Structure and Evolution of Stars

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[For Examples class 2-4pm Monday 25th October] Michaelmas Term, 2010

Examples Sheet I

1. Sirius A has an apparent visual magnitude $m_{\mathrm{V}} = -1.5$ and a spectral type A0V. Its distance is 2.7 pc. Calculate its luminosity and radius in solar units.

Sirius B is a binary companion to Sirius A. It has the same spectral type and $m_{\mathrm{V}} = 8.5$. Calculate its luminosity and radius.

Measurements of the orbits of A and B give a binary period of 50 years, mass ratio $M_{\mathrm{A}}/M_{\mathrm{B}} = 3$ and a semi major-axis of 8 seconds of arc.

Calculate the masses and mean densities of both components. Which star is the main-sequence star and what is the other?

[*For an* A0V *spectrum you may assume* $T_{\mathrm{eff}} = 10{,}000\,\mathrm{K}$, *and Bolometric Correction*, $m_{\mathrm{bol}} - m_{\mathrm{V}} = -0.4$. *An absolute bolometric magnitude* M_{bol} *of zero corresponds to a luminosity of* $3.0 \times 10^{35}\,\mathrm{erg\,s^{-1}}$. *For a binary orbit you may use the fact that the semimajor axis a, total mass M and period P are related by Kepler's law.*

$$\frac{a}{\mathrm{A.U.}} = \left(\frac{M}{M_{\odot}}\right)^{1/3} \left(\frac{P}{\mathrm{years}}\right)^{2/3} \quad]$$

2. In any equilibrium configuration prove that

$$\frac{d}{dr}\left(P + \frac{Gm^2}{8\pi r^4}\right) < 0$$

where $m(r) = \int_0^r 4\pi r^2 \rho dr$. Deduce a lower limit for the central pressure P_{c} in terms of the stellar mass and radius. Evaluate the limit for the Sun.

3. Show that $\Omega = -4\pi \int_0^R \rho \frac{Gm}{r} r^2 dr$. Hence show that the gravitational energy of a polytrope of index n is given by

$$\Omega = -\frac{3}{5-n}\frac{GM^2}{R}$$

The recombination energy of stellar matter is I per unit mass. Show that if the internal structure of a star corresponds to an $n = \frac{3}{2}$ polytrope total disruption of the star is energetically feasible if

$$R > \frac{3}{7}\frac{GM}{I}$$

Given that the ionization energy of a hydrogen atom is 13.6 eV estimate the maximum radius for a star of $1\,M_\odot$ and explain the existence of red giants.

4. By using the virial theorem, or otherwise, show that the period Π of spherically symmetric pulsations of a star satisfying $I = 4\pi\int_0^R r^4\rho dr = kMR^2$ and $\Omega = -\eta GM^2/R$, where k and η are constants, is given by

$$\Pi = \left[\frac{3\pi k}{(3\gamma - 4)\eta G\bar{\rho}}\right]^{\frac{1}{2}},$$

where $\bar{\rho}$ is the mean density of the star.

Comment on the relation of Π to the free-fall time and the acoustic travel time from surface to centre.

5. If ρ decreases outwards, show that $m < \frac{4}{3}\pi r^3\rho_c$.

Prove that $P_c < \frac{1}{2}G\left(\frac{4}{3}\pi\right)^{\frac{1}{3}}\rho_c^{\frac{4}{3}}M^{\frac{2}{3}}$.

Deduce that if $\beta = P_{\rm gas}/P$, then

$$1 - \beta_c \le 1 - \beta^* \text{ where } \beta^* \text{satisfies Eddington's quartic}$$

$$M = \left(\frac{6}{\pi}\right)^{\frac{1}{2}}\left[\left(\frac{R}{\mu_c}\right)^4\frac{3}{a}\left(\frac{1-\beta^*}{\beta^{*4}}\right)\right]^{\frac{1}{2}}G^{-\frac{3}{2}}$$

Evaluate M for $1 - \beta^* = 0.2, 0.5, 0.8$ and comment on what you find.