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Open-Closed String Duality in Field Theory?

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Abstract: I present an open-string description of solitonic domain walls in a semi-classical field theory. I speculate on the possibility for an open-closed string duality in this setting.

Introduction

Everybody loves a good D-brane. These objects have underpinned much of the progress in high-energy theoretical physics for the past ten years, culminating in the miraculous AdS/CFT correspondence which, at its heart, relies on the equivalence of open and closed string descriptions of D-brane dynamics. The purpose of this talk is to describe semi-classical D-brane objects in simple field theories, by which I mean gauge fields interacting with scalars and fermions, decoupled from the complications of gravity. I will then explain how, in certain regimes, an open-string description of the dynamics becomes viable [1].

So what is a D-brane? If we simply define it to be a dynamical surface on which strings can end, then Nature offers several examples, most notably in the arena of fluid dynamics. For example, the AB-interface in superfluid ^{3}He is a D-brane on which a vortex string may end. More prosaically, one may view a cumulonimbus cloud base as a D-brane on which a tornado or a water spout may terminate. You can learn more about this system at the twistor session of this conference.

Figure 1:

However, there is one crucial feature that these D-branes do not share with those found in string theory: there is no regime in which an open-string description holds. One would have a very hard time convincing a meteorologist that, as two clouds approach, their dynamics is governed by quantum effects of virtual tornadoes stretched between them. Yet this is precisely what happens for D-branes in string theory. Indeed, it is the key feature that governs many of the fascinating properties of D-branes. And, as we shall see, it is also what happens for the solitons in this talk.

The Field Theory and its Spectrum

Since we are working with common-or-garden field theories, both the strings and the D-branes must appear as solitonic objects. We will consider a theory in $d = 3 + 1$ dimensions that contains both vortex strings and domain walls in its solitonic spectrum. The simplest example is a $U(1)$ gauge theory, coupled to N complex scalar fields q_i, $i = 1, \ldots, N$, each of charge $+1$. There is also a single, real, neutral scalar field ϕ. The scalar potential is given by

$$V = \frac{e^2}{2}(\sum_{i=1}^{N} |q_i|^2 - v^2)^2 + \sum_{1=1}^{N}(\phi - m_i)^2|q_i|^2 \tag{1.1}$$

This is the truncation of a theory with $\mathcal{N} = 2$ supersymmetry. While the bosonic action above will suffice to describe the classical solitons, when we come to the open string description we will work with the full $\mathcal{N} = 2$ theory, complete with fermions.

When the masses m_i are distinct, the theory (1.1) has N isolated vacua given by $\phi = m_i$ and $|q_j|^2 = v^2\delta_{ij}$ for $i = 1, \ldots, N$. Excitations around each vacuum are gapped. The photon has a mass $M_\gamma = ev$, where e^2 is the gauge coupling. The complex scalars have a mass $M_q = |m_i - m_j|$ for $i \neq j$. The quantum theory also has another, more nefarious, scale: it is the Landau pole $\Lambda \sim ev\,\exp(+1/e^2N)$, above which the theory ceases to make sense. In the following we will take the limit $e^2 \to \infty$[1]. The strong coupling limit $e^2 \to \infty$ is usually considered sick in theories not displaying asymptotic freedom because $\Lambda \to M_\gamma$. To avoid these problems we simply take the UV cut-off of the theory to lie in the region $v, m_i \ll \mu_{UV} \ll M_\gamma$. This means that we ignore all dynamics of the massive photon, and the theory should really be thought of as a proxy for the massive sigma-model of interest. In this limit, the classical theory reduces at energies $E \ll M_\gamma$ to a bunch of interacting massive scalars, described by a sigma-model with potential. Sigma-models in four-dimensions are, of course, non-renormalizable; this is the price we've paid for eliminating the Landau pole. At energies $E \ll v$ the theory is weakly coupled. We will implicitly assume an ultra-violet completion at energies $E \gg v$.

The Solitons

Our theory admits both string and domain wall solitons. Let's start with the strings which are vortices of the type introduced into high-energy physics over 30 years ago by Nielsen and Olesen[3]. The vortices are supported by the phase of the condensed scalar q_i, which winds around the $z = x^1 + ix^2$ plane transverse to the string. They are 1/2-BPS, with tension $T_{\rm vort} = 2\pi v^2$ and width $L_{\rm vort} = 1/ev$. Notice that in the sigma-model limit $e^2 \to \infty$, the vortex becomes singular as befits a fundamental string.

The second soliton is a domain wall whose existence is guaranteed by the multiple, isolated vacuum states. We impose the vacuum $\phi = m_i$ at $x^3 = -\infty$ and $\phi = m_j$ at $x^3 = +\infty$. The tension of the wall is $T_{\rm wall} = v^2\Delta m$ where $\Delta m = |m_i - m_j|$. In the $e^2 \to \infty$ limit of interest, the width of the wall is $L_{\rm wall} = 1/\Delta m$. From now on we order the masses $m_1 > m_2 > \ldots > m_N$.

[1] It is not clear if this limit is necessary, but it evades certain subtleties regarding boojums [2] — energy associated to the end point of the open string — which scale as $1/e^2$ and must otherwise be treated with care when quantizing the open string.

A single domain wall, interpolating between neighboring vacua $\phi = m_i$ and $\phi = m_{i+1}$, has a collective coordinate X^3, describing the center of mass. However, the domain wall also has a second, internal, collective coordinate [4]. To see this note that the original theory is invariant under a $U(1)^{N-1}$ flavor symmetry, in which the phases of the q_i fields are rotated independently. In each vacuum, only a single $q_i \neq 0$, and the rotation $q_i \to e^{i\sigma} q_i$ coincides with the gauge action and does not lead to any new physical state. However, in the center of the domain wall both q_i and q_{i+1} are non zero, and the global action $q_i \to e^{i\sigma} q_i$ with $q_{i+1} \to e^{-i\sigma} q_{i+1}$ does now sweep out a new physical configuration. The center of mass X^3 and the phase collective coordinate σ parameterize the domain wall moduli space $\mathbf{R} \times \mathbf{S}^1$. The low-energy dynamics of the domain wall is determined by promoting the collective coordinates to dynamical degrees of freedom in order to describe long wavelength fluctuations of the position and internal phase orientation.

Wall Dynamics: Bulk Description

We now consider the scattering of two parallel domain walls using the classical equations of motion. We call this the “bulk description”.

In order to have two walls (as opposed to a wall and anti-wall) we need at least three vacua. We therefore choose $N \geq 3$ and set $\phi = m_{i-1}$ at $x^3 = -\infty$ and $\phi = m_{i+1}$ at $x^3 = +\infty$. With these boundary conditions there are domain wall solutions which have the profile of two domain walls, separated by an arbitrary distance [7] R. In between the two walls, the fields lie exponentially close to the middle vacuum configuration $\phi = m_i$.

Figure 2:

The system with two walls has 4 collective coordinates, arising from the position and phase degree of freedom for each wall. The moduli space naturally factors into the center of mass and relative degrees of freedom: $\mathcal{M}_{2-\mathrm{wall}} \cong \mathbf{R} \times (\mathbf{S}^1 \times \mathcal{M}_{\mathrm{cigar}})/\mathbf{Z}_2$, where $\mathcal{M}_{\mathrm{cigar}}$ is a cigar like manifold describing the relative position and phase of the wall. The asymptotic, cylindrical regime of the cigar corresponds to far separated domain walls; the tip of the cigar corresponds to the configuration in which the domain walls sit on top of each other.

The low-energy scattering of the domain walls is described by a sigma-model on $\mathcal{M}_{cigar}$. The metric may be computed explictly [8], but is not required to understand what happens when two walls collide. Geodesics on the moduli space simply round the tip of the cigar, corresponding to two walls approaching and rebounding in finite time, with their relative phase shifted by π. In particular, the key feature of the dynamics is so obvious that it is almost not worth stating: our domain walls cannot pass.

We need to analyze the regime of validity of this bulk calculation. Since the computation was purely classical, we must ensure that quantum effects can be consistently ignored. Higher order quantum corrections to the classical action will appear as an expansion in derivatives (∂/v). For domain walls of width $L_{\rm wall} = 1/\Delta m$, these may be consistently ignored when $\Delta m/v \ll 1$.

Wall Dynamics: Open String Description

We will now present a very different, dual, perspective on domain wall scattering, which we call the "open string description". Recall that the $d = 2+1$ dimensional worldvolume of the domain wall houses a periodic scalar σ. We may exchange this in favor of a photon using the duality map is $\partial_\alpha \sigma \sim \epsilon_{\alpha\beta\gamma} F^{\beta\gamma}$. In these variables, the low-energy effective action for the walls is a $d = 2+1$ $U(1)$ gauge theory. Including the fermion zero modes, this theory has $\mathcal{N} = 2$ supersymmetry (4 supercharges).

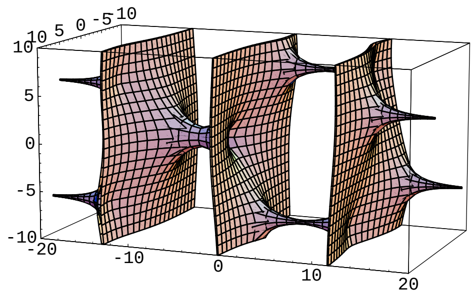

Figure 3:

The existence of a $U(1)$ gauge field living on the domain wall is strikingly reminiscent of D-branes in string theory. But, so far, the derivation of the gauge field is rather cheap because nothing is charged under it. In fact, one can show that the vortex string can terminate on the domain wall, where its end is electrically charged [5, 6]. There are a number of ways to see this. One is to write down the explicit solution for the string ending on the domain wall which can be found analytically in $e^2 \to \infty$ limit [5]. Another is to study the "BIon' spike solution emanating from the wall [5, 6]. Finally, one may find numerical solutions describing multiple strings ending on multiple walls; an example, taken from [9], is shown in the figure.

In the "open string description" of domain wall scattering, we will not consider the classical profile of the domain walls in four-dimensions, nor the bulk field theory interactions between them as they overlap. Instead we will treat the two domain walls as free objects, each described by a $U(1)$ gauge theory, and each free to roam along the x^3 line. In particular, they may move past each other unimpeded. The only interaction between the two walls comes from the quantum effects of open vortex strings stretched between the walls. Note that the quantum open string effects must be very powerful for they must, ultimately, stop the domain walls from passing each other. Let us see how this occurs. (Full details can be found in [1]).

To study the effect of open strings, we must first ask what state the stretched open string gives rise to. The lowest open string mode has electric charge $(+1,-1)$ under the two gauge fields on the domain walls, and bare mass[2] $T_{\rm vort}R$. We also know that the stretched open string is 1/2 BPS in the domain wall worldvolume, ensuring that the lowest mode lies in a short representation of the supersymmetry algebra. There are two possibilities: a vector multiplet or a chiral multiplet. We are used to seeing light vector multiplets, and the associated non-abelian gauge symmetry enhancement, appearing in string theory when two identical branes coincide. However, this cannot be the case for us since our domain walls are not identical. For example, they have different tensions. So we conclude that the lowest mode of the open string gives rise to a charged chiral multiplet in $d=2+1$ dimensions.

I therefore claim that the relative dynamics of two domain walls is governed by a $d=2+1$, $\mathcal{N}=2$ supersymmetric $U(1)$ gauge theory coupled to a single chiral multiplet. However, there is a subtlety. Integrating out a charged chiral multiplet in $d=2+1$ dimensions induces a Chern-Simons coupling. Integrating *in* a charged chiral multiplet must also therefore induce a Chern-Simons coupling, $\kappa A\wedge F$, where $\kappa=-1/2$.

The Chern-Simons term gives a mass to the gauge field on the wall. By supersymmetry, it also gives a mass to the separation of the walls R. This suggests that, in this open string description, the walls cannot be separated. But, of course, we must look at the quantum theory and, in particular, the effect of integrating out the open string mode. This gives a finite renormalization: $\kappa\rightarrow\kappa_{\rm eff}=-\frac{1}{2}+\frac{1}{2}{\rm sign}\,(R)$, where ${\rm sign}\,(R)$ is the sign of the mass of the fermion in the chiral multiplet. We learn that we can separate the walls in the positive $R>0$ direction at no cost of energy. Of course, this is not surprising: we have simply integrated in the chiral multiplet, and immediately integrated it out again, leaving us with no effective Chern-Simons coupling. But if we try to pass the domain walls through each other and separate them in the opposite direction $R<0$, the Chern-Simons coupling bites, and the flat direction is lifted. The final effect of the quantum open strings on the domain wall dynamics is best summarized in the famous words of Gandalf: you cannot pass.

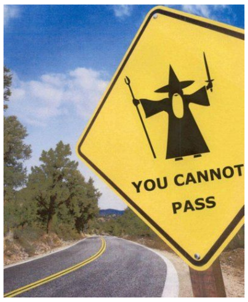

Figure 4:

One may study the low-energy dynamics of the Chern-Simons theory in more detail. After transforming the dual photon σ, one finds that the long wavelength interactions

[2] As stressed in [10], the physical mass of the state is infinite due to a logarithmic divergence associated to both the gauge field and the wall separation R. This is the familiar infra-red divergence in $d=2+1$ dimensions that occurs also for the D2-brane dynamics in string theory. It means that we cannot excite the modes of the open string on-shell, but this does not concern us here. Rather we are interested in the virtual effect of these states on the dynamics of the massless brane modes. And, as we shall see, these are perfectly finite.

of the massless modes R and σ are again described by a sigma-model with target space given by the cigar.

Finally, let us decide when the open-string approach is valid. We have integrated in the lightest mode of an open stretched string, and ignored the tower of higher excitations. This requires $T_{\rm vort}R \ll \Delta m$, where Δm is the excitation gap for internal modes of the string. If we also require that $R \gg L_{\rm wall} = 1/\Delta m$, then we are left with the condition for the validity of the open string description: $\Delta m/v \gg 1$. This is the opposite regime to where we performed the classical bulk computation[3].

Discussion

What are the implications of a field theory soliton whose dynamics admit both a bulk and open string description? Let me start by addressing what is, perhaps, the most familiar aspect of D-branes. Ask the average man on the street what characterizes a D-brane and he will answer that the tension goes as the inverse string coupling, $T \sim 1/g_s$. Is this true for our D-branes? In fact, it's hard to tell since we don't have a good handle on the string coupling g_s in these theories. We may evocatively write the tension of the domain wall as $T_{\rm wall} = (\Delta m/v)\,(1/\alpha'^{3/2})$ with $\alpha' = 2\pi/T_{\rm vortex}$, suggesting that the string coupling is $g_s = v/\Delta m$. Is this plausible? For $g_s \ll 1$, the closed loop of string appears to be the lightest object in the field theory. However, it is precisely in this regime that the sigma-model breaks down and one must study the UV completion of the theory, including potential renormalization to closed loops of string. It certainly seems possible that there exists a 4d little string theory completion of the theory. In this case, the relationship $g_s = v/\Delta m$ is very reasonable and is entirely analogous to similar expressions that appear in 6d little string theories in the double scaled limit.

In fact, the question of a UV completion has bearing on a more important issue. I referred to the classical scattering of domain walls as the"bulk calculation" rather than the more familiar "closed string" regime. Is this latter phrase appropriate? Can the bulk sigma-model fields be thought of as quantized loops of closed vortex string? I don't know the answer, but only if a UV completion exists can we think of the two descriptions of the domain wall dynamics as reflecting an underlying open-closed string duality, with the two methods above corresponding to suitable lowest-mode truncations of a modular invariant annulus partition function.

[3]Indeed, the two calculations differ in the details. For example, in the classical computation, interactions are exponentially suppressed in the separation $\exp(-R\Delta m)$, while the open string approach has power-law interactions $1/R$. In fact, the latter is not to be trusted: the infinite tower of higher open string modes may possibly sum to produce an exponential suppression after all.

If such an open-closed string duality is indeed at play for our solitonic vortex strings, one may try to be more ambitious and study an AdS/CFT-like correspondence in theories without gravity. On a practical level, Maldacena's big breakthrough can be thought of as introducing a factor of N, the number of D-branes, to allow two, seemingly mutually exclusive, regimes of validity to overlap. In the present case it seems difficult to implement this. On a trivial level, our domain walls are not identical objects and bringing many of them together does not improve the validity of the bulk calculation. On a more fundamental level, questions of entropy matching between the two sides appear much more of a hurdle in theories without gravity and the associated thinning of degrees of freedom.

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