Two Stories of Magnetic Catalysis

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Magnetic Catalysis

Klimenko; Gusynin, Miransky
and Shovkovy, '94

e.g. in $d = 2+1$

$$\lim_{m\to 0^\pm} \langle \bar{\Psi}\Psi \rangle = \mp \frac{B}{4\pi}$$

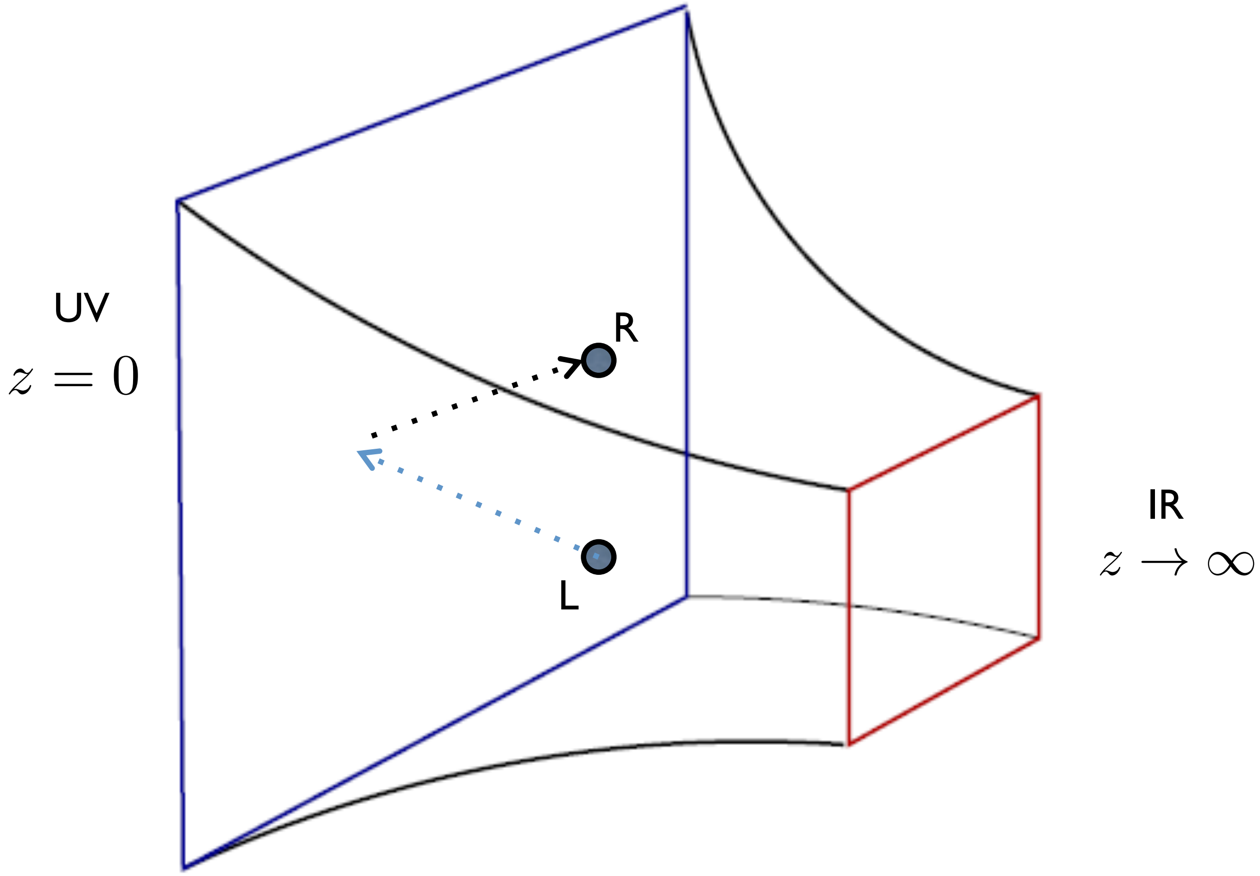
Story 1: Magnetic Catalysis in the bulk

Massless Dirac Fermion in AdS$_4$

$$S = \frac{i}{2}\int_{\mathcal{M}} \sqrt{-g}\;\bar{\psi}\;\overset{\leftrightarrow}{\not{D}}\;\psi + \frac{i}{2}\int_{\partial\mathcal{M}} \sqrt{-h}\;\bar{\psi}\psi$$

4 component Dirac spinor

Flipping Helicity at the Boundary

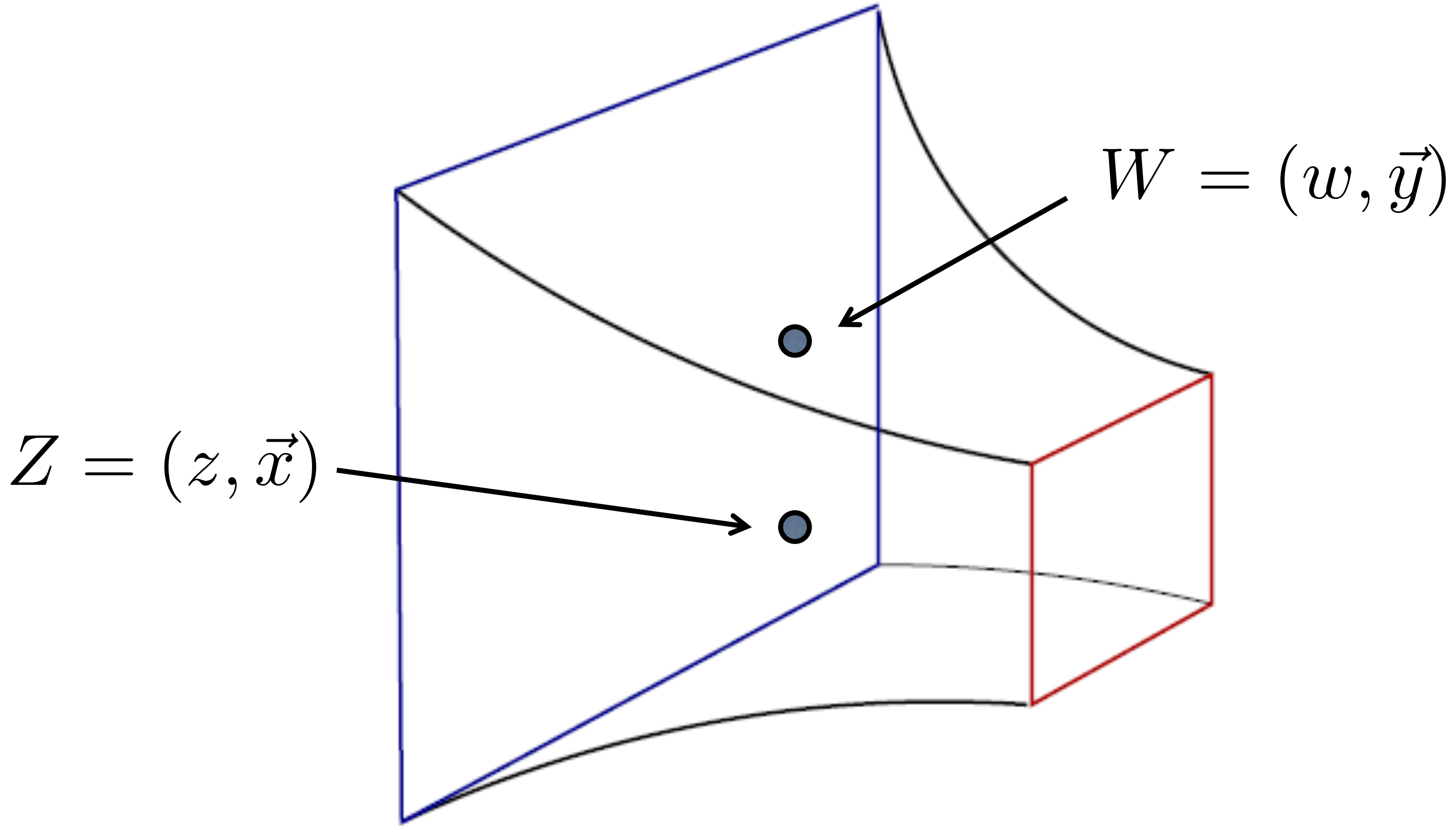


$$\psi_L = \frac{1}{2}(1-\gamma^5)\psi \;\longrightarrow\; \psi_R$$
$$\psi_R = \frac{1}{2}(1+\gamma^5)\psi \;\longrightarrow\; \psi_L$$

$$ds^2 = L^2\,\frac{dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu}{z^2}$$

Chiral Symmetry Breaking in the Bulk

Allen and Lutken, ‘86

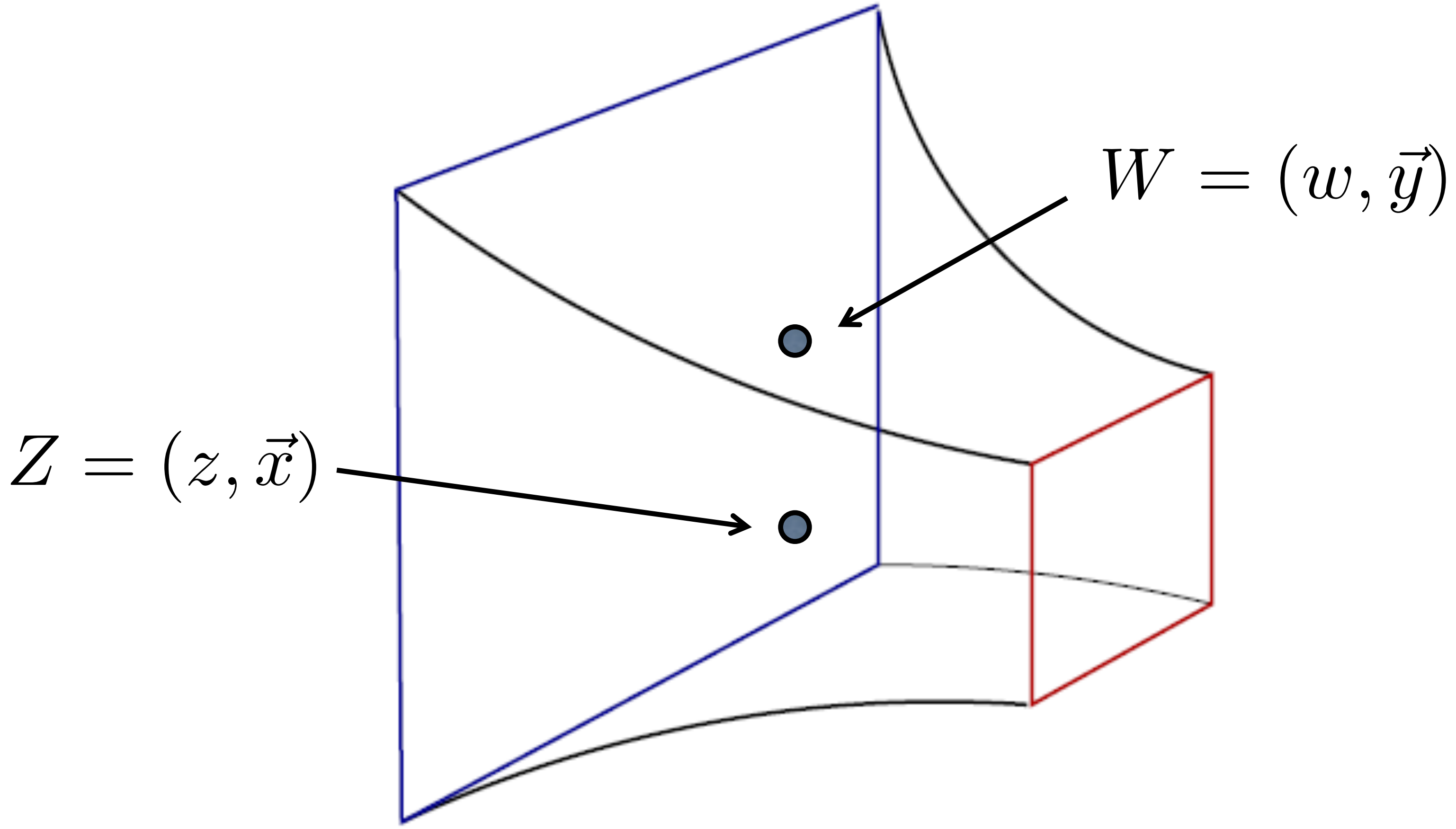


Bulk to bulk propagator for *massive* spinor

$$S(Z,W) \quad = \quad [\text{Stuff}] \times {}_2F_1[\text{Horrible Blah}]$$

Chiral Symmetry Breaking in the Bulk

Allen and Lutken, '86



Bulk to bulk propagator for *massless* spinor

$$S(Z,W) = \frac{1}{2\pi^2}\left(\frac{zw}{L^2}\right)^{3/2}\left[\frac{\not{Z} - \not{W}}{[(z-w)^2 + (\vec{x}-\vec{y})^2]^2} + \frac{\not{Z}\gamma^z + \gamma^z \not{W}}{[(z+w)^2 + (\vec{x}-\vec{y})^2]^2}\right]$$

Chiral Symmetry Breaking in the Bulk

Allen and Lutken, ‘86

$$S(Z,W) \quad = \quad \frac{1}{2\pi^2}\left(\frac{zw}{L^2}\right)^{3/2}\left[\frac{\not{Z}-\not{W}}{[(z-w)^2+(\vec{x}-\vec{y})^2]^2}+\frac{\not{Z}\gamma^z-\gamma^z\not{W}}{[(z+w)^2+(\vec{x}-\vec{y})^2]^2}\right]$$

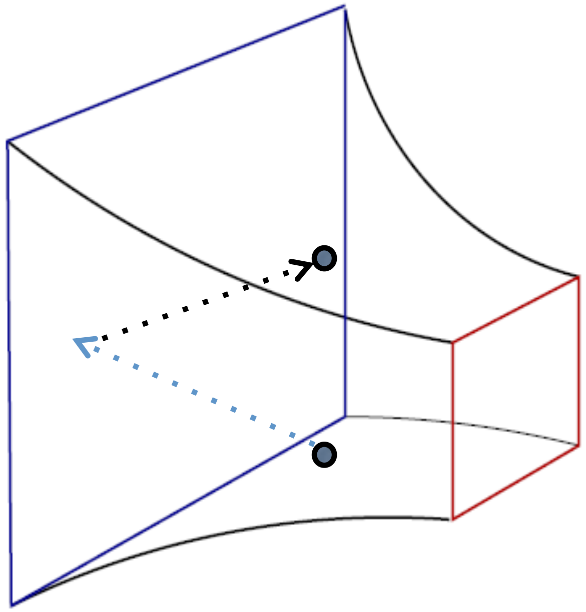
$$\Longrightarrow \qquad \langle\bar{\psi}\psi\rangle = \lim_{Z\to W}\,\mathrm{Tr\ S(Z,W)} = \frac{1}{4\pi^2\mathrm{L}^3}$$

Changing the Boundary Conditions

Porrati & Giradello, Rattazzi and Redi, '09

Chiral symmetry broken by boundary conditions ⟹ circle of boundary conditions

$$S_{\text{bdy}} = \bar{\psi}\psi \longrightarrow \bar{\psi}e^{i\alpha\gamma^5}\psi$$



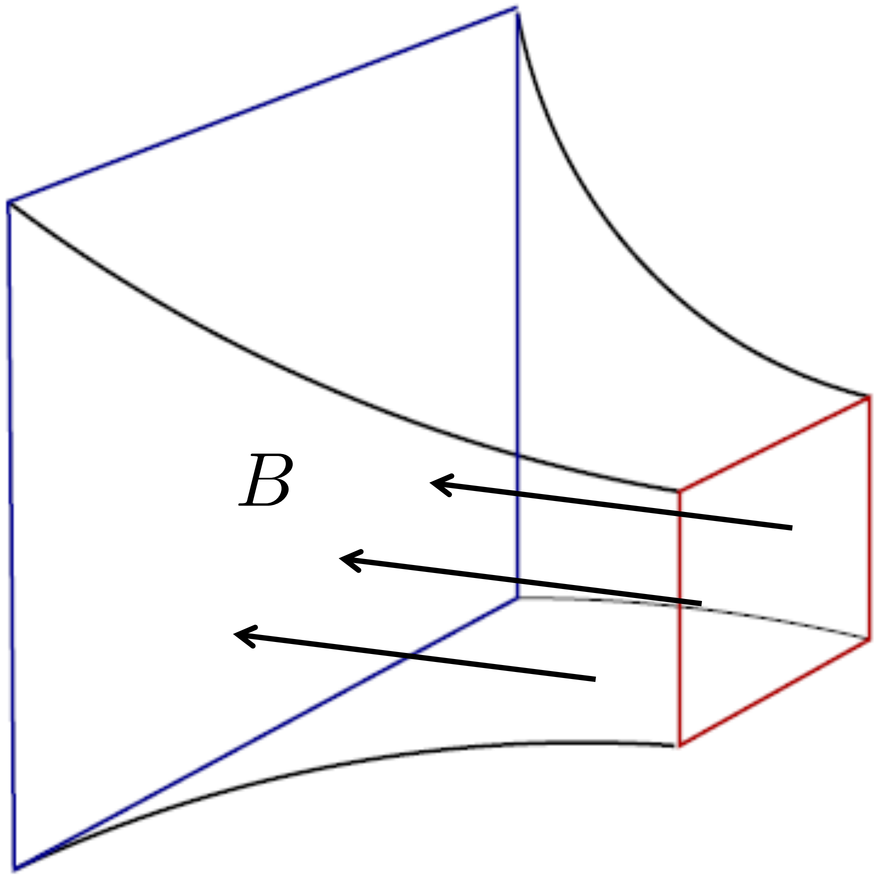
$$\psi_L = \frac{1}{2}(1-\gamma^5)\psi \;\longrightarrow\; e^{-i\alpha}\psi_R$$

$$\psi_R = \frac{1}{2}(1+\gamma^5)\psi \;\longrightarrow\; e^{i\alpha}\psi_L$$

$$\Longrightarrow \quad \begin{aligned} 2\langle\bar{\psi}_L\psi_R\rangle &= \langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}\gamma^5\psi\rangle \\ &= \frac{e^{i\alpha}}{4\pi^2 L^3} \end{aligned}$$

A Magnetic Field in AdS

(Should work in Reissner-Nordstrom black hole; ignore backreaction for now)



A Magnetic Field in AdS

Solve the Dirac Equation in background *B* field

<u>Stragey</u>

- Ignore radial z direction
- Landau levels are given

$$E = \sqrt{2Bn} \qquad\qquad n \in \mathbf{Z}^{+}$$

- Zero modes (*n=0*) sector are ξ_α (2 component spinor)
- Promote $\xi_\alpha \to \xi_\alpha(z,t)$
- Effective 2d action

$$S_{\text{eff}} = \int dtdz\ \sqrt{-g}\ i\bar{\xi}\,\not{D}_{2\text{d}}\xi$$

Condensate in Magnetic Field

$$\langle \bar{\xi}\xi \rangle = \frac{e^{i\alpha}}{2\pi}\,\frac{z^2}{L^3}$$

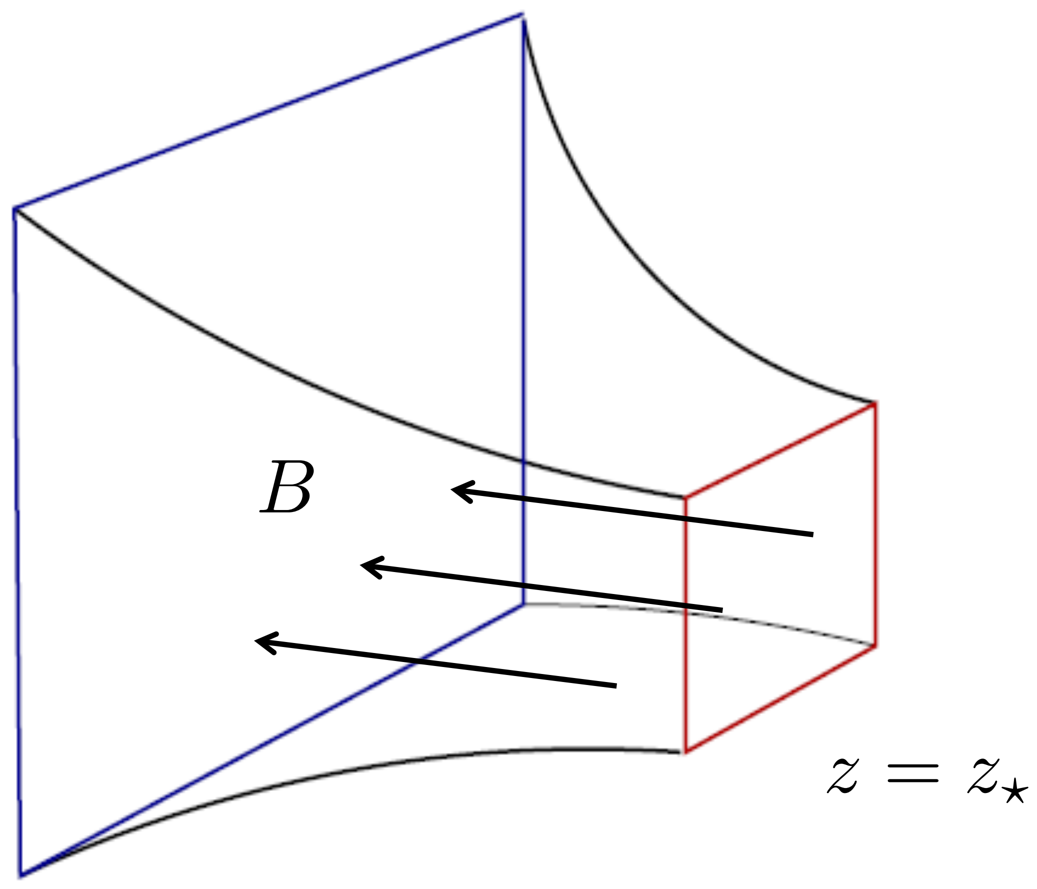
$$\langle \bar{\psi}_L \psi_R \rangle = \frac{B}{2\pi}\,\langle \bar{\xi}\xi \rangle$$

Density of states in lowest Landau level

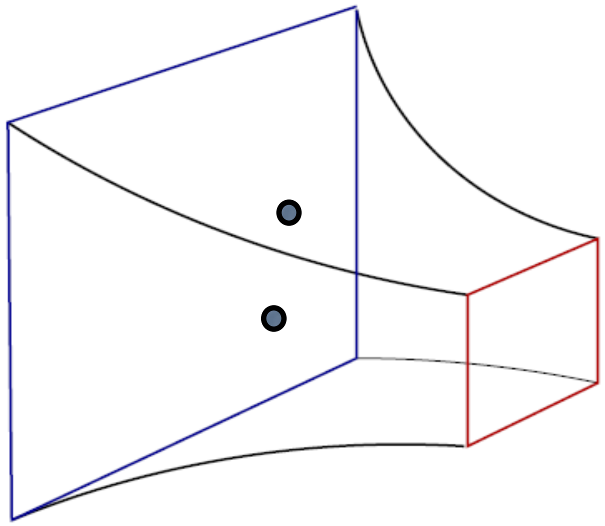
<u>Caveat</u>

- Gap to higher levels is warped in AdS to $\sim \sqrt{B}Lz$
- Higher Landau levels important near boundary

Condensate in Magnetic Field with Hard Wall



Condensate in Magnetic Field with Hard Wall



First going left: 1 bounce, 3 bounces, 5 bounces

$$\langle \bar{\xi}_L \xi_R \rangle = \frac{1}{\pi}\left(\frac{z}{L}\right)^3 \Bigg[\underset{\uparrow \text{ 1 bounce}}{\frac{e^{i\alpha}}{2z}} + \underset{\uparrow \text{ 3 bounces}}{\frac{e^{2i\alpha}}{2z+2z_\star}} + \underset{\uparrow \text{ 5 bounces}}{\frac{e^{3i\alpha}}{2(z+2z_\star)}} + \dots \Bigg]$$

$$-\frac{1}{\pi}\left(\frac{z}{L}\right)^3 \Bigg[\underset{\uparrow \text{ 1 bounce}}{\frac{1}{2(z_\star - z)}} + \underset{\uparrow \text{ 3 bounces}}{\frac{e^{-i\alpha}}{2(2z_\star - z)}} + \underset{\uparrow \text{ 5 bounces}}{\frac{e^{-2i\alpha}}{2(3z_\star - z)}} + \dots \Bigg]$$

First going right: 1 bounce, 3 bounces, 5 bounces

Condensate in Magnetic Field with Hard Wall

$$\langle \bar{\psi}_L \psi_R \rangle = e^{i\alpha} \frac{B}{4\pi^2} \left(\frac{z}{L}\right)^3 \sum_{n=-\infty}^{+\infty} \frac{e^{in\alpha}}{z + n z_\star}$$

Series is conditionally convergent, but not absolutely convergent.

$$\lim_{\alpha \to 0^\pm} \langle \bar{\psi}_L \psi_R \rangle = \text{real} \ \pm \ \frac{i}{2\pi} \frac{B z^3}{L^3 z_\star}$$

c.f. $\displaystyle \int_{-\infty}^{+\infty} dn \ \frac{e^{i\alpha n}}{z + n z_\star} = \text{real} \ \pm \ \frac{i\pi}{z_\star} \text{sign}(\alpha)$

Punchline of Story 1

$$\lim_{\alpha\to 0^\pm} \langle \bar{\psi}\gamma^5\psi\rangle = \pm\frac{1}{2\pi}\,\frac{Bz^3}{L^3 z_\star}$$

“Chiral” Symmetry Breaking in d=3+1

	B	$\bar{\psi}\psi$	$\bar{\psi}\gamma^5\psi$
P	-	$+$	-
C	-	$+$	$+$
CP	$+$	$+$	-

Spontaneous breaking of *CP*

Story 2: Magnetic Catalysis in the boundary

Usual AdS/CFT Dictionary

$$\psi \longleftrightarrow \Psi$$

Four component bulk spinor

Two component boundary spinor

$$\Delta[\Psi] = \frac{3}{2} + mL = \frac{3}{2}$$

But…

$$\bar{\psi}\psi \longleftrightarrow ?$$

$$\bar{\psi}\gamma^5\psi \longleftrightarrow ?$$

Our Claim

$$\bar{\psi}\gamma^5\psi \;\longleftrightarrow\; \bar{\Psi}\Psi$$

Symmetries

	B	$\bar{\psi}\psi$	$\bar{\psi}\gamma^5\psi$	$\bar{\Psi}\Psi$
P	-	$+$	-	-
C	-	$+$	$+$	$+$
CP	$+$	$+$	-	-

Implication

$$\bar{\psi}\gamma^5\psi \;\longrightarrow\; \frac{\sin\alpha}{4\pi^2L^3} + \frac{1}{2\pi}\,\frac{B}{L^3 z_\star}\,z^3 + \dots$$

Source for $\bar{\Psi}\Psi$
(Arises due to boundary terms)

Expectation value $\langle\bar{\Psi}\Psi\rangle$

Punchline of Story 2

$$\langle \bar{\Psi}\Psi \rangle = \pm \frac{3}{2} B M$$

with $$M = \frac{1}{z_\star}$$

Summary and Open Questions

- Hard Wall + Magnetic Field = Magnetic Catalysis
  - Spontaneous *CP* breaking
- $\bar{\psi}\psi \longleftrightarrow ?$
- Other effects due to bulk condensates?

Comments and Remarks

Caveat

- Gap to higher levels is warped in AdS to $\sim \sqrt{B}Lz$
- Higher Landau levels important near boundary

Note: RN black hole

- Non-extremal: condensate diverges at the horizon
- Extremal: condensate constant at the horizon