What is String Theory?

David Tong
University of Cambridge

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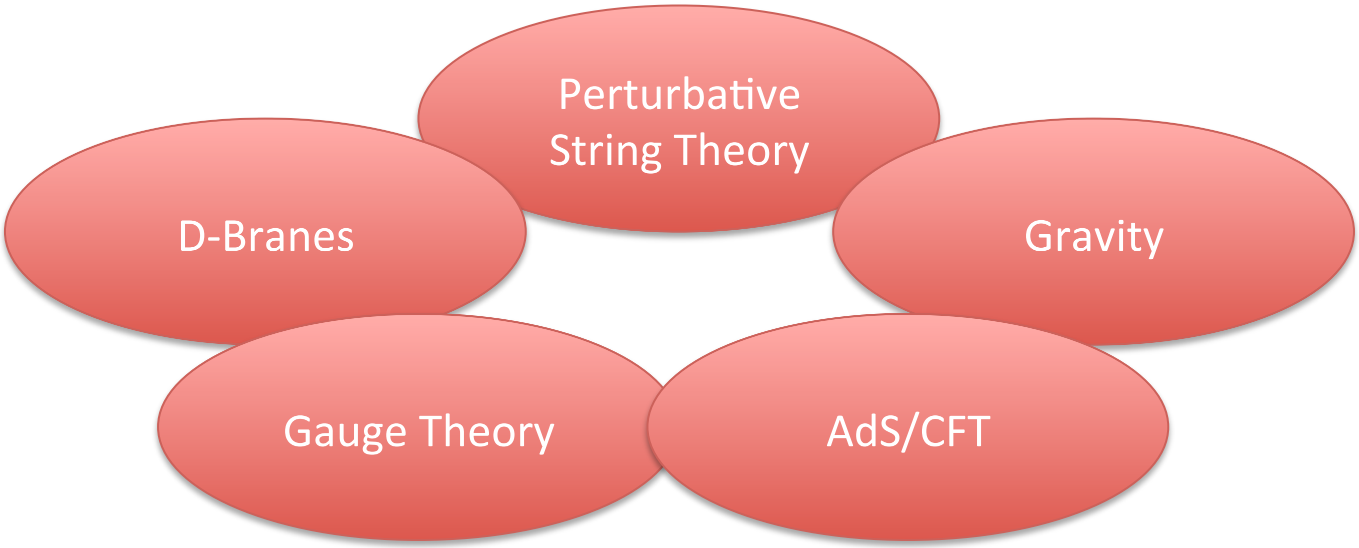
The Faces of String Theory



Quantum Gravity

$$Z = \sum_{\text{topologies}} \int \mathcal{D}g \, \mathcal{D}\phi \; \exp\left(\frac{i}{\hbar} \int d^4x \sqrt{-g} R + \mathcal{L}_{\text{matter}}[\phi] \right)$$

Can we make sense of this?

A Simpler Toy Model

$$Z = \sum_{\text{topologies}} \int \mathcal{D}g\,\mathcal{D}\phi\ \exp\left(-\frac{1}{\hbar}\int d^2x\sqrt{g}R + \mathcal{L}_{\text{matter}}[\phi]\right)$$

- Quantum field theory in d=1+1 is often more tractable.
- We have also gone to Euclidean space to make life easier.

$$\int_{\mathcal{M}} d^2x\sqrt{g}R = 4\pi\chi(\mathcal{M})$$

An Even Simpler Toy Model

$$Z = \sum_{\text{topologies}} \int \mathcal{D}g\,\mathcal{D}\phi\; \exp\left(-\frac{1}{\hbar}\int d^2x\sqrt{g}R + \mathcal{L}_{\text{matter}}[\phi]\right)$$

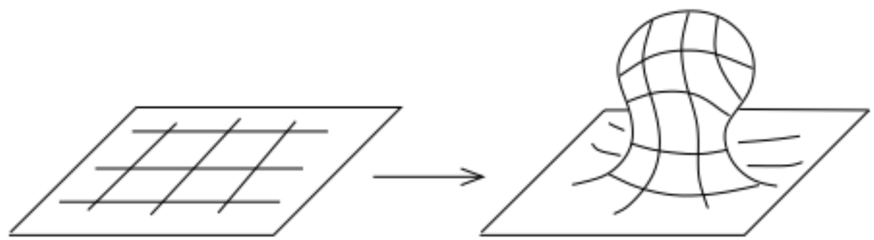
$$\mathcal{L}_{\text{matter}}[\phi] = \sqrt{-g}g^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi$$

i.e. a *massless* scalar field. This is special because it has a new symmetry

The Symmetries

$$\mathcal{L}_{\mathrm{matter}}[\phi] = \sqrt{-g}g^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi$$

- Diffeomorphisms (in two dimensions)
- Weyl Symmetry: $g_{\alpha\beta} \to \Omega^2(x)\,g_{\alpha\beta}$

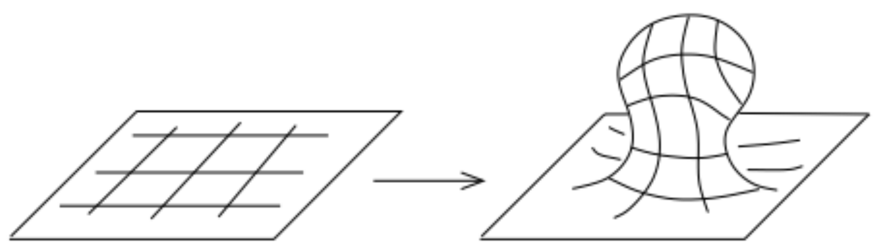


- Both diffeomorphisms and Weyl symmetry are gauge symmetries.
  - i.e. they are redundancies of our description
- This wouldn't work if we included a potential for the scalar
  - And it doesn't work in higher dimensions.

Conformal Field Theories

$$\mathcal{L}_{\text{matter}} = \mathcal{L}_{\text{CFT}}$$

Any conformal field theory (CFT) coupled to 2d gravity has *classical* Weyl symmetry



The *central charge, c*: This characterises the number of degrees of freedom of a CFT

Perturbative string theory is the study of quantum field theories with Weyl symmetry.

Anomalies

Anomalies are symmetries of classical field theories that do not survive quantisation.

- Anomalies in global symmetries are interesting
  - Most of the mass in the Universe
  - Chiral anomaly in QCD (pion decay, eta-prime mass)

- Anomalies in gauge symmetries are fatal
  - Cancellation of gauge (and gravity!) anomalies in Standard Model

Weyl Anomaly

Polyakov

The Weyl symmetry suffers an anomaly. This is bad. In fact, it suffers two anomalies

Trace of stress tensor should vanish in a conformal field theory

But it doesn't typically vanish in a curved background

$$\langle T^{\alpha}{}_{\alpha} \rangle = -\left(\frac{c}{12} - \frac{13}{6}\right) R$$

c is the *central charge* of the CFT. It counts the number of degrees of freedom.

But you get an extra term coming from the dynamical metric. (Strictly speaking, from ghosts)

The Punchline

Quantum field theories with Weyl symmetry only make sense if

$$c = 26$$

e.g.
- 26 free scalars
- 52 copies of the Ising model
- Many more interesting possibilies…

An Example of a CFT with *c=26*

$$\mathcal{L}_{CFT} = \sqrt{g} g^{\alpha\beta} \, \partial_\alpha \phi^\mu \partial_\beta \phi^\nu \, G_{\mu\nu}(\phi)$$

This has the interpretation of a two-dimensional object (i.e. string) moving in a *d=26* dimensional space with metric *G*.

This is where the extra dimensions of string theory come from.

An Example of a CFT with *c=26*

$$\mathcal{L}_{CFT} = \sqrt{g} g^{\alpha\beta}\ \partial_\alpha \phi^\mu \partial_\beta \phi^\nu\, G_{\mu\nu}(\phi)$$

But….this isn't a CFT for any choice of background metric *G*. It needs to obey

$$R_{\mu\nu}[G] = 0$$

Friedan

These are the vacuum Einstein equations in d=26!

Other Examples of CFTs with *c=26*

$$\mathcal{L}_{CFT} = \sqrt{g}\left(g^{\alpha\beta}\ \partial_\alpha\phi^\mu\partial_\beta\phi^\nu\, G_{\mu\nu}(\phi) + \epsilon_{\alpha\beta}\partial_\alpha\phi^\mu\partial_\beta\phi^\nu B_{\mu\nu}(\phi) + \alpha'\Phi(\phi)R\right)$$

The theory is a CFT only if the functions *G,* B and *D* obey the equations of motion arising from the action

$$S = \int d^{26}X\sqrt{G}e^{-2D}\left(R[G] - H_{\mu\nu\rho}H^{\mu\nu\rho} + 4\partial_\mu D\partial^\mu D\right)$$

$$H = dB$$

Some Comments

- c=26 is also special for another reason: the whole set-up works in Lorentzian signature. e.g.

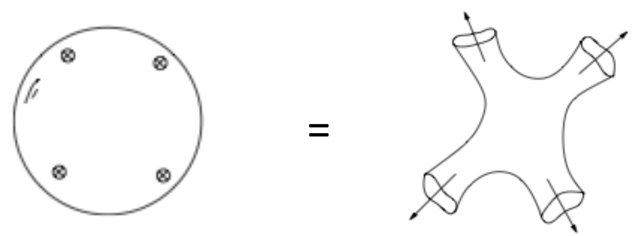
$$\mathcal{L}_{CFT} = \sqrt{-g} g^{\alpha\beta}\ \partial_\alpha \phi^\mu \partial_\beta \phi^\nu\, G_{\mu\nu}(\phi)$$

- Include fermions in d=1+1 dimensional theory and you find that the critical dimension is d=10. Now CFT gives you equations of d=10 supergravity.

Main Lesson: Geometry is replaced by CFT

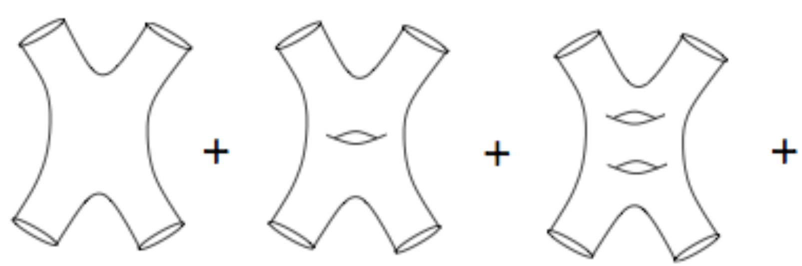
Finding Gravitons

- Vacuum state of theory = Einstein equations
  - Excited states in d=1+1 = Particles in higher dimensions
  - Correlation functions in d=1+1 = S-Matrix in higher dimension



- Note: In this way, can change the background spacetime. It's not fixed!

- Summing over topologies gives quantum corrections to gravity…



Open Problems

- It’s a first quantised theory. What is the more general formulation?
    - Note: first quantised but not fixed particle number!
- Time dependent backgrounds poorly understood
- Finite perturbation theory?

Thank you for your attention