Extrasolar Planets - Atmospheres and Interiors

Cambridge Part-III Mathematics/Astrophysics, Michaelmas 2015

Example Sheet 1

Problem 1: Transmission Spectra of Transiting Planets. Figure 1 shows an observed low-resolution near-infrared transmission spectrum of a transiting exoplanet along with a model fit to the data. Answer the following questions using the figure.

(a) Estimate the radius of the planet assuming it orbits a sun-like star.

(b) Estimate the scale height of the atmosphere and the characteristic atmospheric temperature at the day-night terminator of the planet. Assume that the mass and atmospheric composition of the planet are the same as those of a solar system planet with a radius closest to that estimated in part (a).

(c) Given the observational uncertainties in the data points, what is the approximate signal-to-noise (S/N) of the H_2O absorption feature near 1.4 μm. Assuming the same observational uncertainty, what is the radius of the smallest 'hypothetical' planet around the same star whose transits can be detected at S/N = 5 using the same instrument in the same spectral region.

(d) Now, assuming that the star is instead an M Dwarf with a radius of 0.2 $R_\odot$, what are the radius and temperature of the planet? Make similar assumptions as in part (b).

Problem 2: Thermal Emission Spectra of Transiting Planets.

(a) An observation of thermal emission from a transiting exoplanet, WASP-12b, in the IRAC 8 μm channel of the Spitzer Space Telescope obtained an eclipse depth of 0.7 ± 0.09 %. For simplicity, assume that all the emission is coming from the central wavelength of the channel.
(i) Estimate the brightness temperature of the planetary atmosphere in the observed channel. The stellar and planetary properties for this system can be obtained from one of the online exoplanet catalogues linked on the course site. State and/or justify any assumptions you make about the stellar and planetary fluxes.
(ii) If the brightness temperature derived above is in fact the temperature of the whole atmosphere (i.e. assuming an isothermal atmosphere), at what wavelength is the peak of the planetary spectrum? And, what is the planet-star flux ratio at that wavelength?

(b) Assuming that future observations can obtain precisions of 0.01 % at any wavelength, what is the radius of the smallest hypothetical planet for which thermal emission can be detected around the same star at a S/N $\geq$ 3? Assume a maximum possible planetary temperature of 3000 K.

(c) Calculate the wavelengths corresponding to the peaks of thermal emission for the following objects: Earth, Jupiter, Sun, three hot Jupiters (at 1000, 2000, and 3000 K), and three hot Neptunes (at 1000, 2000, and 3000 K). Assume that all these objects emit as blackbodies. For Earth, Jupiter, and Sun, adopt the equilibrium temperatures from the sheet distributed in class (also available on the course site).

Problem 3: Direct Imaging of Exoplanets

(a) What are the two primary challenges in direct imaging of exoplanets?

(b) What is the sensitivity on the planet-star flux ratio and the angular resolution required to image an Earth-like planet around a sun-like star at a distance of 10 parsecs? Assume that you are observing in thermal emission, and ignore reflected light in this exercise. Repeat the same exercise for a Jupiter analogue.

(c) One of the first directly-imaged exoplanetary systems, HR 8799, at a distance of 39.4 pc from Earth, was discovered in 2008 using the 10-m Keck Telescope in Mauna Kea, Hawaii. The discovery image, shown in Fig. 2, in the near-infrared showed three planetary companions in the system at angular separations of 0.63", 0.95", and 1.73" from the central star. Estimate the projected orbital separations of the three planets HR 8799b,c,d in the sky plane? Assuming the images were obtained at a wavelength of 1.6 micron, what is the diffraction limit of the telescope in arcsecs ("), and how does that compare with the angular separation of the innermost planet? What limits the minimum angular separation at which planets can be detected, i.e. the Inner Working Angle (IWA)?

(d) Consider a telescope that is able to confidently detect planet-star flux ratios $\geq 0.01\%$ at an IWA of 0.4" in the mid-infrared, at say 10-μm. What are the orbital separations and temperatures of the coolest and hottest planets that can be detected with such a telescope, for systems at a distance of 40 pc? Assume Earth and Jupiter to be the two extremes in planet sizes. Assume the stellar temperature and radius to be 7400 K and 1.34 $R_\odot$, respectively.

Problem 4: Radiative Transfer

(a) Show that the specific intensity is invariant of distance (d) in free space whereas the flux falls off as $1/d^2$.

(b) Derive the flux received by a detector at distance d from a spherical isotropic blackbody. Derive the equilibrium temperature of a planet at an orbital separation a from the star, assuming that the star is a blackbody with a temperature T_s and that the planet has an albedo A. Why do you think planets generally have higher 'surface' temperatures than their equilibrium temperatures, as shown in the data sheet distributed in class (also on course site)? Compute the equilibrium temperature for a typical hot Jupiter, making reasonable assumptions.

(c) The emergent spectrum of an exoplanet was found to be of a double-peaked structure, with the two peaks occurring at wavelengths of 0.5 μm and 1.0 μm respectively. What could be causing the two peaks? The planet-star flux ratio of the spectrum in the far infrared was found to be 0.1% and the star was found to be a sun-like star. What are the radius and the approximate orbital separation of the planet? Make and state any assumptions required. Which detection method is most likely to find such a planet and why?

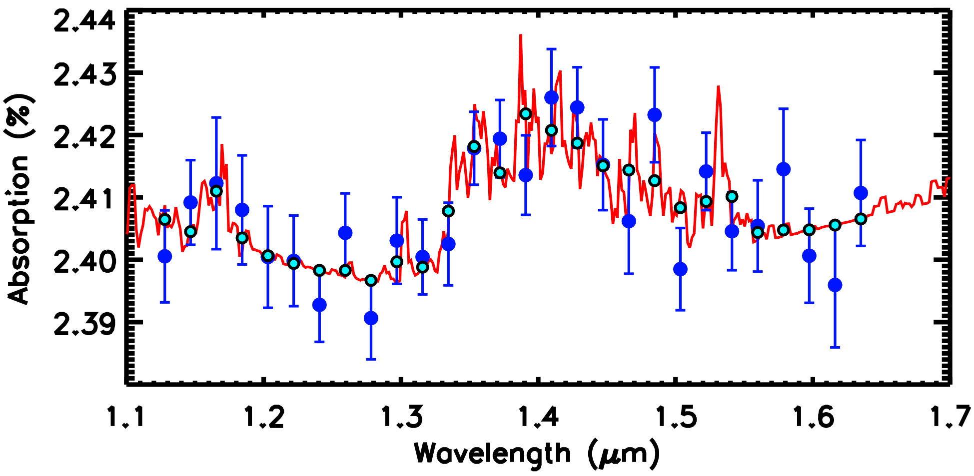

Figure 1: A transmission spectrum of an exoplanet observed using the Hubble Space Telescope when the planet was passing in front of its parent star as viewed from Earth. The blue circles with error bars show the observed data. The red curve shows a model matching the data. The cyan dots represent the red model curve binned to the same resolution as the data.

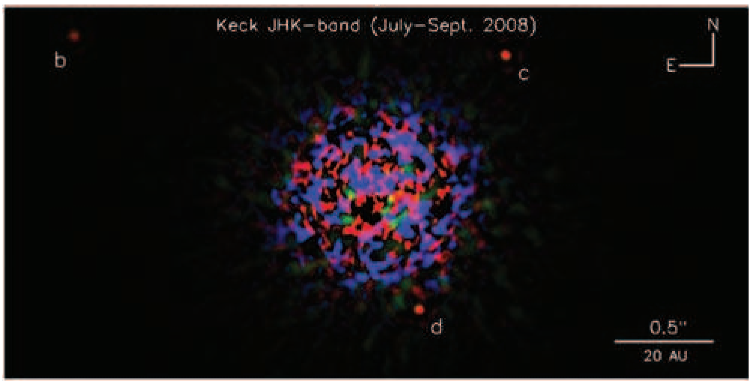

Figure 2: A direct image of the planetary system HR 8799 showing the three planets (b,c,d) originally discovered in 2008 (Marois et al. 2008, Science, 322, 1348) using the Keck Telescope in Hawaii.