \Omega^{1,0}(\Sigma)$ and $e \in \Omega^{0,1}(\Sigma, T_\Sigma)$

- complexification of worldline, or chiral string
- e imposes the constraint $P^2 = 0$, so P_μ is null
- this constraint generates the gauge symmetry

$$\delta X^\mu = \alpha P^\mu \qquad \delta P_\mu = 0 \qquad \delta e = \bar{\partial} \alpha$$

where $\alpha \in \Omega^{0,1}(\Sigma, T_\Sigma)$. Thus $X \sim X + \alpha P$.
P has no fixed scale; target is space of light rays

The space of light rays, called *projective ambitwistor space* $P\mathbb{A}$, has a rich geometric structure:

- it is a non-degenerate $2d-3$ dimensional contact manifold with contact form $\theta = p_\mu \mathrm{d}x^\mu$ upto scaling

π_1 projects along null geodesics

- space-time point x corresponds to a quadric surface $Q_x \subset P\mathbb{A}$ *e.g.* in four dimensions $Q_x \cong \mathbb{CP}^1 \times \mathbb{CP}^1$

- contact form determines complex structure via its kernel: $\bar{V}$ antiholomorphic iff $\imath_{\bar{V}}(\theta \wedge \mathrm{d}\theta^{d-2}) = 0$

LeBrun proved that if given contact structure on $P\mathbb{A}$, can recover space-time metric. No clear field equations.

To describe (type II super-)gravity, we also add in two sets of chiral fermions

$$S = \int_{\Sigma} P_\mu \bar{\partial} X^\mu - \frac{e}{2} P^2 + \sum_{r=1}^{2} \frac{1}{2} \psi_{r\mu} \bar{\partial} \psi_r^\mu + \chi_r P_\mu \psi_r^\mu$$

In the absence of vertex operators, we can impose the gauge $e = 0$, $\chi_r = 0$. The BRST operator is then

$$Q = \oint cT + \tilde{c} P^2 + \sum_r \gamma_r P \cdot \psi_r + \text{ghosts}$$

where c is the usual (chiral) reparametrization ghost, and $(\tilde{c}, \gamma_r)$ are ghosts associated with the gauge symmetries generated by $(P^2, P_\mu \psi_r^\mu)$

- central charge vanishes in ten dimensions

In this gauge we have the simple OPEs

$$X^\mu(z)\,X^\nu(w) \sim 0 \qquad\qquad P_\mu(z)\,X^\nu(w) \sim \frac{\delta_\mu{}^\nu}{z-w}$$

$$\psi_1^\mu(z)\,\psi_2^\nu(w) \sim 0 \qquad\qquad \psi_1^\mu(z)\,\psi_1^\nu(w) \sim \frac{\delta_{\mu\nu}}{z-w}$$

Fixed vertex operators are similar to RNS string:

$$U = c\tilde{c}\,\delta^2(\gamma)\,\psi_1^\mu\psi_2^\nu\,\delta g_{\mu\nu}(X)$$ in ‘NS-NS’ sector

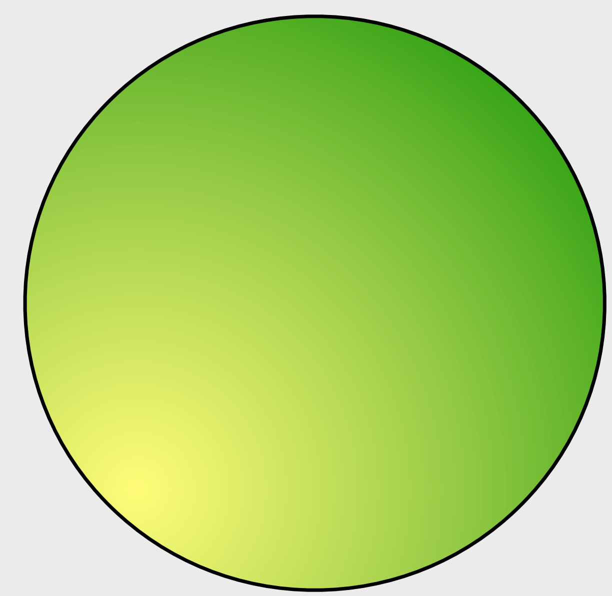
Because $XX \sim 0$, the wavefunction $\delta g_{\mu\nu}$ *never* has anomalous conformal weight: the mass-shell and transversality conditions come from imposing gauge constraints rather than from the stress tensor.

There are no massive states in the spectrum.

In the absence of vertex operators, we can impose the gauge $e = 0$.

Usually in string theory, worldsheet oscillations lead to an infinite (Regge) tower of extra states, entering at a scale set by the string tension $1/\alpha'$

These chiral strings cannot oscillate



$$S = \int_\Sigma P_\mu \bar{\partial} X^\mu$$

since P_μ constrains $\bar{\partial} X^\mu = 0$, implying X^μ is constant

Inserting n such vertex operators has two main geometric effects:

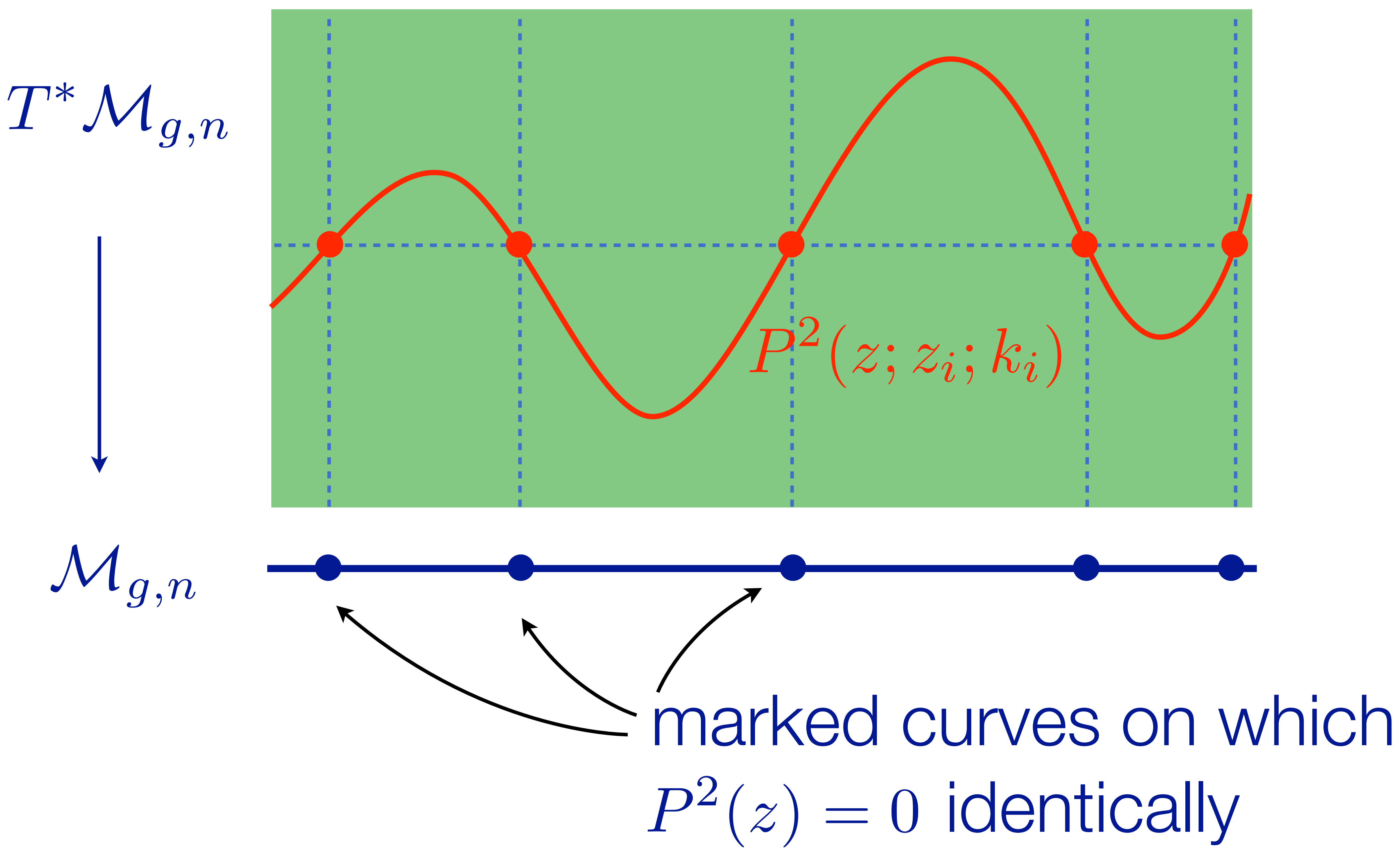
- can no longer use $\delta e = \bar{\partial}\alpha$ to move to gauge $e = 0$ as, like conformal structure, e now has moduli
- for momentum eigenstates, $\delta g_{\mu\nu}(X) = \epsilon_\mu \tilde{\epsilon}_\nu \, \mathrm{e}^{\mathrm{i}k\cdot X}$ so the X dependence of the path integral is

$$\int_\Sigma \left(P_\mu \bar{\partial} X^\mu + \sum_{i=1}^{n} \delta(z - z_i) k_i \cdot X \right)$$

Integrating out the non-zero modes of X leads to

$$P_\mu(z) = \sum_{i=1}^{n} \frac{k_{i\mu} \mathrm{d}z}{z - z_i} \quad \Rightarrow \quad P^2(z) = (\mathrm{d}z)^2 \sum_{i,j} \frac{k_i \cdot k_j}{(z - z_i)(z - z_j)} \neq 0$$

In the presence of vertex operators, $P^2(z)$ is a meromorphic quadratic differential, *ie*, a cotangent vector to the moduli space of the marked curve



Moduli of e are $n-3$ Beltrami differentials μ_α. We introduce the BRST trivial term $\left\{Q, \tilde{b}\left(e-\sum_\alpha s_\alpha \mu_\alpha\right)\right\}$ leading to

$$\sum_\alpha \int_\Sigma \left(P^2 s_\alpha \mu_\alpha + \tilde{b}\, q_\alpha \mu_\alpha\right)$$

s_α: bosonic moduli

q_α: BRST partners

in the action. Integrating over the moduli parameters & their fermionic BRST partners then gives the measure

$$\prod_\alpha \int_\Sigma (\tilde{b}, \mu_\alpha) \prod_\alpha \delta\left(\int_\Sigma P^2 \mu_\alpha\right)$$

on the moduli space of the gauge field e.

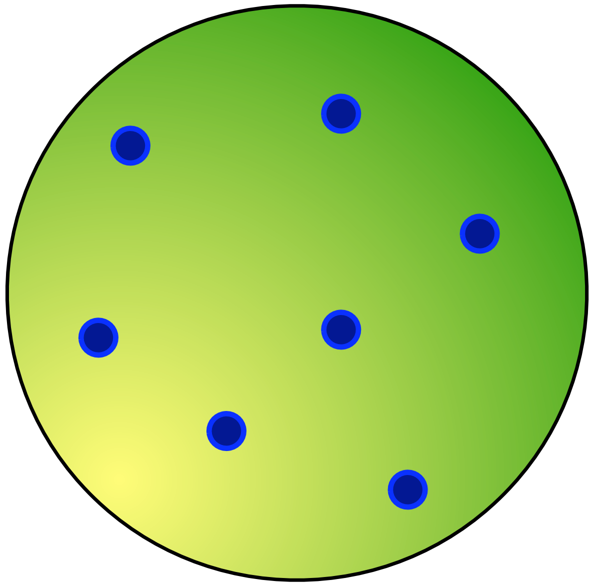
With a standard choice of Beltrami differentials,

$$\delta\left(\int_\Sigma \mu_\alpha P^2\right) = \bar\delta\left(\mathrm{Res}_{z_\alpha} P^2(z)\right)$$

At genus zero, a meromorphic quadratic differential vanishes identically iff $n-3$ of its residues vanish

For momentum eigenstates, $\mathrm{Res}_{z_i} P^2(z) = \mathrm{d}z \sum_{j\neq i} \frac{k_i \cdot k_j}{z_{ij}}$ whose vanishing is the scattering equations!

The scattering equations precisely ensure that the theory does indeed live on the space of light rays, even in the presence of vertex operators



With vertex operators

$$U = c\tilde{c}\,\delta^2(\gamma)\,\psi_1^\mu\psi_2^\nu\,\epsilon_\mu\tilde{\epsilon}_\nu\,\mathrm{e}^{\mathrm{i}k\cdot X}$$

$$\int_\Sigma V = \int_\Sigma (P^\mu + \psi_1^\mu\, k\cdot\psi_1)\,(P^\nu + \psi_2^\nu\, k\cdot\psi_2)\;\bar{\delta}(k\cdot P)\epsilon_\mu\tilde{\epsilon}_\nu\mathrm{e}^{\mathrm{i}k\cdot X}$$

the n-point correlation function at genus zero gives exactly the CHY formula for tree-level gravity

- Pfaffians of Ψ and $\tilde{\Psi}$ come from fermion correlators
- V actually represents the Penrose transform of a linearized graviton to the space of light rays
- States in Ramond sectors flesh out type II supergravity multiplets, as in RNS string

Curved background

While much of the story is parallel to RNS strings, these chiral strings cannot oscillate and amplitudes have no α' corrections. *Field theory*, not string theory.

- On curved background, Einstein eqns should be *exact* conditions for consistency

- Linearized eoms for vertex operators came from BRST algebra

$$G(z)\,\tilde{G}(z) = \frac{H}{z-w}$$

$$G = P_\mu(\psi_1^\mu + \mathrm{i}\psi_2^\mu)$$

$$\tilde{G} = P_\mu(\psi_1^\mu - \mathrm{i}\psi_2^\mu)$$

$$H = P^2$$

  rather than OPE with T.

- Expect Einstein eqs from anomaly here, rather than from β-functions

Unlike twistor space, *ambi*twistor space exists for any (geodesically complete) manifold.

$$S = \int_\Sigma P_\mu \bar\partial X^\mu + \tilde\psi_\mu \bar D \psi^\mu \; = \int_\Sigma \Pi_\mu \bar\partial X^\mu + \tilde\psi_\mu \bar\partial \psi^\mu$$

$$\bar D \psi^\mu = \bar\partial \psi^\mu + \Gamma^\mu_{\nu\lambda} \psi^\nu \bar\partial X^\lambda \qquad \Pi_\mu = P_\mu + \tilde\psi_\nu \gamma^\nu_{\mu\lambda} \psi^\lambda$$

Though the action remains free, the *currents* are deformed

$$\mathcal{G}^0 = \psi^\mu \Pi_\mu \qquad \tilde{\mathcal{G}}^0 = g^{\mu\nu} \tilde\psi_\mu \left(\Pi_\nu - \Gamma^\rho_{\nu\sigma} \tilde\psi_\rho \psi^\sigma \right)$$

$$\mathcal{H}^0 = g^{\mu\nu} \left(\Pi_\mu - \Gamma^\kappa_{\mu\lambda} \tilde\psi_\kappa \psi^\lambda \right)\left(\Pi_\nu - \Gamma^\rho_{\nu\sigma} \tilde\psi_\rho \psi^\sigma \right) - \frac{1}{2} R^{\kappa\lambda}{}_{\mu\nu} \tilde\psi_\kappa \tilde\psi_\lambda \psi^\mu \psi^\nu$$

Classically, the only non-trivial Poisson bracket is $\{\mathcal{G}^0, \tilde{\mathcal{G}}^0\} = \mathcal{H}^0$, reflecting the existence of ambitwistors

This theory is actually an example of a 'curved $\beta\gamma$-system'. Bosonic versions are subtle, due to anomlies in chiral determinants, but easier with SUSY.

[Malikov,Schechtman,Vaintrob; Nekrasov; Witten; Frenkel,Nekrasov,Losev]

- Operator $\mathcal{O}_V = V^\mu(X)\Pi_\mu + \partial_\nu V^\mu \, \tilde{\psi}_\mu \psi^\nu$ obeys OPE $\mathcal{O}_V(z)\, \mathcal{O}_W(w) \sim \dfrac{\mathcal{O}_{[V,W]}}{z-w}$ with no higher poles, so generates space-time diffeomorphisms

While all the basic fields transform correctly, $\mathcal{O}_V$ shows that the composite currents $(\mathcal{H}^0, \mathcal{G}^0, \tilde{\mathcal{G}}^0)$ have anomalous behaviour under target space diffeomorphisms.

To get something sensible, we must add quantum corrections to the currents.

The required modifications of $\mathcal{G}^0$ & $\tilde{\mathcal{G}}^0$ turn out to be

$$\mathcal{G} = \mathcal{G}^0 + \partial(\mathcal{L}_{\psi^\mu \partial_\mu} \log \Omega) \qquad \tilde{\mathcal{G}} = \tilde{\mathcal{G}}^0 + \partial(\mathcal{L}_{g^{\mu\nu} \tilde{\psi}_\mu \partial_\nu} \log \Omega)$$

where $\mathcal{L}$ is the Lie derivative and $\Omega = \sqrt{g}\, \mathrm{d}X^1 \wedge \cdots \wedge \mathrm{d}X^{10}$ or $\Omega = \mathrm{e}^{-2\Phi(X)} \sqrt{g}\, \mathrm{d}X^1 \wedge \cdots \wedge \mathrm{d}X^{10}$ in the presence of a dilaton.

These modified currents have the desired OPEs

$$\mathcal{O}_V(z)\, \mathcal{G}(w) \sim \cdots + \frac{\mathcal{L}_V \mathcal{G}}{z-w} \qquad \mathcal{O}_V(z)\, \tilde{\mathcal{G}}(w) \sim \cdots + \frac{\mathcal{L}_V \tilde{\mathcal{G}}}{z-w}$$

respecting target space diffeomorphisms.

To ensure these modified currents remain worldsheet primaries, we must also modify the stress tensor

$$T = T^0 - \frac{1}{2}\partial^2 \log(\mathrm{e}^{-2\Phi}\sqrt{g})$$

$$S \to S + \frac{1}{8\pi}\int_\Sigma R_\Sigma \log(\mathrm{e}^{-2\Phi}\sqrt{g})$$

Again this is similar, but not identical, to the way a dilaton is incorporated into the usual string.

- $R_\Sigma = 0$ locally, so short distance OPEs unchanged
- Stress tensor OPE takes usual form provided only $d = 10$. No field equations imposed here.

Because the curved space action is trivial, we can compute OPEs *exactly*. One finds

$$\mathcal{G}(z)\,\mathcal{G}(w) \sim 0 \qquad\qquad \tilde{\mathcal{G}}(z)\,\tilde{\mathcal{G}}(w) \sim 0$$

just requires the usual Bianchi identities on $R^{\kappa}{}_{\lambda\mu\nu}$, while (also allowing for a B-field)

$$\mathcal{G}(z)\,\tilde{\mathcal{G}}(w) \sim \frac{2}{(z-w)^3}\left(R + 4\nabla_\mu\nabla^\mu\Phi - 4\nabla_\mu\nabla^\mu\Phi - \frac{1}{12}H^2\right)$$
$$+2g^{\nu\lambda}\frac{(\Gamma^\mu_{\kappa\nu}\partial X^\kappa + \psi^\mu\tilde{\psi}_\nu)}{(z-w)^2}\left(R_{\mu\lambda} + 2\nabla_\mu\nabla_\lambda\Phi - \frac{1}{4}H_{\mu\rho\sigma}H_\lambda{}^{\rho\sigma}\right)$$
$$+\frac{\psi^\mu\psi^\nu - \tilde{\psi}^\mu\tilde{\psi}^\nu)}{(z-w)^2}\left(\nabla_\kappa H^\kappa{}_{\mu\nu} - 2H^\kappa{}_{\mu\nu}\nabla_\kappa\Phi\right) + \frac{\mathcal{H}}{z-w}$$

where $\mathcal{H}$ ($= \mathcal{H}^0$+ quantum corrections) generalizes P^2

The Einstein, B-field and dilaton eqns are the *exact* conditions for a consistent background

In flat space, $G(z)\tilde{G}(w) \sim \dfrac{H}{z-w} = \dfrac{P^2}{z-w}$ and requiring this algebra to hold in the presence of vertex operators imposed the scattering equations

Vertex operators are infinitesimal deformations of these currents. Requiring the same algebra to hold nonlinearly amounts to the full nonlinear field equations

Quantum Scattering Equations $\leftrightarrow$ Einstein Field Equations

Further directions

Given the worldsheet theory, can look at higher genus.

- At genus g, still have $\bar{\partial} P_\mu(z) = \sum_{i=1}^{n} k_{i\mu}\, \delta(z - z_i)\, \mathrm{d}z$ but this now implies

$$P_\mu(z) = \sum_{\alpha=1}^{g} \ell_\mu \omega_\alpha - \sum_{i=1}^{n} k_{i\mu}\, \partial \ln E(z - z_i; \tau)$$

prime form

basis of $H^0(\Sigma, K_\Sigma)$

- Again $P^2(z)$ is a meromorphic quadratic differential, now with $3g - 3 + n$ moduli.

Higher genus analogue of the scattering equations is

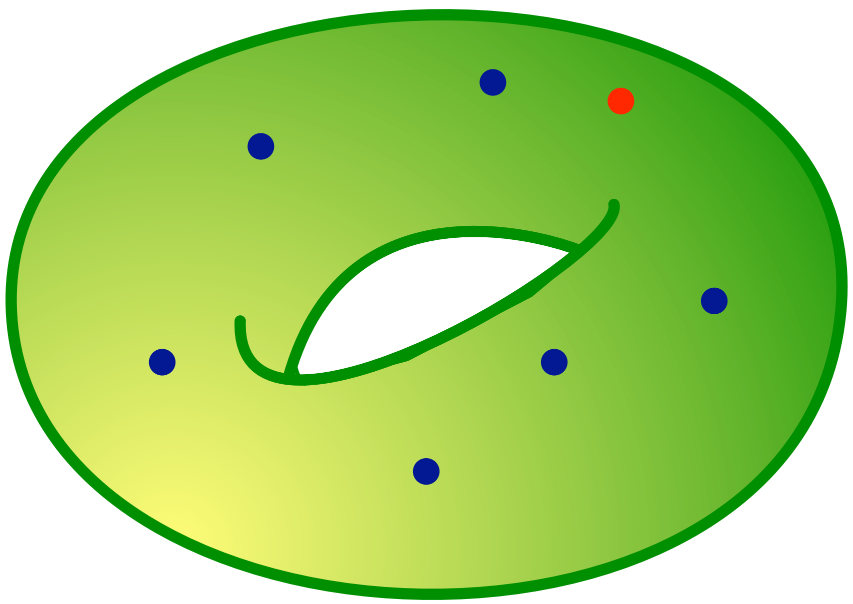
$$\mathrm{Res}_{z_i} P^2 = 0 \qquad\qquad P^2(z_j) = 0$$

at exactly enough points to ensure $P^2(z) = 0$ identically

For example, at genus 1

$$P_\mu(z) = \left(\ell_\mu + \sum_{i=1}^{n} k_{i\mu} \, \frac{\theta_1'(z - z_i, \tau)}{\theta_1(z - z_i, \tau)}\right) \mathrm{d}z$$

and the scattering equations enforce



$\mathrm{Res}_{z_i} P^2 = 0$ for $i \in \{1, 2, \ldots, n-1\}$

$P^2(z_j) = 0$ at any one point z_j

- Not an arbitrary prescription, but dictated by the BRST procedure at higher genus (with moduli)
- Agrees with Gross-Mende saddle point when $n = 4$

[Casali, Tourkine]

The genus 1 amplitude is (even spin structure part)

$$\mathcal{M}^1 = \int \mathrm{d}^{10}\ell \; \mathrm{d}\tau \; \bar{\delta}(P^2(z_n;\tau)) \prod_{i=1}^{n} \bar{\delta}(k_i \cdot P(z_i;\tau))$$
$$\times \sum_{\alpha,\beta} (-1)^{\alpha+\beta} Z_{\alpha,\beta}(\tau) \, \mathrm{Pfaff}'(\Psi) \, \mathrm{Pfaff}'(\tilde{\Psi})$$

where the Pfaffians come from free fermion correlators on the torus. We've checked the amplitude has:

- correct behaviour in separating degeneration
- correct R^4 tensor structure at four points and is the integral of a rational function [Casali, Tourkine]
- correct behaviour in IR limit

but do not yet have a complete proof this is gravity.

Conclusions

I’ve argued that the CHY formulation of massless amplitudes really means there’s an underlying theory in the space of light rays

- Hybrid form of complexified worldline & infinite tension chiral string
- Spectrum contains only massless states. Amplitudes are CHY formula for gravity
- Scattering equations are the avatars of nonlinear field equations

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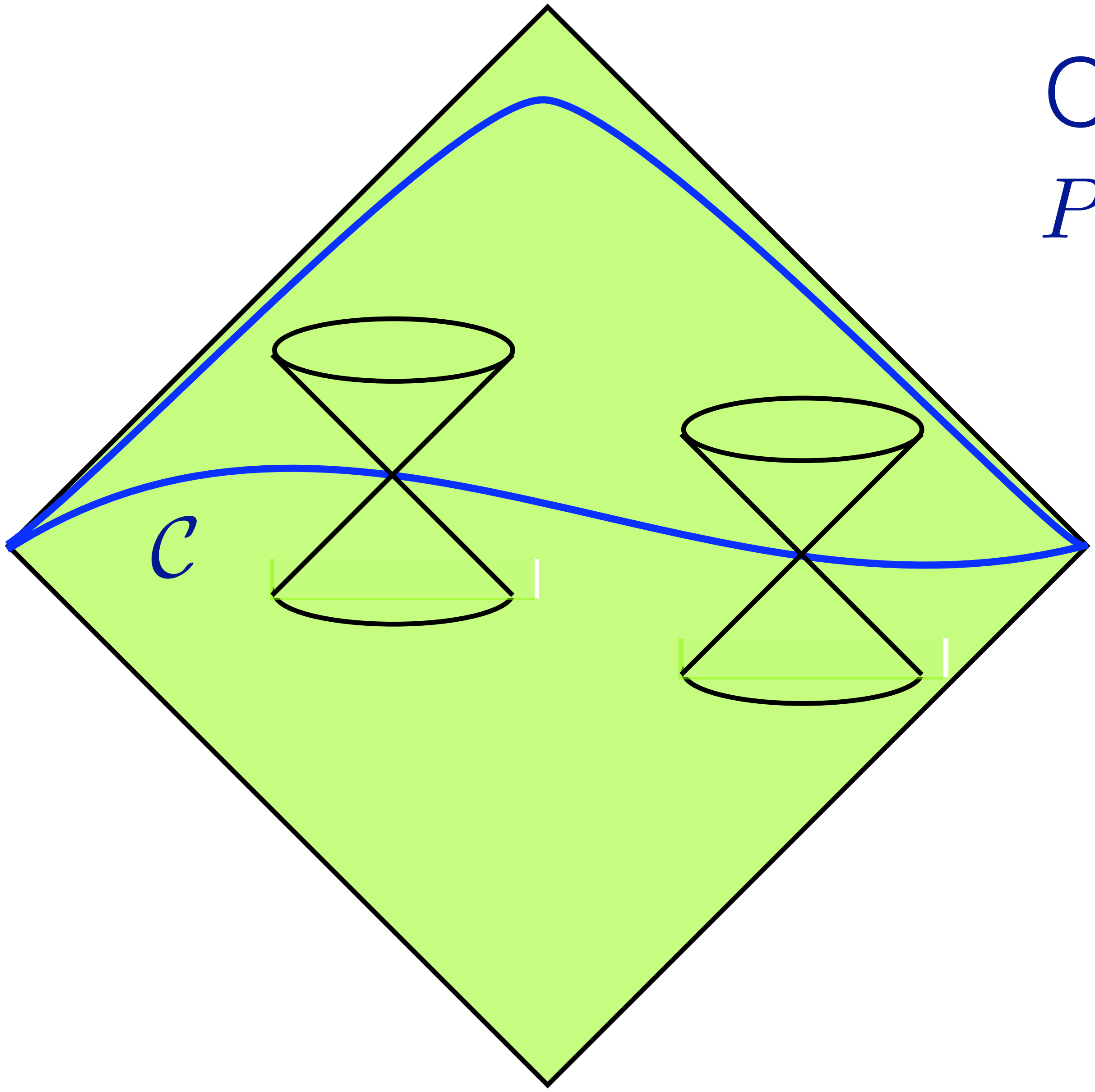
There are many open questions

- Can we prove loop formula is correct?
- How many other models exist? (*e.g.* Einstein-Yang-Mills)
- Compute on other backgrounds? Are there worldsheet instantons?

Thank you



Can also regard ambitwistors as $PT^*\mathcal{C}$ for any Cauchy surface $\mathcal{C}$