Are Black Holes Like Metals?

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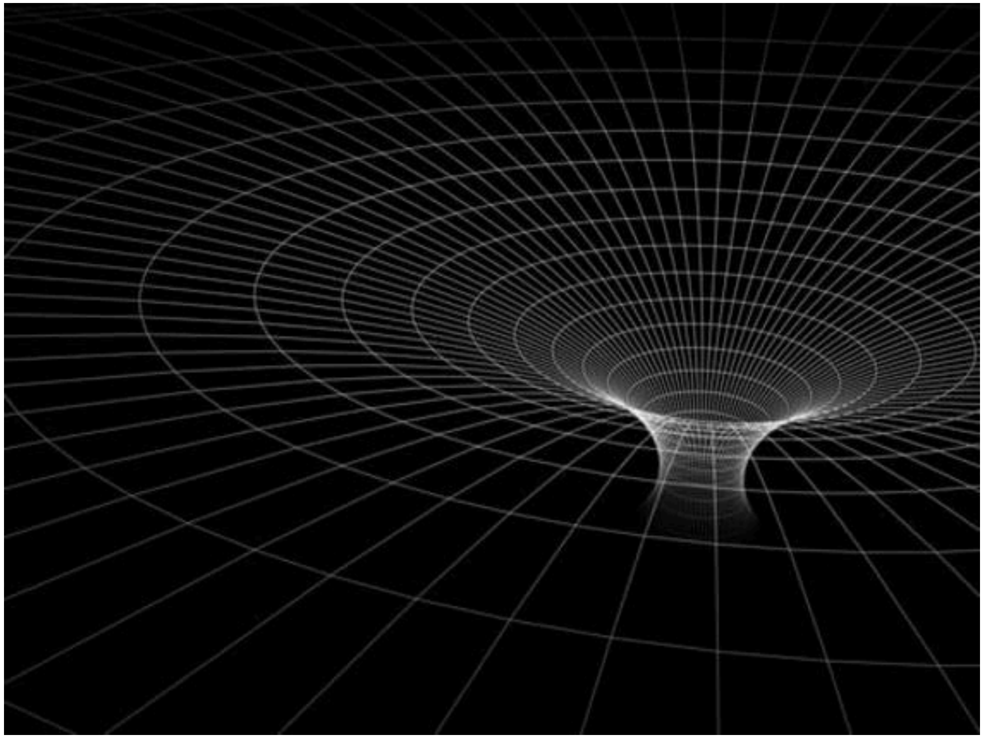
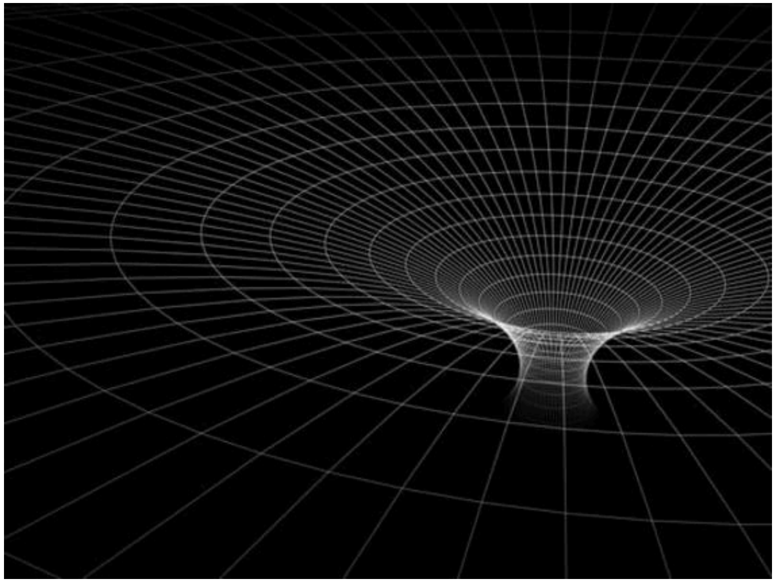
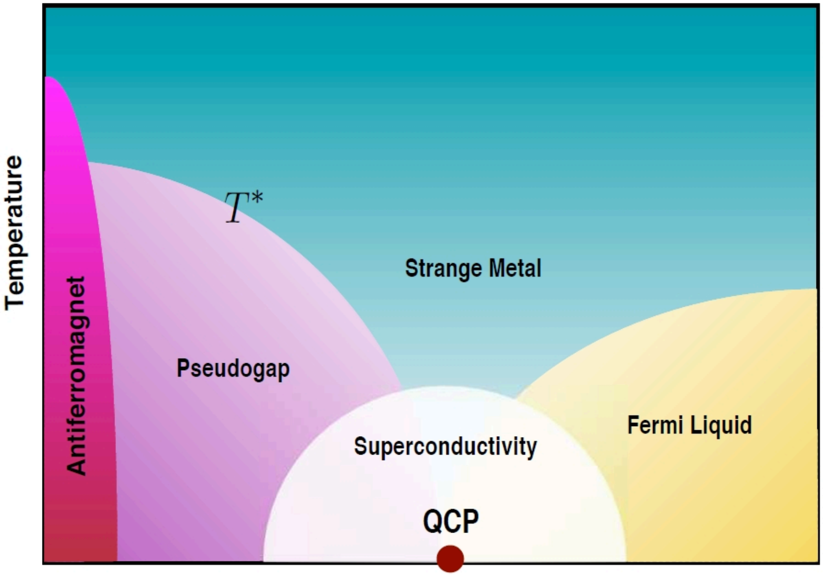
Black Holes



A strongly correlated quantum system

Are black holes like metals?





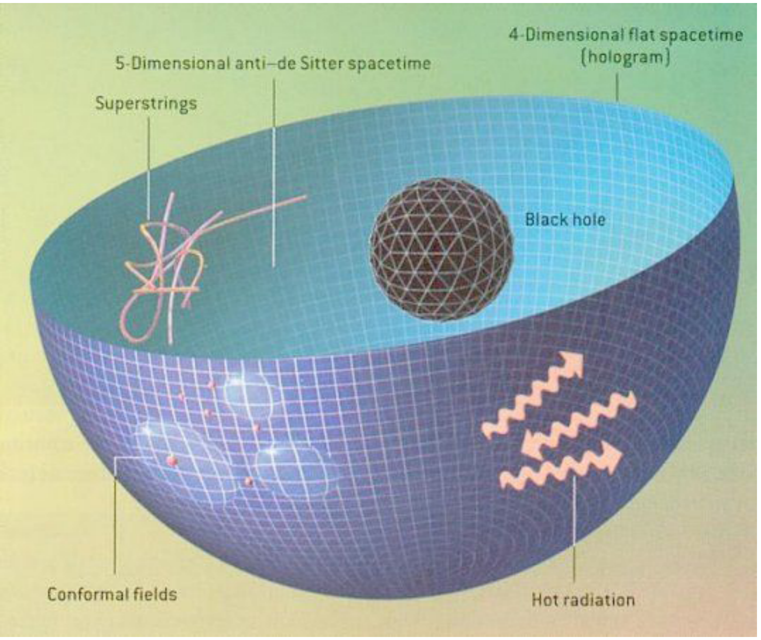
The Context: Holography

also known as: gauge gravity duality, or AdS/CFT correspondence

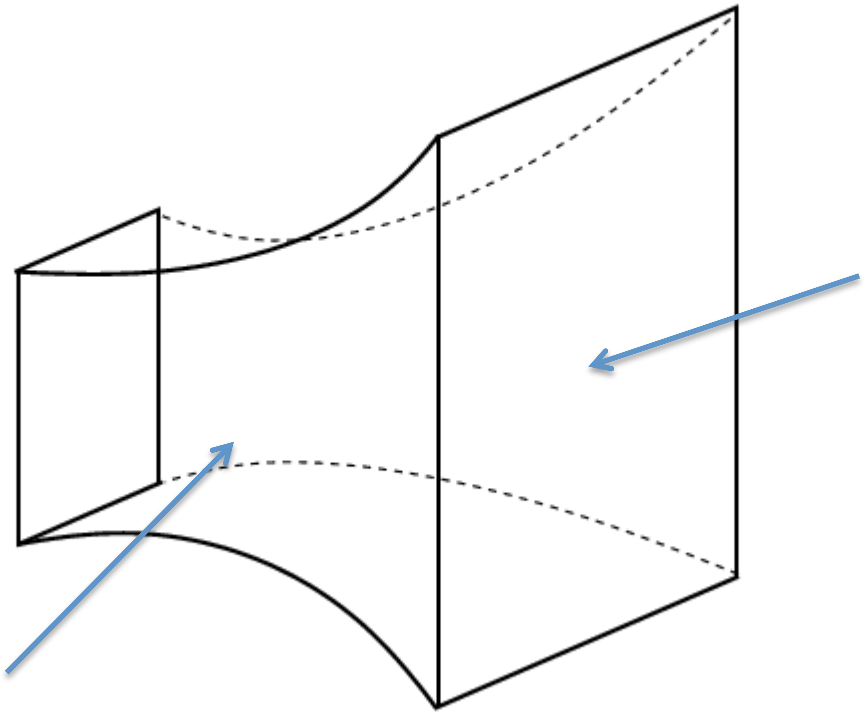
Strongly interacting quantum field theory

↔

Gravity in (at least) one dimension higher



The Vacuum State

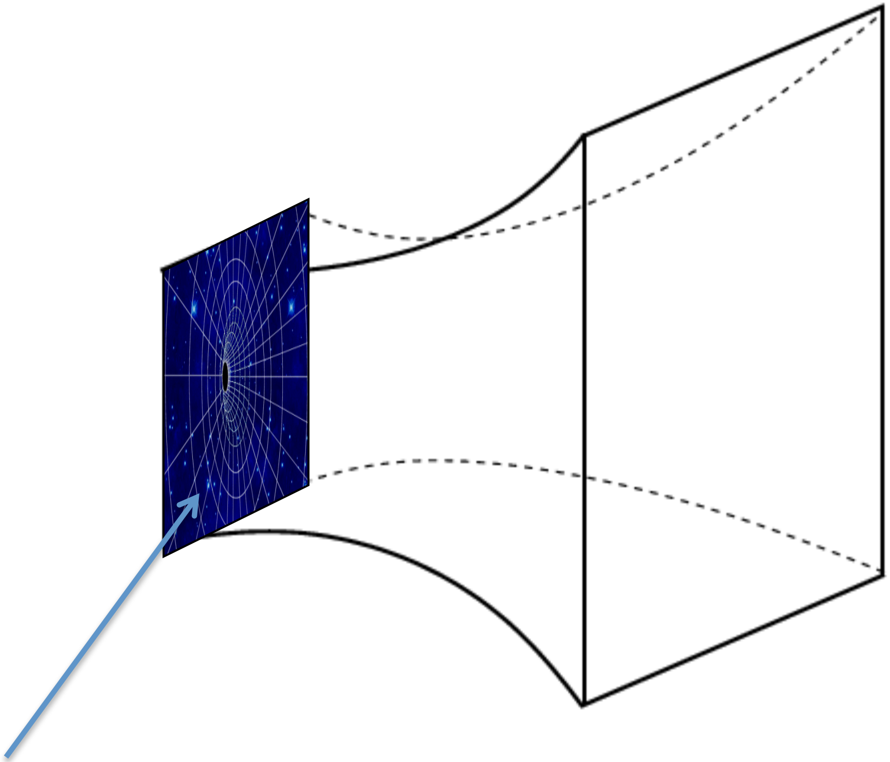


Boundary field theory

My attempt at drawing
anti de-Sitter spacetime

$$ds^2 = \frac{L^2}{r^2}\left(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

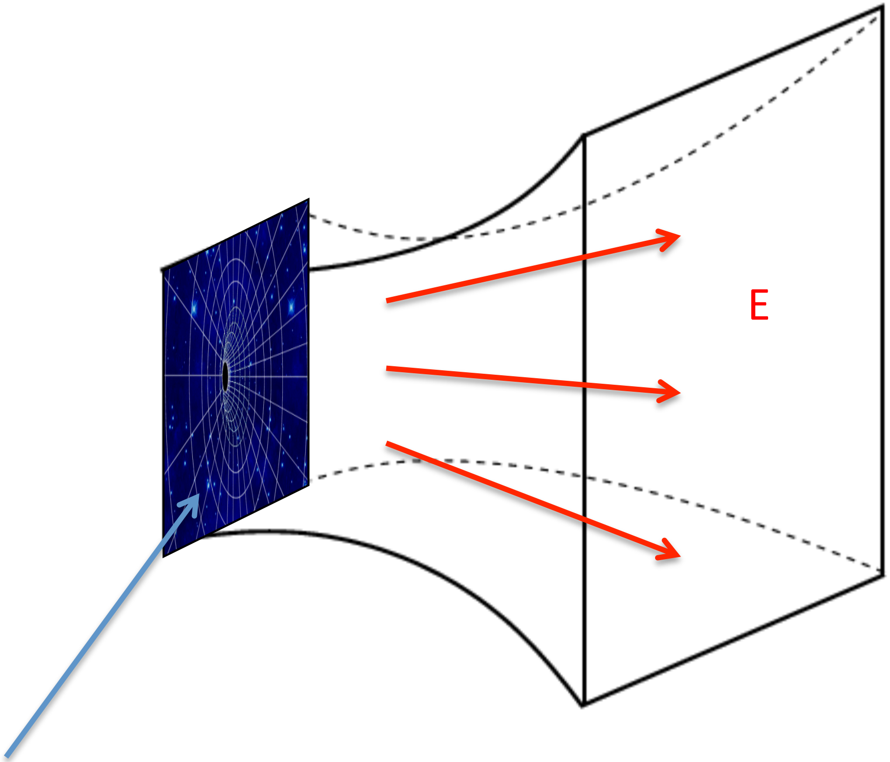
Heating up the Boundary Theory



Boundary field theory

- Black hole
- Hawking radiation = finite temperature, *T*

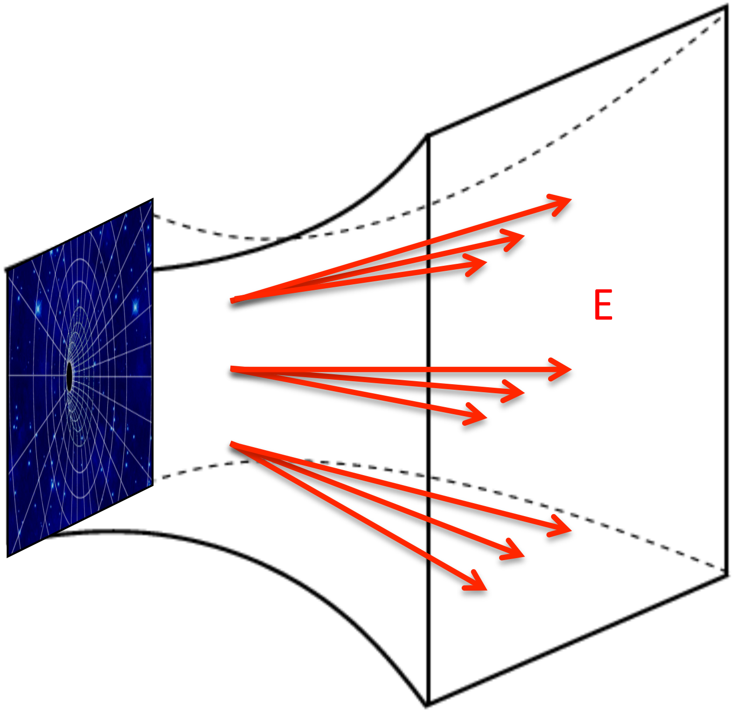
Finite Density Matter



Boundary field theory

- Reissner-Nordstrom black hole
- Hawking radiation = finite temperature, T
- Electric field = chemical potential, μ

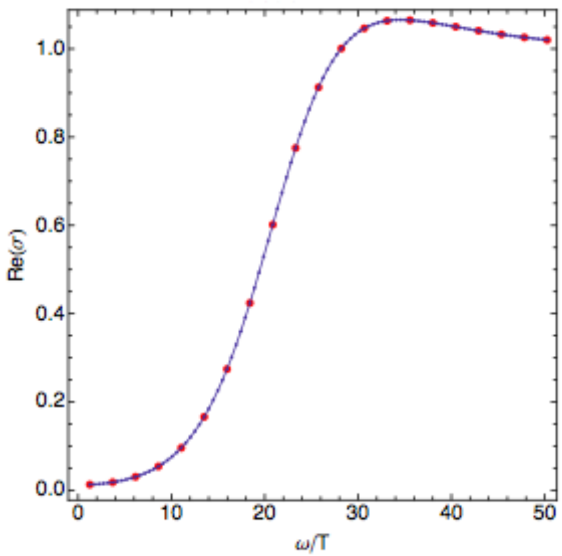
Ohm’s Law



Boundary field theory

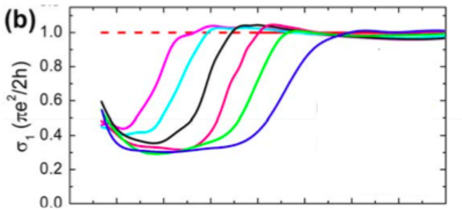
$$j(\omega) = \sigma(\omega)E(\omega)$$

Optical Conductivity in d=2



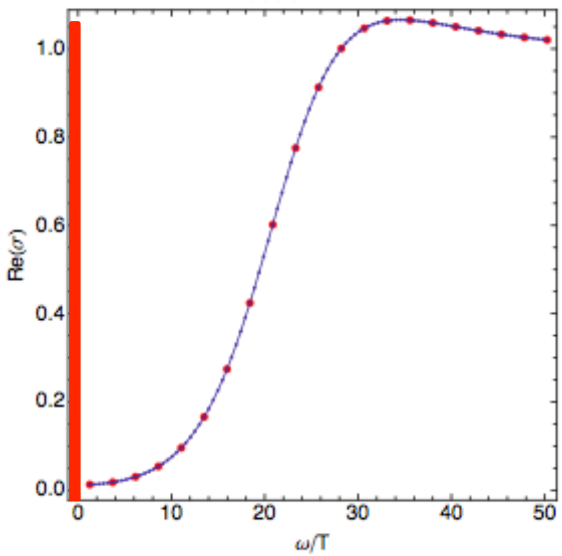
Herzog, Kovtun, Sachdev, Son
Hartnoll, 2007

$$j(\omega) = \sigma(\omega) E(\omega)$$



c.f. graphene

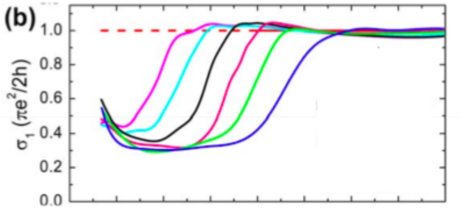
Optical Conductivity in d=2



Herzog, Kovtun, Sachdev, Son
Hartnoll, 2007

$$\mathrm{Re}\,\sigma(\omega) \sim K\,\delta(\omega)$$

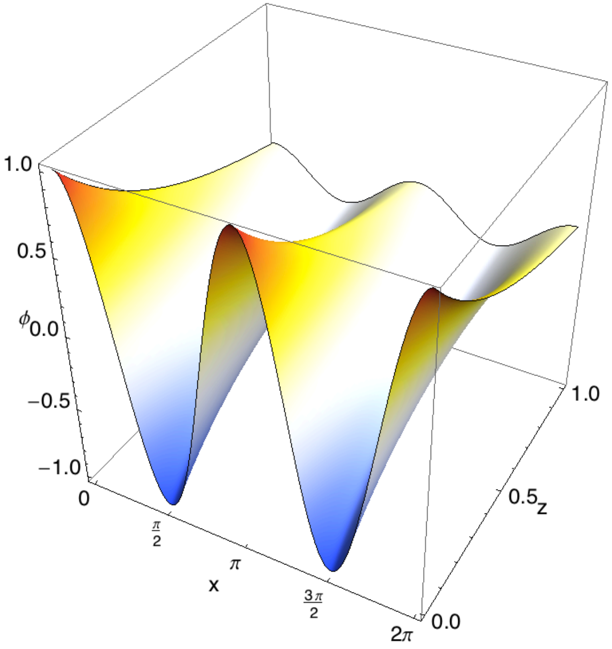
Finite charge density + momentum conservation



c.f. graphene

Breaking Translational Invariance

$\mu = \mu(x, y)$: spatially varying chemical potential with wavevector k_L

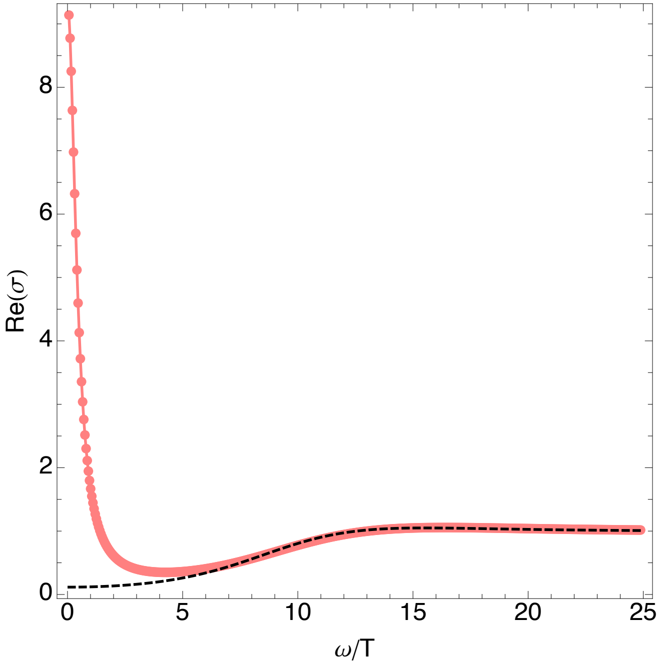


Horowitz, Santos, Tong, 2012

A rippled black hole

Optical Conductivity

$$j(\omega) = \sigma(\omega)E(\omega)$$

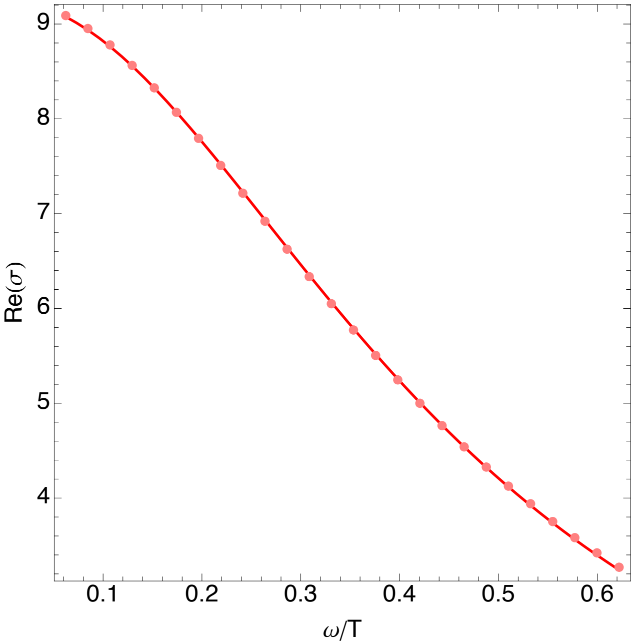


Horowitz, Santos, Tong, 2012

Delta function spreads out

Low Frequency Behaviour

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$



Horowitz, Santos, Tong, 2012

Classical Drude model from general relativity!

DC Conductivity

Can compute analytically:

$$\sigma_{\rm DC} = \frac{Q^2}{\mathcal{E}+P}\tau + \sigma_0$$

Blake, Tong
Blake, Tong and Vegh
Donos and Gauntlett, 2013

DC Conductivity

Can compute analytically:

charge density

scattering time

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

energy density

pressure

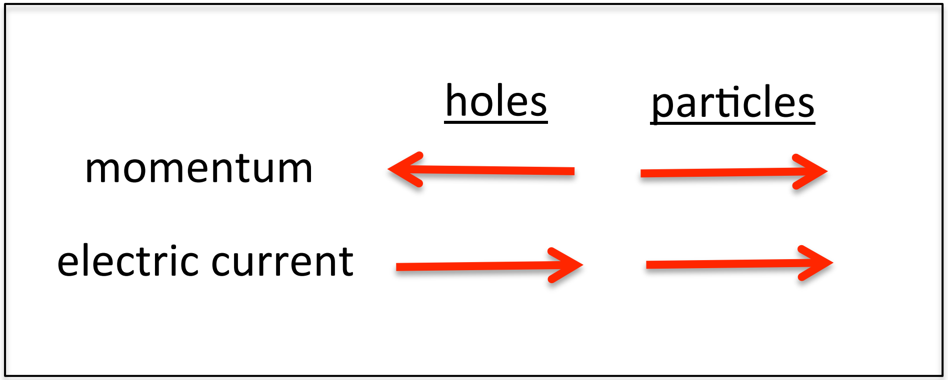
Blake, Tong
Blake, Tong and Vegh
Donos and Gauntlett, 2013

DC Conductivity

Due to scattering of charged stuff

Due to “pair creation”

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$



	holes	particles
momentum	←	→
electric current	→	→

σ_0 gives charge current with no heat current

Damle and Sachdev, 1997

DC Conductivity

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

There are three surprising things about this formula!

DC Conductivity: Surprise 1

The two contributions add

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

This is the inverse of Matthiessen’s rule

DC Conductivity: Surprise 2

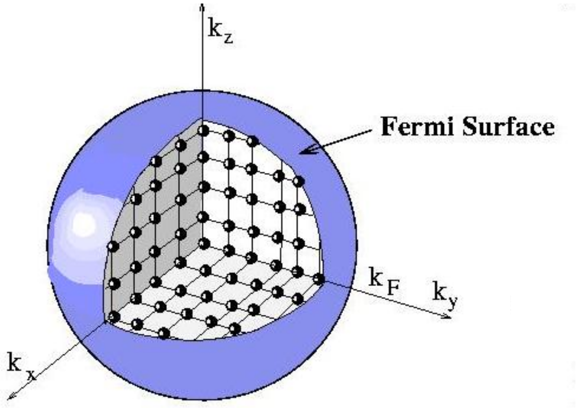
The first term varies as a power-law in temperature.

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E} + P}\tau + \sigma_0$$

$$\tau(T) \sim T^{-2\Delta(k_L)}$$

There must be low-energy degrees of freedom at finite momentum *k*

Low-Energy Excitations in a metal



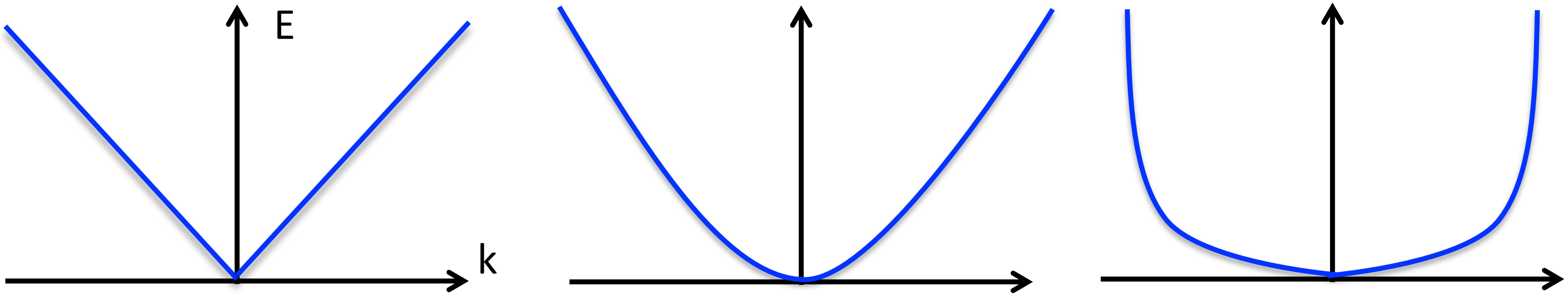
$$\tau(T) \sim T^{-2}$$

Low-energy, finite momentum excitations are the defining property of a metal

Low-Energy Excitations in a black hole

Finite momentum excitations arise in a more exotic way. Consider dispersion relations

$E \sim k$ $\quad$ $E \sim k^2$ $\quad$ $E \sim k^z \quad z > 2$



relativistic $\quad$ non-relativistic $\quad$ unusual!

Excitations around the black hole have:

$$\boxed{E \sim k^z \qquad z \to \infty}$$

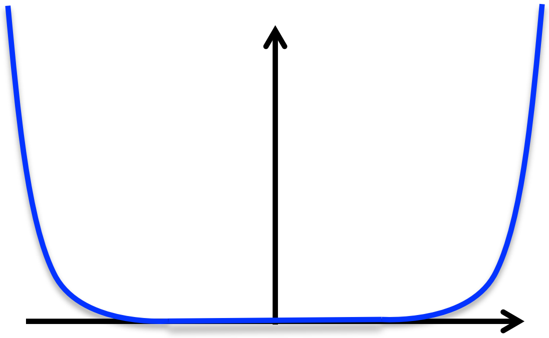
This is known as *local criticality*.

Faulkner, Liu, McGreevy, Vegh, 2009

Si et al. 2001

Low-Energy Excitations in a black hole

$E \sim k^z \qquad z \to \infty$



The DC conductivity for such a system is

$$\sigma_{\mathrm{DC}} \sim T^{-2\Delta(k_L)}$$

ugly function depending
on lattice spacing

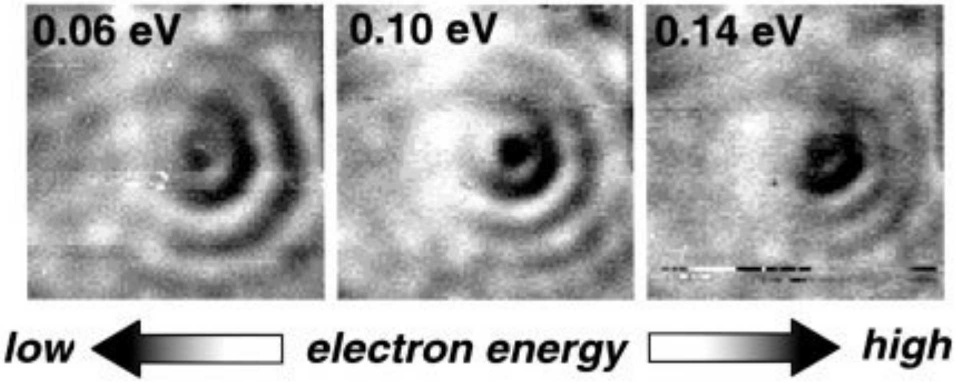
Hartnoll and Hofman, 2012

This agrees with the first term of the black hole calculation

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E}+P}\tau + \sigma_0$$

Friedel Oscillations

In metals, you can *see* the Fermi surface



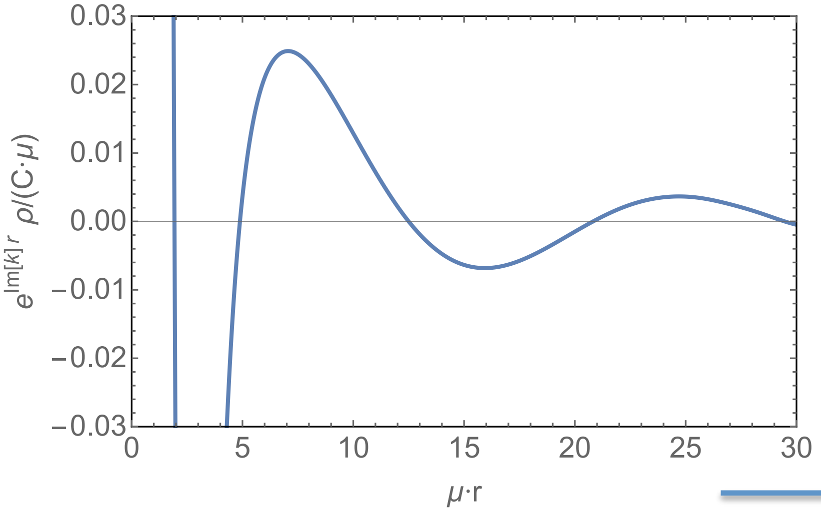
A charged impurity in a metal makes a rippling effect. The wavelength is $2k_F$.

What about for black holes?...

Friedel Oscillations for Black Holes

Place a charged impurity near a black hole. How does the horizon respond?

charge density
on horizon



Blake, Donos
and Tong, 2014

distance from
impurity

But the black hole has *complex* k_F

Is this a lesson for strongly coupled electron systems?

DC Conductivity: Surprise 3

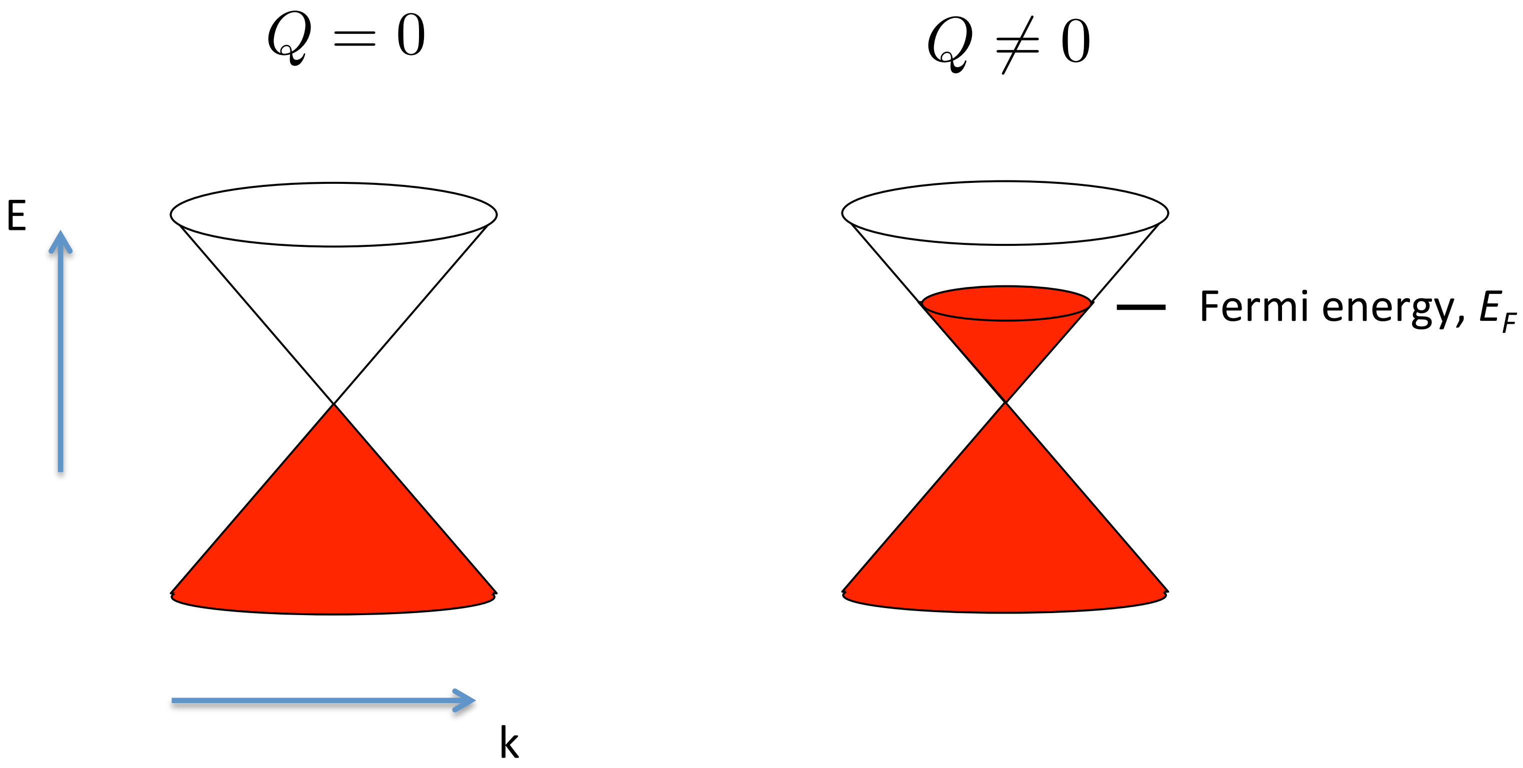
The second term also varies as a power-law in temperature

$$\sigma_{\rm DC} = \frac{Q^2}{\mathcal{E}+P}\tau + \sigma_0$$

$$\sigma_0(T) \sim T^{\#}$$

Expected at *Q*=0 but surprising at finite charge density

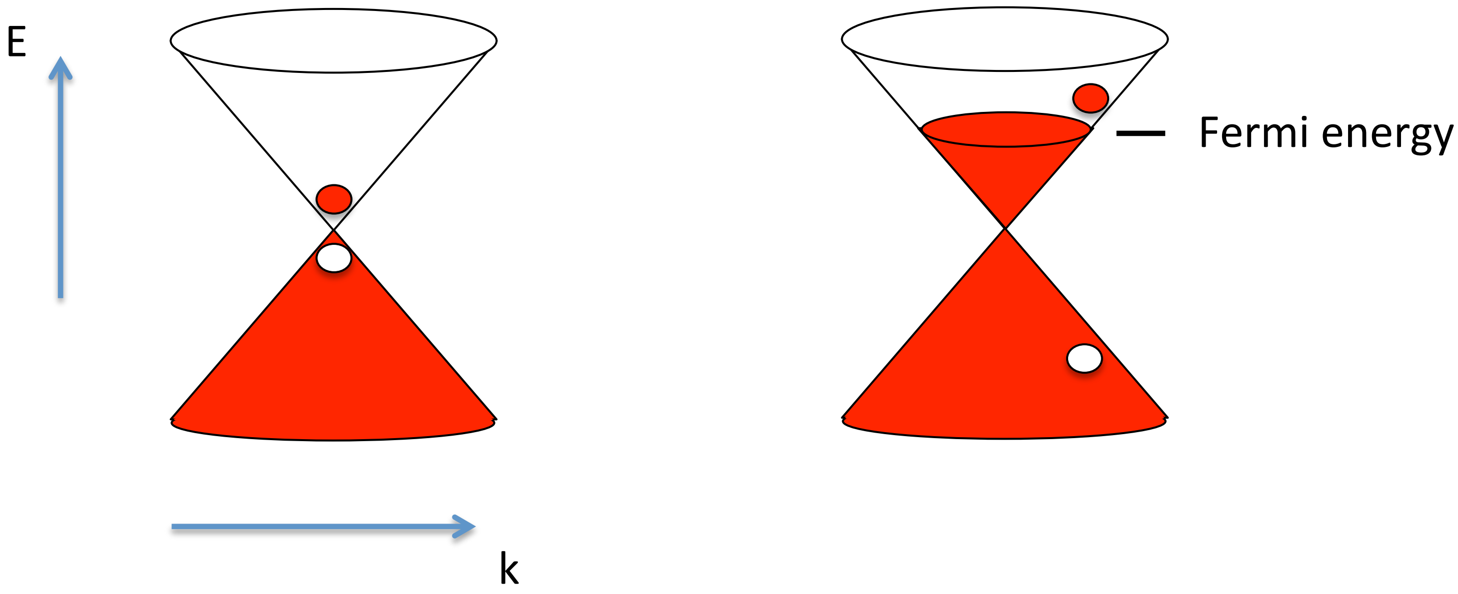
Pair Creation at Weak Coupling



c.f. graphene

Pair Creation at Weak Coupling

$Q = 0$ $\qquad\qquad$ $Q \neq 0$



$\sigma_{DC} \sim T^{d-2}$ $\qquad\qquad$ $\sigma_{DC} \sim e^{-2E_F/T}$

c.f. graphene

“Pair Creation” at Strong Coupling

$$Q \neq 0 \quad \Longrightarrow \quad \sigma_0(T) \sim T^{\#}$$

Intuition behind this remains unclear.

Is there also a lesson here for strongly coupled electron systems?

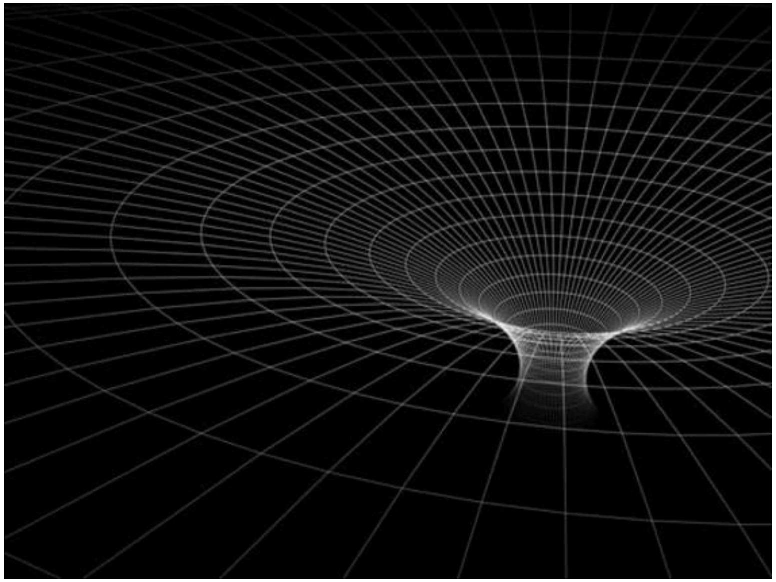
Summary of Black Hole Conductivity

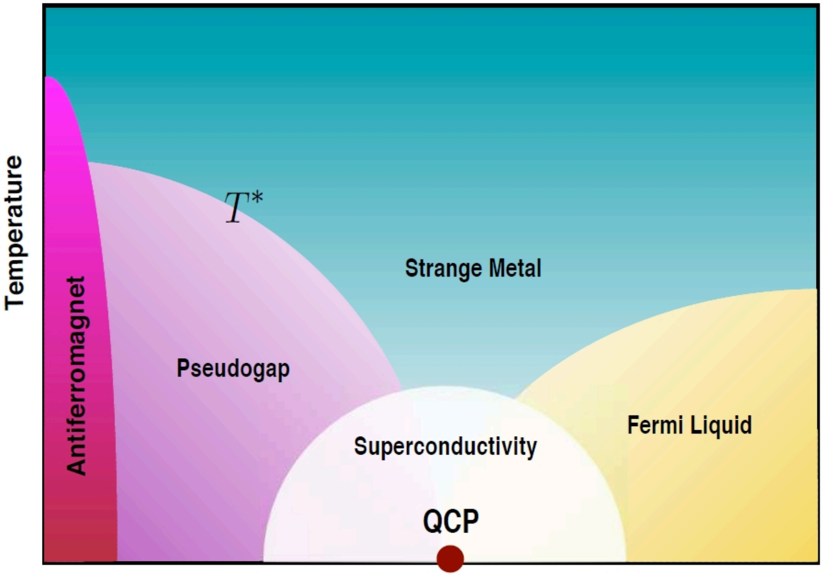
Two Processes

- Low energy modes at finite momentum
  - But not a Fermi surface

- Low energy pair creation even at finite Q

Are there any similarities?

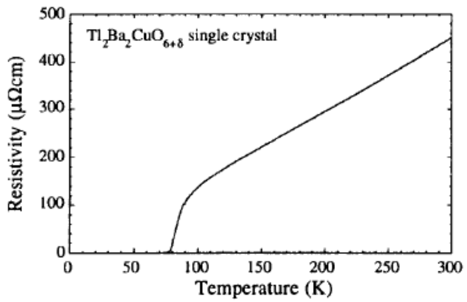




Strange Properties of Strange Metals

DC Conductivity

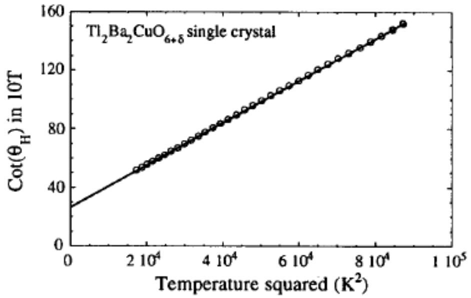
$$\sigma_{DC} \sim \frac{1}{T}$$



Mackenzie et al 1997

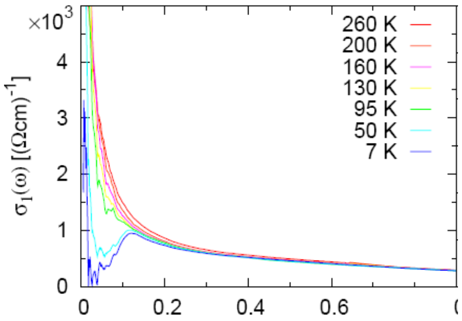
Hall Conductivity

$$\frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$



AC Conductivity

$$\sigma(\omega) \sim 1/(i\omega)^{2/3}$$



Van der Marel et al 2001

Lesson 1: Hall Angle

Drude model
(or Fermi liquid theory)

$$\sigma_{DC} \sim \frac{\sigma_{xy}}{\sigma_{xx}} \sim \tau$$

Experimental data
on strange metals

$$\sigma_{DC} \sim \frac{1}{T} \qquad \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$

Suggests two time scales at play?

Anderson, 1991
Coleman, Schofield, Tsvelik, 1996

"Over broad regions of doping, the two kinds of relaxation rates, the one for the conductivity and the one for the Hall rotation, seem to add as inverses: conductivity is proportional to 1/T + 1/T^2. That is, it obeys an anti-Matthiessen law"

P.W. Anderson

Lesson 1: Hall Angle

Drude model
(or Fermi liquid theory)

$$\sigma_{DC} \sim \frac{\sigma_{xy}}{\sigma_{xx}} \sim \tau$$

Experimental data
on strange metals

$$\sigma_{DC} \sim \frac{1}{T} \qquad \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$

Black Holes

$$\sigma_{DC} = \sigma_0 + \frac{Q^2}{\mathcal{E} + P}\tau \qquad \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{BQ}{\mathcal{E} + P}\tau$$

Blake and Donos, 2014

If this term dominates DC transport,
we get two time scales

Lesson 2: (In)coherent Transport

There is another interpretation of these two terms*

Hartnoll, 2014

$$\sigma_{\mathrm{DC}} = \frac{Q^2}{\mathcal{E}+P}\tau + \sigma_0$$

<u>Coherent Transport</u>

due to (almost) conserved momentum

$$\tau^{-1} \gg T$$

<u>Incoherent Transport</u>

due to charge diffusion

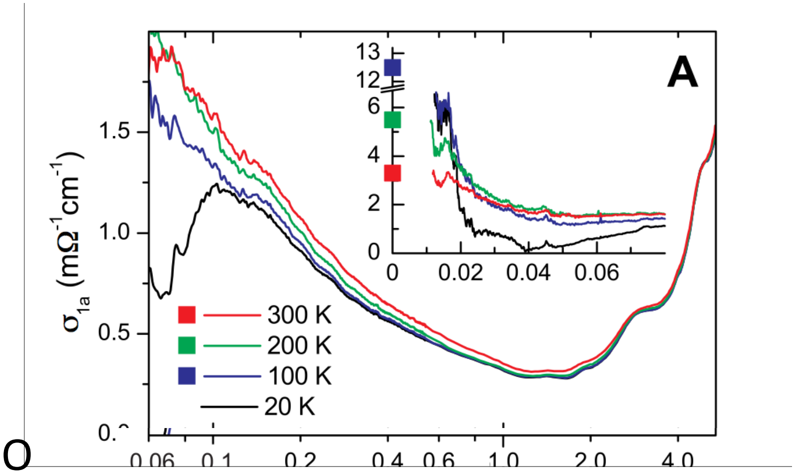
$$\tau_{\mathrm{eff}}^{-1} \sim T$$

which of these processes describes actual materials?

*actually it's slightly more complicated

Davison and Gouteraux (last week)
Blake (today)

Lesson 2: Incoherent Transport



LSCO

Takenaka et al

$$\tau^{-1} \sim \frac{k_B T}{\hbar}$$

Suggests incoherent transport

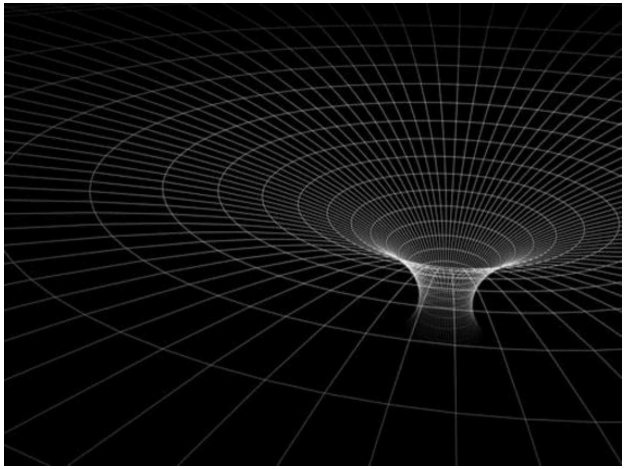
Conjecture: there is a quantum bound for diffusion $\sigma_0 \gtrsim \frac{\hbar}{k_B T}$

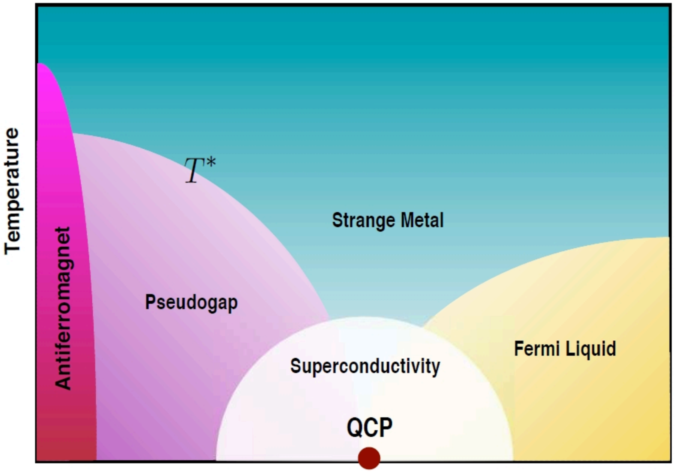
Hartnoll 2014 (see also Bruin et al; Sachdev, Zaanen)

Does this explain linear relativity? Evidence far from conclusive

Summary

- We're understanding better the conductivity properties of black holes
- Are there lessons here for strongly interacting electrons?





The End

Thank you for your attention