# The DBI Model of Inflation

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ABSTRACT

A novel mechanism of inflation based on the Dirac-Born-Infeld (DBI) action is described. The model generates a distinctive spectrum of density perturbations, with a lower bound on non-Gaussianity. It is falsifiable and observationally distinct from traditional slow-roll inflation.

## 1. Introduction

In this talk I will describe a model of inflation based on ingredients from string theory. The resulting spectrum of density perturbations in the CMBR includes a distinctive non-Gaussian signal at a level observable in upcoming satellite experiments, and a strong preference for observable tensor modes. The work presented here was done in collaboration with Mohsen Alishahiha and Eva Silverstein [1,2]

## 2. Cosmology with a Speed Limit

Our starting point is the brane inflation scenario, with a D3-brane in type IIB string theory moving down a warped throat [3,4]. At the UV end, the throat joins smoothly onto a Calabi-Yau compactification which acts as an Randall-Sundrum Planck brane. This ensures that four-dimensional gravity is dynamical. In the IR, the throat is smoothly capped off at a scale $\phi_{IR} \sim g_s m$.

Usually in such inflationary scenarios, the D3-brane is taken to move slowly down the throat which maps to the condition for slow-roll in the 4d effective Lagrangian. We will consider a somewhat different regime in which the brane moves down the throat approaching the speed of light. Let $\phi$ denote the position of the brane. Then for $\phi_{IR} < \phi < \phi_{UV}$, we can approximate the throat by an $\mathrm{AdS}_5$ of radius $R$. (A better approximation would be the Klebanov-Strassler throat, but this makes little difference to the ensuing physics). The dynamics of the brane is captured by the Dirac-Born-Infeld (DBI) action coupled to gravity,

$$\mathcal{L}_{\mathrm{brane}} \sim -\frac{1}{g_s}\sqrt{-g}\left(\frac{\phi^4}{\lambda}\sqrt{1+\lambda\frac{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{\phi^4}}+m^2\phi^2\right) \tag{1}$$

Here $\lambda \sim (R/l_s)^4$. Note that we have included a mass term $m \sim \phi_{IR}/g_s$ for $\phi$. Such a mass is generically generated for a scalar field which is protected by conformal invariance only up to the scale of the capped off throat.

A crucial point about the action (1) is that it imposes a speed limit on how quickly $\phi$ can change. For a homogeneous field, reality of the action requires $\dot{\phi} \leq \phi^2/\sqrt{\lambda}$. From the perspective of the 5d geometry, this is nothing but Einstein's causal speed limit in the radial direction. However, when viewed from the dual 4d field theory, it is rather novel: it is a speed limit on how fast an order parameter (the vev) may change [5]. Usual applications of brane inflation take a very slow moving brane, neglecting the higher derivative terms in (1). In contrast, we shall exploit these higher derivative terms and the resulting speed limit on $\phi$. To see how this works, let us couple (1) to the Einstein-Hilbert action and examine the resulting dynamics of $\phi(t)$ in a flat FRW ansatz: $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$. It was shown in [1] that there exists a late time attractor solution of this system,

$$\phi(t) \sim \frac{\sqrt{\lambda}}{t} \quad , \quad a(t) \sim a_0 t^{1/\epsilon} \qquad \text{with } \frac{1}{\epsilon} \approx \sqrt{\frac{\lambda}{3g_s}}\frac{m}{M_p} \tag{2}$$

where $M_p = 2.4 \times 10^{18}$ GeV is the reduced Planck scale. Let us pause a minute to examine this solution. We see that we have power-law inflation if $\epsilon < 1$ which translates into $\sqrt{\lambda}m \gg \sqrt{g_s}M_p$. We get accelerated expansion only if the inflaton mass is suitable large! The parameter $\epsilon$ will play a role similar to the slow-roll parameter in standard power-law inflation. Meanwhile, the scalar field $\phi$ appears to be slowing down – this effect gave rise to the name D-cceleration for this mechanism. However, from the perspective of the 5d geometry, a measure of the velocity of the brane in the radial direction is given by the analog of the special relativistic $\gamma$ factor,

$$\gamma \equiv \sqrt{1 - \lambda\dot{\phi}^2/\phi^4} \sim \tfrac{2}{3}\epsilon m^2 t^2 \tag{3}$$

where in the second equality we have evaluated $\gamma$ on our solution (2). We see that the brane is approaching the speed of light as inflation proceeds. In summary, we have a fast moving brane moving down a steep potential – a very counter-intuitive situation from the perspective of traditional slow roll inflation. The reason the mechanism drives accelerated expansion is because the potential does not affect the dynamics of $\phi$; it is controlled almost entirely by the speed limit. In contrast, the potential is the dominant contribution to the dynamics of the scale factor $a(t)$.

Since the throat is finite, inflation ends when the brane reaches $\phi \sim \phi_{IR}$, either by oscillating around the bottom of the throat or, more dramatically, by annihilating with an anti-brane. The number of e-foldings that the universe undergoes in a single pass from $\phi_{UV}$ to $\phi_{IR}$ is $N_e \sim \sqrt{\lambda/g_s}(m/M_p)\log(\phi_{UV}/\phi_{IR})$

## 3. Density Perturbations

The power spectrum of density perturbations in this model was computed in [2], following the work of Garriga and Mukhanov in the general context of k-inflation [6]. The standard gauge invariant combination of the scalar field and the metric perturbations is denoted $\zeta \equiv (H/\dot{\phi})\delta\phi + \Phi$ where $\Phi$ is the Newtonian potential and $H = \dot{a}/a$ is, of course, the Hubble parameter. An interesting aspect of the fluctuations $\zeta$ in our model is that high momentum ripples do not travel at the speed of light, but rather at a lower sound speed $c_s \sim 1/\gamma$. This means that the fluctuations of wavenumber $k$ do not freeze out at the causal horizon $aH = k$ as in traditional inflation, but rather when they cross the smaller *sound horizon* $aH = c_s k$. From equation (3) we see that the speed of sound is monotonically decreasing in our model. Taking this small subtlety into account, the power in the fluctuation of the scalar modes can be computed to be

$$P^{\text{scalar}} = \frac{1}{2\pi^2}|\zeta_k|^2 k^3 = \frac{1}{4\pi^2}\frac{g_s}{\epsilon^4\lambda} \sim 10^{-9} \tag{4}$$

where the numerical value is the COBE normalisation. When computing the tilt of this spectrum, an interesting cancellation occurs: the deviation of the background from a scale invariant de Sitter space is compensated by the shrinking sound horizon which means modes are freezing out on ever smaller scales and, to leading order in $\epsilon$, the tilt vanishes. We have $n_s \sim 1 + \mathcal{O}(\epsilon^2)$. The power in the tensor modes can be computed using standard techniques; the fluctuations now freeze at the causal horizon, and yield the simple ratio

$$r = \frac{P^{\text{tensor}}}{P^{\text{scalar}}} = \frac{16\epsilon}{\gamma} \tag{5}$$

while the tensor tilt is $n_t = -2\epsilon$. As discussed in [6], the low sound speed alters the usual consistency relations between $r$, $n_s$ and $n_T$ from that found in weakly coupled, slow-roll inflation.

Finally, we come to the non-Gaussianity as measured by the 3-point function $\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle$. Recall that standard, single field, slow-roll inflation robustly predicts negligible deviation from a Gaussian spectrum [7]. Since our model contains an infinite series of higher derivative terms, all important to the dynamics, one may expect to find large non-Gaussianities. Indeed, this is the case. A accurate estimate of their magnitude can be found by expanding the Lagrangian (1) to quadratic $\mathcal{L}_2$ and cubic $\mathcal{L}_3$ order and evaluating on the solution,

$$\mathcal{L}_3/\mathcal{L}_2 \sim \gamma^2\sqrt{P^{\text{scalar}}} \tag{6}$$

From which we see that the non-Gaussianity is intimately tied to our D-cceleration effect as measured by $\gamma$.

Detection of primordial non-Gaussianity in the CMBR is an exciting prospect, offering a window on the short distance physics involved in the inflationary regime. Indeed, if non-Gaussianity is observed, it provides a wealth of information about the early universe since it is a function of any triangle $\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$ that can be drawn on the sky. The full momentum dependence of the 3-point function $\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle$ for our model can be found in [2] and distinguishes the DBI model of inflation from others on the market. (In fact, the same $\vec{k}$ dependence of the 3-point function is also generated by higher derivative terms, albeit at a lower scale [10]). A detailed discussion of the shape of the non-Gaussianities that arise in different models was given by Babich, Creminelli and Zaldarriaga in [8]. For example, any non-Gaussianity that is generated outside the horizon – as in the curvaton scenario, or models with fluctuations in reheating efficiency – is peaked on tall, thin triangles with, say, $k_3 \to 0$. This long wavelength mode creates a background in which shorter wavelengths propagate. The resulting non-Gaussianity is parameterised by a number $f_{NL}$, arising as a non-linear correction to a Gaussian ensemble: $\zeta = \zeta_g - \frac{3}{5} f_{NL}(\zeta_g^2 - \langle\zeta\rangle^2)$ where $\zeta_g$ is Gaussian. Tests for non-Gaussianity of this form have been performed on WMAP data [9] resulting in the bound $|f_{NL}| \leq 100$. The forthcoming Planck satellite will improve this bound to $|f_{NL}| \leq 5$. However, for other shapes of non-Gaussianity, rigorous tests of WMAP data remain to be done. In particular, the non-Gaussianity of our DBI model is peaked on equilateral triangles, $k_1 = k_2 = k_3$. This is also true for other models which alter the kinetic term of the inflaton, such as higher derivative interactions [10] and ghost inflation [11]. Clearly it would be exciting to analyse the data for this specific form of 3-point correlations.

To finish, we determine the parameters and scales involved in our model. As discussed above, limits on our specific form of non-Gaussianity are not available. If instead we take the WMAP limit on $f_{NL}$ [9] it translates into $\mathcal{L}_3/\mathcal{L}_2 \leq 10^{-2}$. Equation (6) then provides an upper bound on the speed of our brane $\gamma \leq 40$. In fact, it turns out that with this measure of non-Gaussianity, to keep $f_{NL}$ suppressed even for the 10 e-foldings that we see in the CMBR requires us to start with a value for $\phi$ slightly above the Planck scale, a somewhat undesirable feature. Matching to COBE normalisation now gives us a mass of the inflaton in the range $m \sim 10^{13} \rightarrow 10^{14}$ GeV. Like all inflationary models, ours is fine-tuned: we require $\lambda \sim 10^{12}$. While this looks ridiculous when viewed as a 't Hooft coupling of the 4d theory, it becomes very reasonable when seen from the dual 5d perspective: recall that $R = \lambda^{1/4} l_s$, so we simply require a throat radius $R$ $10^3$ times larger than the string scale. Finally, note from (5) that the upper bound on $\gamma$ results in a lower bound on the tensor modes; we favour a large, observable component of tensor modes with $0.02 \leq r \leq 0.4$.

## 4. References


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