Monopoles, Lattices and Holography

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Based on work with Stefano Bolognesi
arXiv:1010.4178

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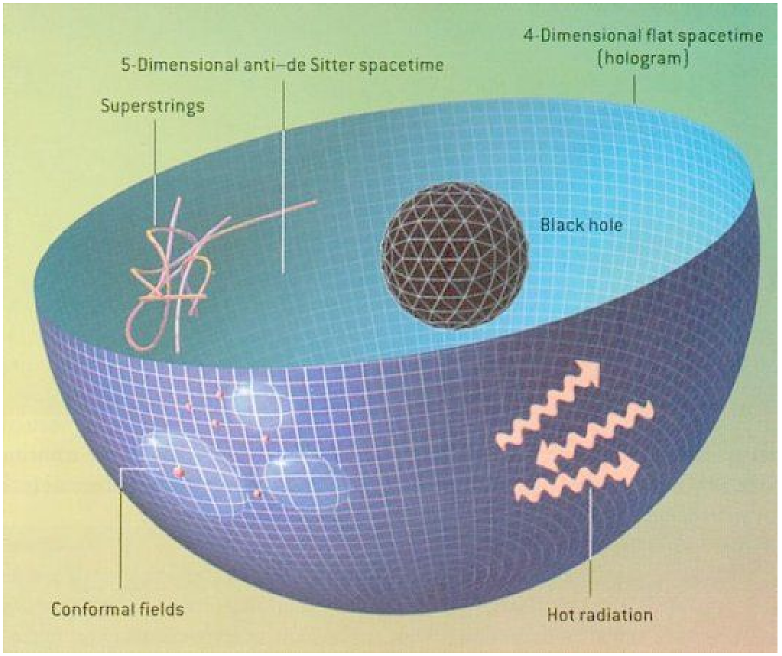
What can strongly interacting matter do?

- Goal: Understand the map between physical phenomena in the bulk and boundary



AdS/CFT and Monopoles

What is the boundary signature
of bulk magnetic monopoles?

Review: ‘t Hooft-Polyakov Monopoles

- SU(2) Yang-Mills + adjoint Higgs

$$\mathcal{L} = -\frac{1}{2e^2}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \frac{1}{e^2}\mathrm{Tr}\, \mathcal{D}\phi^2 - \lambda \mathrm{Tr}(\phi^2 - v^2)^2$$

- $\phi^a\phi^a \to v^2$ as $x \to \infty$
- Map $S^2_\infty \to S^2_{\mathrm{vacuum}}$ labelled by winding.
- This is magnetic charge

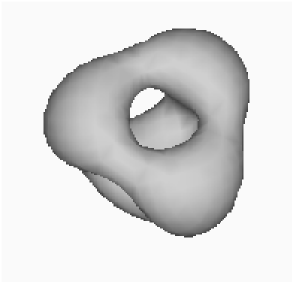
BPS Monopoles

- Nice things happen when the potential vanishes

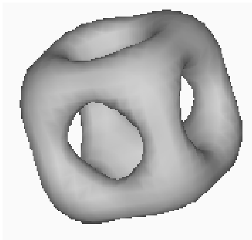
$$B_i = \mathcal{D}_i \phi$$

- The force between monopoles cancels
  - multi-monopole solutions

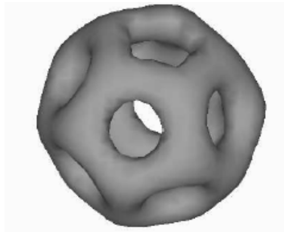
Multi-Monopoles



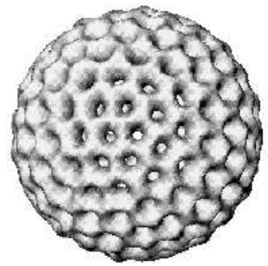
n=3



n=4



n=7



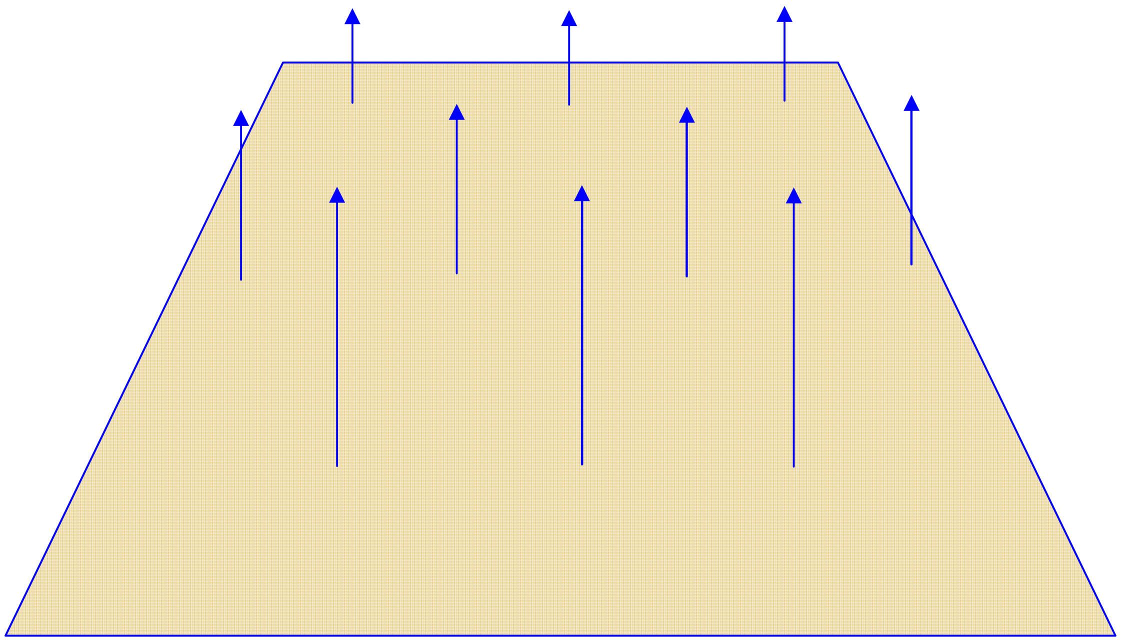
n=97
(skyrmions)

$$\text{radius} \sim \frac{n}{v}$$

Images by Paul Sutcliffe

The Monopole Wall

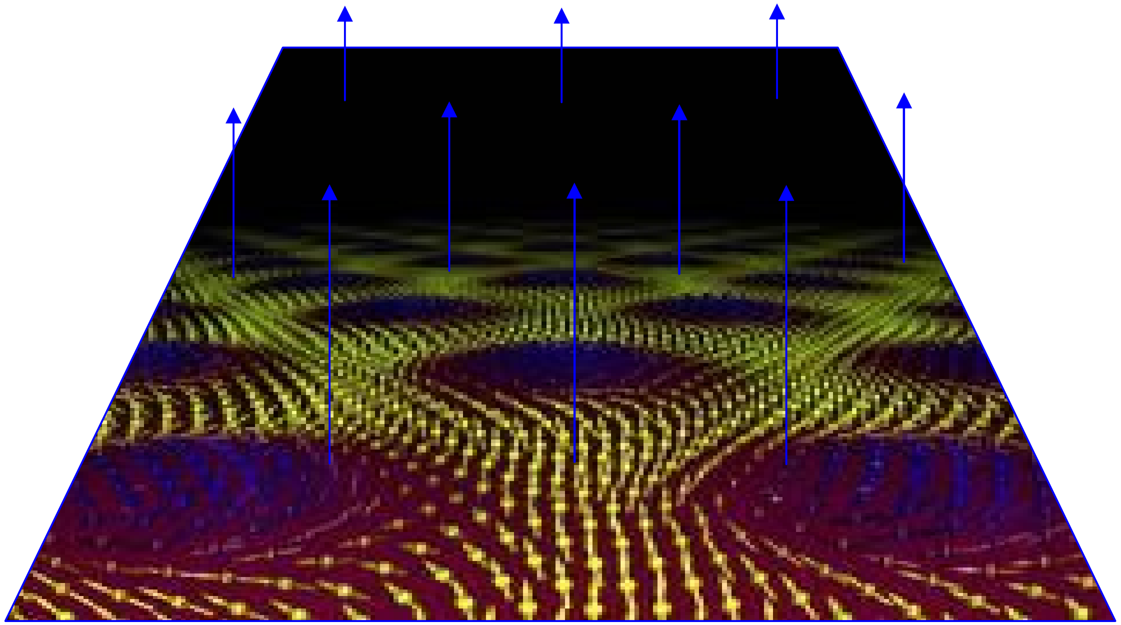
$\vec{B} \neq 0$ $\qquad$ $\phi \sim \sigma^3 z$



$\vec{B} = 0$ $\qquad$ $\phi = 0$

The Monopole Wall

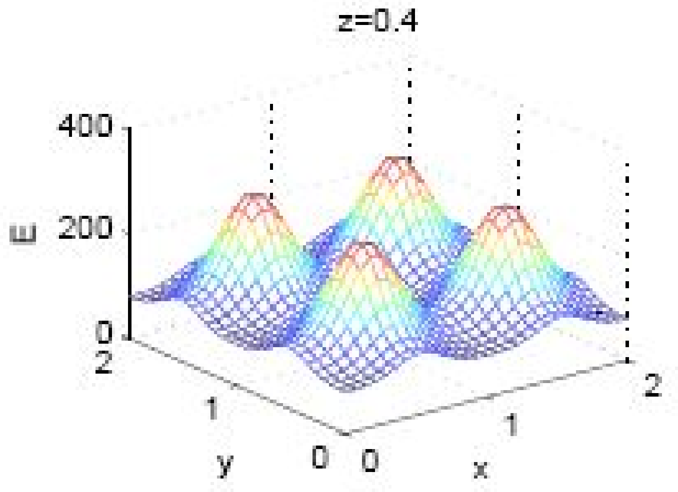
$\vec{B} \neq 0 \qquad \phi \sim \sigma^3 z$



$\vec{B} = 0 \qquad \phi = 0$

The Lattice Structure

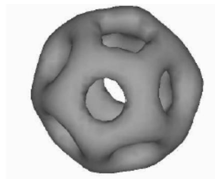
Richard Ward 2006



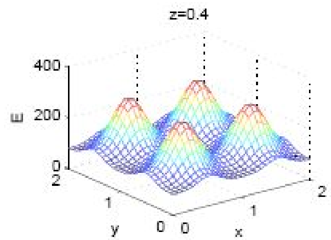
- Explicit solutions are not currently known.
- Lattice structure is a *moduli* of Bogomolnyi equations

Monopoles in AdS$_4$

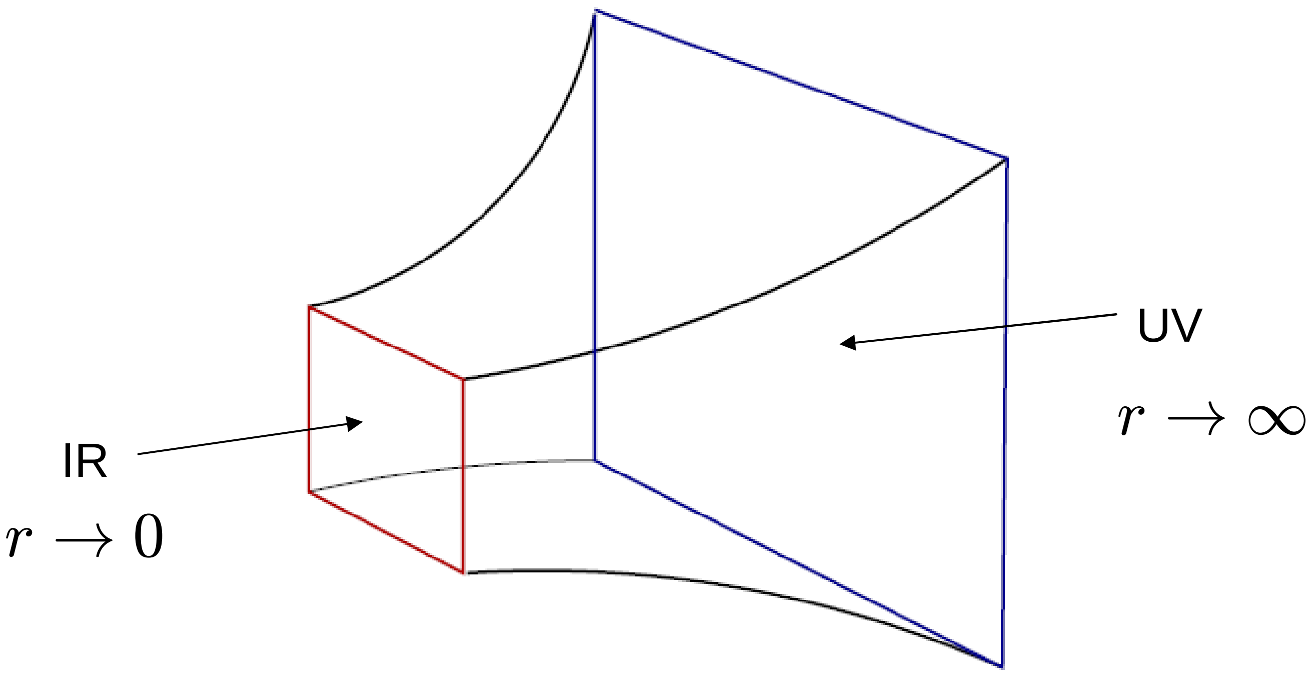
- Global AdS has boundary $\mathbb{R} \times \mathbf{S}^2$
  - Natural place for magnetic monopoles



- Planar AdS has boundary $\mathbb{R}^{2,1}$
  - Natural place for the monopole wall



Planar AdS



$$ds^2 = \frac{r^2}{L^2}(-dt^2 + dx^2 + dy^2) + \frac{L^2}{r^2}dr^2$$

Yang-Mills-Higgs in AdS

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R - \frac{L^2}{6}\right) - \frac{1}{2e^2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu} + \mathcal{D}\phi^2\right)\right]$$

- Dictionary: $\mathrm{A}^a_\mu \leftrightarrow J^a_\mu$ global *SU(2)* symmetry

  $\phi^a \leftrightarrow \mathcal{O}^a$ triplet of scalar operators

- Asymptotics: $$\phi^a \to v^a + \frac{\langle \mathcal{O}^a \rangle}{r^3} + \ldots$$

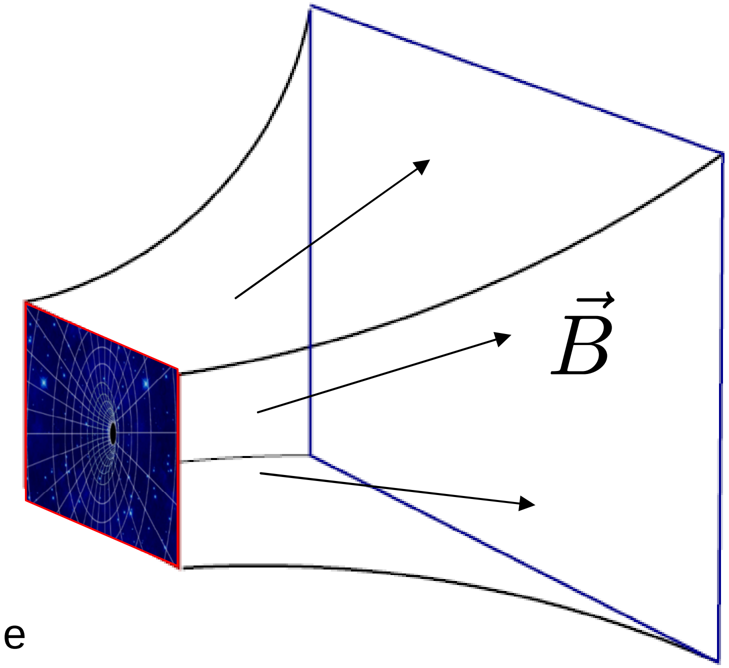
  source

- *Explicit* breaking of global *SU(2):* $$\partial^\mu J^a_\mu = \epsilon^{abc} v^b \mathcal{O}^c$$
  - Unbroken *U(1)* global symmetry

Background Magnetic Field

$$A_i \to \tfrac{1}{2} B \epsilon_{ij} x^j + \ldots \quad \Longrightarrow \quad \mathcal{L}_{QFT} = A_i J^i + \ldots$$

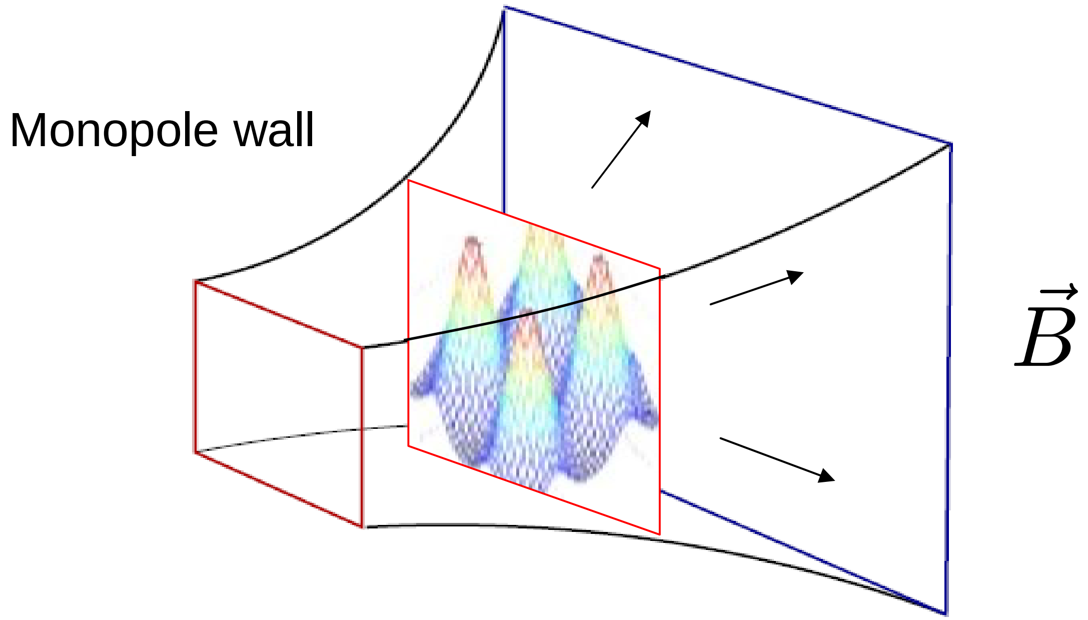
source



Magnetic Reissner-
Nordström Black Hole

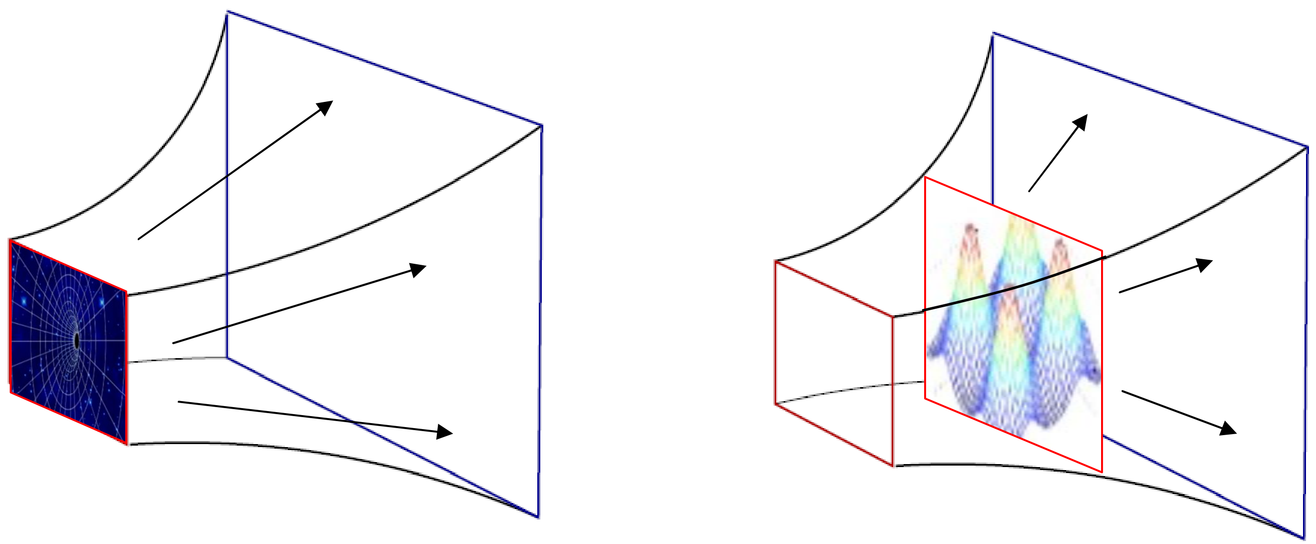
Usually this is the ground state. But…

Another Candidate Ground State



Which Wins?

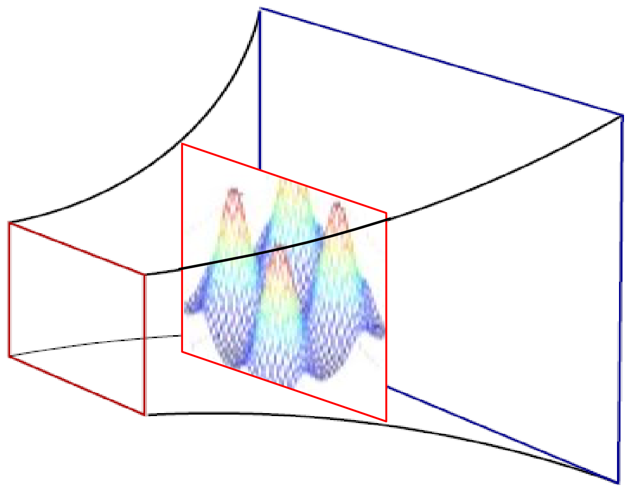
- Both have the same asymptotics (i.e. B field)



A Model for the Wall

Bolognesi, 2005

- Neglect lattice structure (for now)
- Energy is in long range tails…treat using an Abelian approximation



$$\phi = v\left(1 - \frac{R_{\rm wall}^3}{r^3}\right)$$

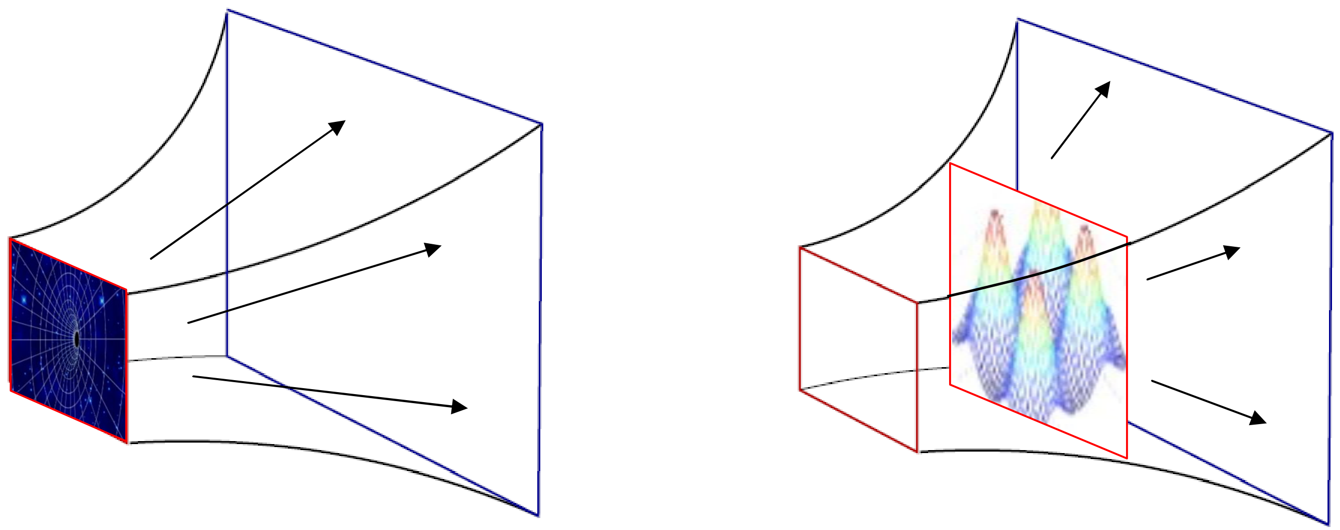
$$R_{\rm wall} = \sqrt{\frac{BL^3}{3v}}$$

The Winner is…

- At zero temperature, the monopole wall wins for $vL \gg 1$, $v^2 < \frac{e^2}{\kappa^2}$

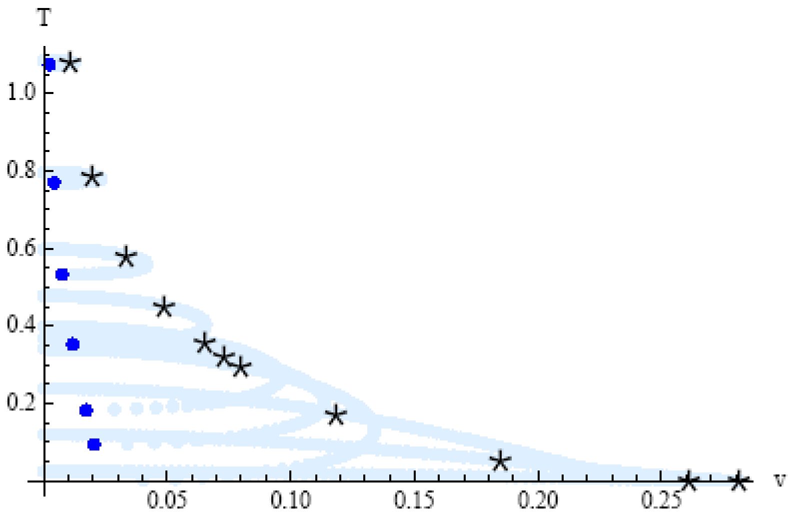
Flat space monopole wall sits in AdS

No backreaction to black hole



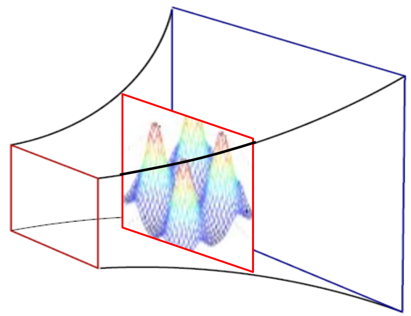
Higher Temperatures…

- First order phase transition to the black hole at ★



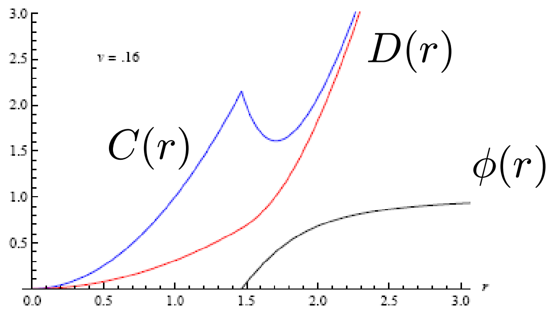
Physics of the Boundary Theory

- Infra-Red
  - Magnetic Field Screened $\vec{B} = 0$
  - SU(2) global symmetry restored $\phi = 0$
  - Lower speed of light



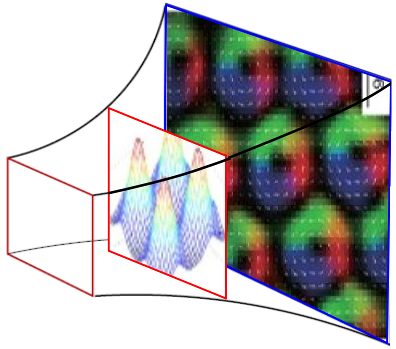
$$ds^2 = -C(r)dt^2 + \frac{dr^2}{D(r)} + \frac{r^2}{L^2}(dx^2 + dy^2)$$

$$c_{\text{eff}}^2 = \frac{C(R)}{D(R)}$$



More Physics of the Boundary Theory

- Expectation Values
  - Spontaneously broken translational symmetry (Skyrme lattice)
  - p-wave breaking of U(1) by W-bosons



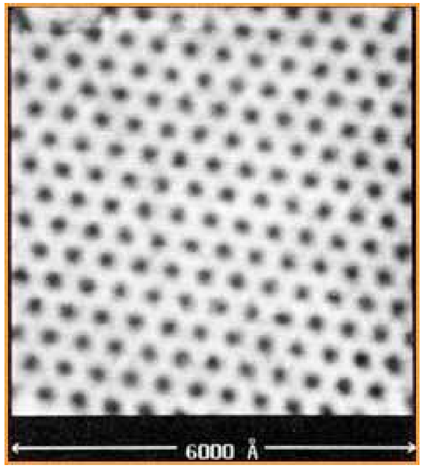
$$\langle \mathcal{O} \rangle \sim B^{3/2} \exp\left(-\sqrt{vL}\right)\, \sin(x/\Gamma)\, \sin(y/\Gamma)$$

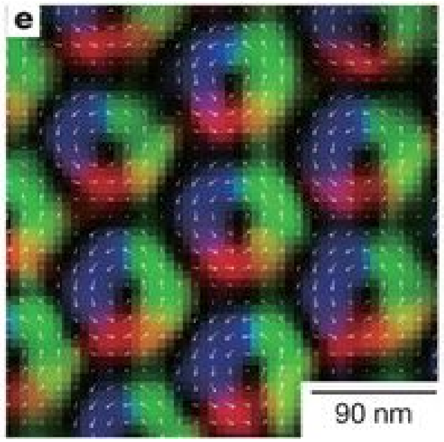
$$\Gamma \sim 1/\sqrt{B}$$

  - Lattice structure not currently known

Dynamical Lattice Formation

- Known or predicted to occur in several systems
  - Abrikosov lattice; chiral magnets (Skyrme lattice)





- Both very different physics

Lattice Formation in QH System

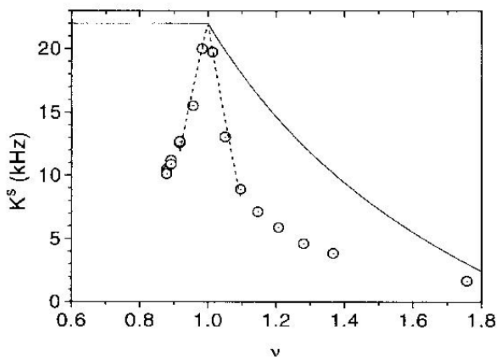
Sondhi et. al. (1993)

- Spin polarized $\nu = 1$
  - Low energy excitations from magnetic orientation
  - Berry Phase ensures Skyrmion has charge 1

$$\nu = 1 \pm \epsilon$$

- Formation of Skyrmions in ground state
  - Skyrme lattice
  - Different scaling

$$\Gamma \sim \left(\frac{e^2}{\epsilon g \mu_B B n_0}\right)^{1/3}$$



Barrett et. al. (1995)

Future Questions

- Better understanding of solution…numerics
- Realization in Nature?
- Transport
- Is this the strongly coupled holographic superconductor?
  - Suggestions that true ground state should be a crystal of spins