Extrasolar Planets - Atmospheres and Interiors

Cambridge Part-III Mathematics/Astrophysics, Michaelmas 2015

Examples Sheet 2

Problem 1: Energy Transport and Temperature Profiles in Atmospheres.

(a) Derive the expression for the temperature gradient of a plane-parallel atmosphere in radiative equilibrium and heated from below due to an internal energy source. Assume an optically thick atmosphere and an appropriately averaged gray opacity. (Hint: The temperature gradient will be of the form:

$$\frac{dT}{dz} = -\frac{3\bar{\kappa}\rho}{16\sigma T^3}F. \tag{1}$$

Make any required assumptions and explain.) Use the above result to also derive an expression for the temperature profile as a function of the optical depth.

(b) Show using simple scaling arguments that for a highly irradiated atmosphere the lower layers of the observable atmosphere approaches an isotherm.

(c) Derive the condition for the temperature gradient that makes an atmosphere unstable against convection, i.e. that allows energy transport by convection.

(d) Consider three categories of planets: (a) Earth and larger planets in the solar system, (b) irradiated giant exoplanets (e.g. hot Jupiters), and (c) less irradiated giant exoplanets on distant orbits (e.g. directly imaged planets or isolated objects). Draw qualitative sketches of atmospheric pressure-temperature (P-T) profiles expected for the three categories of planets. Comment on which parts of the atmospheres are dominated by radiative versus convective temperature gradients, and at what pressures are the typical radiative-convective boundaries in each case.

(e) Derive the expressions for pressure and density as functions of altitude in an isothermal atmosphere. Define the scale height.

Problem 2: Theory of Thermal Inversions in Planetary Atmospheres.

(a) Draw qualitative sketches for the P-T profile of a hot Jupiter atmosphere with and without a thermal inversion, and compare it to an isotherm.

(b) Discuss the key developments in the theory of thermal inversions in highly irradiated giant exoplanets, i.e. hot Jupiters. What is the current state-of-the-art in observational evidence for thermal inversions?

(c) Figure 1 shows model spectra of two hot Jupiter atmospheres. Use the figure to answer the following questions. (i) Which of the two models has a thermal inversion and which does not? (ii) What are the approximate brightness temperatures of the two models in the photometric bandpasses at 3.6 μm and 4.5 μm? (iii) What are the maximum and minimum atmospheric temperatures of each model?

(d) Do solar system planets host thermal inversions? If yes, which of them do and what is responsible for the thermal inversion in each of those planets? How do they differ from our expectations of thermal inversions in hot Jupiters?

(e) Show that the part of a temperature profile with a thermal inversion is necessarily radiative, i.e. it cannot be convective.

(f) Demonstrate that temperature profiles without (with) thermal inversions give rise to absorption (emission) features in thermal emission spectra of hot Jupiters. Hint: Start with the expression for specific intensity for emergent flux, and consider just a single layer of atmosphere with a constant temperature.

Problem 3: Chemical Equilibrium

(a) Derive the condition for chemical equilibrium in a planetary atmosphere. Derive also the expression for the total Gibbs free energy (G) of a closed chemical system in gas phase. Hint: G should be in the following form:

$$\frac{G}{RT} = \sum_i N_i \left[\frac{G_i^0(T)}{RT} + \ln\Big(P\Big) + \ln\Big(\frac{N_i}{N}\Big) \right] \tag{2}$$

(b) List the five most dominant molecules expected in hydrogen-rich atmospheres, and discuss their dependance on temperature, metallicity, and the C/O ratio. Assume a nominal pressure of 1 bar. Compare the equilibrium chemistry expectations in solar system giant planets versus hot Jupiters.

(c) Consider a reaction A + B → C + D. The rate coefficient for the forward (reverse) reaction is given as k_f (k_r), and the Gibbs free energies of the chemical species are known. Show that in equilibrium the following condition holds:

$$\frac{k_f}{k_r} = e^{(G_f - G_r)/RT}, \tag{3}$$

where, $G_f = G(A) + G(B)$ and $G_r = G(C) + G(D)$. The above expression demonstrates that the ratio of the reaction rates are related to the Gibbs free energy of the system. So, in practice if only one of the reaction rates is known experimentally the other can be obtained using the Gibbs free energies and the above expression.

Problem 4: Non-equilibrium Chemistry

(a) Draw a rough sketch of the key chemical processes in planetary atmospheres as a function of altitude or pressure. Hint: Show the regions of an atmosphere which are expected to be in chemical equilibrium and those where different disequilibrium processes dominate.

(b) Consider the following reaction for CO conversion in the atmosphere of a gas giant planet: $CO + 3H_2 \rightarrow CH_4 + H_2O$. Given that the chemical timescale ($\tau_{\rm chem}$) for the forward reaction at 1 bar pressure is 10^5 s, and that the atmospheric temperature profile is an isotherm at 1200 K. Estimate the vertical eddy mixing coefficient (K_{zz}) that would be required to quench CO in the atmosphere at 1 bar, and loft it to higher regions of the atmosphere. Make reasonable assumptions as needed and state them.

(c) What is the evidence for vertical mixing in the atmosphere of Jupiter in the solar system? Which molecules would be useful for inferring vertical mixing on giant exoplanets, and for which ranges of planetary temperatures?

(d) What are the two key effects of photochemistry on the chemical compositions of giant exoplanetary atmospheres? And, what are the effects on their temperature profiles, if any?

(e) Derive the expression for ozone concentration in the Earth's atmosphere, and discuss why there is an ozone "layer". What are the effects of the ozone layer on the temperature profile of Earth's atmosphere and on the biosphere?

(f) What is the effect of photochemistry on the atmospheric chemistry and temperature profile of Jupiter in the solar system?

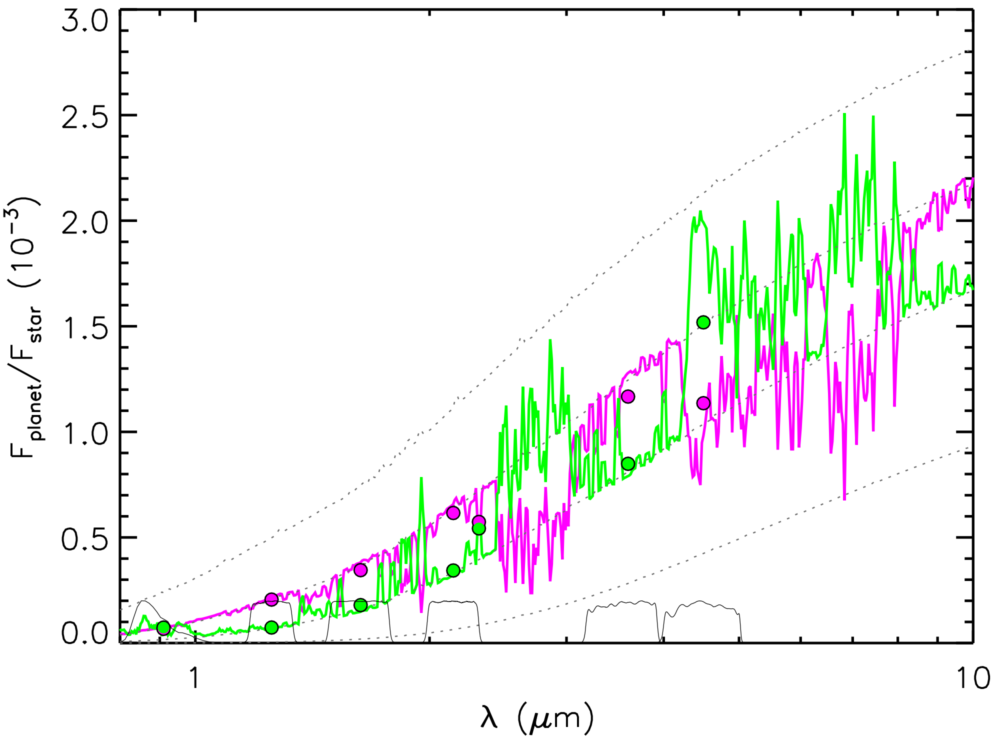

Figure 1: Model thermal emission spectra of a hot Jupiter. The two solid curves in green and magenta show spectra of planet-star flux ratios for two atmosphere models of the same planet. Both models have the same chemical composition, but different temperature profiles. One model has a thermal inversion whereas another does not. The four black dotted curves show spectra assuming the planet is a blackbody, e.g. has an isothermal temperature profile, with temperatures of 1300 K, 1900 K, 2300 K, and 2800 K. The colored circles are the models binned in the photometric bandpasses shown at the bottom of the figure in black solid curves.