Extrasolar Planets - Atmospheres and Interiors

Cambridge Part-III Mathematics/Astrophysics, Michaelmas 2015

Example Sheet 4

Problem 1: Interiors of Rocky Exoplanets

(a) Seismic measurements are used to infer the density profile of the Earth's interior. Two types of seismic waves are particularly useful for this purpose - the primary or pressure (P) waves and the secondary or shear (S) waves. The velocities of the P and S waves are given by $v_p = \sqrt{(K_m + 4\mu_s/3)/\rho}$ and $v_s = \sqrt{\mu_s/\rho}$, respectively. K_m and μ_s are the bulk modulus and shear modulus of the material, respectively, and ρ is the density. Show that the interior density profile can be determined using measurements of v_p and v_s. *Hint: Using the equation of hydrostatic equilibrium and the definition of bulk modulus, $K_m = \rho(dP/d\rho)$, derive the Adams–Williams equation which is given by*

$$\frac{d\rho}{dr} = -\frac{GM}{r^2}\frac{\rho^2}{K_m} \tag{1}$$

(b) Using the above result and the equation for mass conservation show that the mass profile of the interior can also be determined.

(c) Given that only the mass and radius have been measured for a transiting super-Earth with high precision. Can any constraints be placed on the interior composition of the planet? What are the various factors that influence the constraints? What additional observations might be useful to improve on the constraints?

(d) What are the sources of energy in the Earth's interior? What additional sources might be expected in the interiors of terrestrial exoplanets?

Problem 2: Habitable Planets

(a) The Greenhouse effect: Derive the equilibrium temperature (T_e) of a terrestrial exoplanet with a surface Bond albedo A_B orbiting a star with temperature $T_\star$ and radius $R_\star$ at an orbital separation a. Using a simple one-layer atmosphere model, show that the surface temperature (T_s) of the planet due to greenhouse effect is given by

$$T_s = \left[\frac{2}{2-\alpha}\right]^{1/4} T_e \tag{2}$$

(b) What is the habitable zone? What is the extent of the habitable zone in the solar system? What are the factors governing the inner edge and outer edge of the habitable zone in a given system. How does the location of habitable zone depend on the stellar type?

(c) List and discuss four factors affecting the habitability of rocky exoplanets in the habitable zones of their host stars.

Problem 3: Plate Tectonics and Biosignatures

(a) Some GPS measurements of the Earth's continental plates showed lateral velocities of $\sim$2 cm per year. Assuming the plates have always moved at this rate, estimate the approximate duration (to within a factor of 5) of the Wilson cycle over which the continents merge and separate. What constraint does the above measurement place on the energy transport mechanism in the Earth's interior?

(b) What factors influence the possibility of plate tectonics on rocky exoplanets? Why don't terrestrial planets other than Earth in the solar system have plate tectonics?

(c) What are the characteristics of an ideal biosignature gas? Name such gases present in the Earth's atmosphere. Are H_2O, CH_4, and CO_2 biosignatures? Why?

(d) Discuss the difference between primary and secondary metabolic byproducts.

(e) Discuss any three observational facilities in the next decade that will help in the detection of super-Earths and/or in characterizing their atmospheres.