Galaxies

Lectures 22-23: seeds of galaxy formation

- Goal: to fill in the early era of galaxy formation up to the point of gas cooling in DM halos
- Growth of structure
- Spherical collapse
- Press-Schechter

Cosmology

- Universe described by Friedmann equation -> particularly relevant as it is the same equation that describes growth of dark matter overdensities
- Review Preliminaries:
  - General relativity describes the Universe as a four dimensional spacetime.
  - The relationship between matter/energy and geometry is provided by the Einstein field equation and the metric tensor, where $T_{\mu\nu}$ is the stress-energy tensor (i.e. matter and energy) and $G$ is Newton’s constant in relativistic ($c = 1$) units. We realize $\Lambda$ has to come into this in a proper treatment.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}.$$
The Friedmann equation

- The Friedmann equation describes the time variation of the scale factor $a(t)$.
- It can be derived by substituting the metric terms described by the RW line element into the Einstein field equation.
- However, we will review the Friedmann equation using a Newtonian analogue:
  - Consider a universe with coordinate distances defined by the RW line element.
  - The universe is populated with galaxies with a space density $\rho$ and pressure $P = 0$ (i.e., the galaxies do not interact with each other).
  - Consider a spherical volume of the universe of radius $l$ and mass $M$.
  - We further consider the dynamical behaviour of a test particle (a single galaxy if you like) of mass $m$ located on the surface of this spherical shell.
  - Birkhoff’s theorem states that the mass within the sphere will act upon the test particle as if the entire mass $M$ were concentrated at the centre of the sphere.

- From the Newtonian equation of motion of the test particle we therefore obtain
  - Multiplying the equation by $dl/dt$:
  - Integrating yields
    $$ \frac{1}{2} \frac{dl}{dt} = \frac{GMm}{l^2} $$
    where the integration constant has units of energy.
    - Note: this equation may be interpreted as a basic energy relation of the form, $\text{Kinetic} + \text{Potential} = \text{Total}$.

The Friedmann equation

- $E < 0$: The potential term is proportional to $1/a(t)$ and always dominates. Hence $a(t)$ cannot increase without limit, instead it must reach some maximum value (at which point $d^2a/dt^2 < 0$) and decrease.
  - $E = 0$: $a(t)$ increases throughout time, tending toward (but never reaching) an asymptotic maximum scale as $t \to \infty$. The critical density value corresponding to this case is
    - The current value of the Hubble parameter is $H_0 = 70 \pm 10$ km s$^{-1}$ Mpc$^{-1}$ and the corresponding value of the critical density is $\rho_c = (2.7 \pm 0.3) \times 10^{-27}$ kg m$^{-3}$.
    - Current estimates of the space density of galaxies are of order $10^{-27}$ Mpc$^{-3}$.
  - If one assumes that the mass of a typical galaxy is $10^{11}$ M$_\odot$, then one concludes that visible galaxies contribute about 1% of the value of $\rho_c$.
- $E > 0$: $a(t)$ increases monotonically for all time. The universe expands forever.

The Friedmann equation

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The Friedmann equation

- Returning to the form of Friedmann’s equation, one may redefine the constant of integration to be
  $$ \frac{2E}{\Omega} = -k \Omega. $$
- Re-writing the Friedmann equation yields
  $$ \frac{2E}{\Omega} = \frac{k \Omega}{3}. $$
- Note that this form of the equation was that obtained by Friedmann (and re-discovered by Lemaitre) by inserting the corresponding elements of the metric tensor applicable to the RW line element into the EFQ.
- The parameter $k$ takes one of three values
  - $E < 0$ : $k = -1$
  - $E = 0$ : $k = 0$
  - $E > 0$ : $k = 1$

- We have now obtained the complete link between the time dependence of the universal scale factor $a(t)$ and the curvature constant $k$ first encountered in the RW line element.
- Proceeding further, one may redefine the Friedmann equation in terms of a new dimensionless density parameter
  $$ \Omega(t) = \frac{\rho(t)}{\rho_c} $$

- $\Omega_0 = 0.5$ (open)
- $\Omega_0 = 1$ (flat)
- $\Omega_0 = 2$ (closed)
The Friedmann equation
- If we divide the Friedmann equation by $H(t)^2$ and use the above identity, we obtain

$$\frac{H^2}{H_0^2} = \frac{8\pi G}{3} \rho = \Omega_\Lambda + \Omega_m + \Omega_k.$$ 

- This expression has the following consequences:
  1. For a closed universe ($k > 0$), $H(t)$ decreases with time, leading to an infinite age.
  2. For a flat universe ($k = 0$), $H(t)$ decreases as $t^{-1}$, leading to a finite age.
  3. For an open universe ($k < 0$), $H(t)$ increases with time, leading to an infinite age.

- Therefore, the total matter/energy content of the universe determines the overall spatial geometry, the time variation of $a(t)$, and the ultimate fate of the universe.
- Importantly, these equations indicate that the universe has a well-defined characteristic matter density that marks the limit of each of the above cases.
- Therefore, to determine this critical density, one requires $H(t)$ or $H_0$ (cosmological distance scale—previous lecture).
- The determination of the total matter content of the universe became an immediate challenge for early observational cosmologists.

The seeds of galaxy formation: The cosmological horizon and Inflation
- The cosmological horizon may be defined as the maximum distance a photon could travel within the lifetime of the universe.
- It is a convenient definition of the largest region of the universe that could exist in causal contact at any particular epoch.
- The horizon is defined by considering a radial null geodesic within the RW line element, i.e.

$$\int_0^{\infty} \frac{c dt}{a(t)} = \int_0^{r_H} \frac{dr}{\sqrt{1 - k r^2}}.$$

- Where $t_0$ indicates the current age of the universe and $r_H$ is the horizon distance.

Seeds of Galaxy Formation
- For an $\text{EdS}$ Universe (universe containing a non-interacting gas of galaxies) with $k = 0$ (flat) we may write:

$$a(t) = a_0 t^{2/3(1+w)},$$

- The combination of a finite age of the universe with a finite speed of light ensures that we can only observe photons from a finite region of the universe.
- However, computation of the cosmological horizon gives rise to the horizon problem.
- When we look at the universe at the cosmological horizon, it is isotropic, i.e., large scale structure and CMB temperatures are statistically identical even though separated by 180° on the sky.
- However, these two regions, though in casual contact with us, are themselves causally isolated.
- How could the CMB and LSS have developed in exactly the same manner?

Structure Formation:
- Cosmic inflation postulates that the universe underwent a rapid phase of expansion at early times.
  - Regions in the early universe were originally in causal contact and were subsequently inflated to scales much larger than the horizon during the inflationary epoch.
  - And inflation must therefore produce initial density fluctuations.

- Define the density contrast:

$$\delta = \frac{\rho - \rho_0}{\rho_0},$$

- Density perturbations may be modified by:
  - Amplification due to gravitational instability
  - Pressure
  - Dissipation

Short history of early Universe
- The energy density in the early universe is dominated by radiation, $p = w\rho c^2$, $a(t) = a_0 t^{2/3(1+w)}$.
- Therefore, we may write

$$a(t) \propto t^{1/2}; T(t) = a(t)^{-1}; T(t) \propto t^{-1/2}.$$ 

- Using radiation-matter equality provides constant of proportionality:

$$T(t) = 10^{10} K (t/1\text{sec})^{1/2}; k T(t) = 1 \text{ MeV} (t/1\text{sec})^{1/2}; E_{\text{mean}} = 2.7k T(t) = 3 \text{ MeV} (t/1\text{sec})^{1/2}.$$ 

- These formulae provide us with the energy scale as a function of epoch in the early universe.
- A comparison to the energy associated with particle interactions provides an understanding of the physical conditions associated with each epoch.
Short history of early Universe

1) The hadron era: \( t_{\text{rec}} < t < 10^{-4} \, \text{sec} \).
   - This marks the earliest Universal epoch where experimental physics can be applied with any confidence.
   - Almost all matter, including electrons, protons, neutrons, neutrinos and their associated anti-particles are in thermal equilibrium with the photon radiation field.
   - The disparity between particles and anti-particles is thought to be 1 part in \( \sim 10^{7} \) and is ultimately responsible for all matter in the present day universe.
   - The exact physics (e.g. equation of state) of this epoch is not known.
   - Thus the dependence of the scale factor \( a(t) \) and the temperature \( T(t) \) upon cosmic time is not known.

2) The lepton era: \( t < 10^{-6} \, \text{sec} \).
   - The temperature decreases such that \( kT \) is significantly lower than the rest mass energy of the proton (\( m_p = 938 \, \text{MeV} \)).
   - Proton-antiproton pairs, in addition to other hadrons present, annihilate.
   - The lepton era begins with photons in thermal equilibrium with electrons and positrons, muons, neutrinos and antineutrinos.
   - The energy released by hadron annihilation is thus shared between all of these particle families.
   - Each of the relativistic particles (photons, neutrinos and electrons – plus antiparticles) contributes an energy density \( \sim T^4 \).
   - The lepton era ends when the radiation temperature drops significantly below \( T = 5 \times 10^9 \, \text{K} \) (i.e. \( kT = m_e^2 c^2 = 511 \, \text{keV} \)).
   - Electron-positron pairs annihilate and temperatures begin to decrease to levels where a protons fall out of equilibrium with neutrons.

3) The plasma era: \( 10 \, \text{sec} < t < 10^{-3} \, \text{sec} \).
   - The universe consists of photons, neutrinos, electrons, protons and neutrons (the discussion assumes that at this stage Dark Matter particles do not interact).
   - The early stages of the plasma era remain sufficiently hot and dense to produce light nuclear elements from hydrogen.
   - Matter and radiation are coupled to form a photon-baryon fluid: photons are coupled to electrons via Thompson scattering, and electrons are coupled to protons via Coulomb interactions.
   - Matter and radiation remain in thermal equilibrium until the photon temperature drops below \( T = 3 \times 10^{4} \, \text{K} \) where electrons combine with protons to form atomic hydrogen.
   - The radiation field continues unimpeded to the present day where it is observed as the Cosmic Microwave Background.

4) The post-recombination era: \( t > 10^{-3} \, \text{sec} \). Various astrophysical processes combine to produce the present day universe.
   - Understanding this cosmic history preceding the formation of galaxies provides insight into two important questions.
   - Why do galaxies contain so much helium? Gaseous nebulae in galaxies (HI and HII regions) contain some 25-27% of \( ^4\text{He} \) by mass.
   - Simple calculation indicates that this abundance could not have been created by stellar nucleosynthesis.
   - The helium is primordial in that it was created in the earliest (first few minutes) of the Universe’s history.

I) What were the first baryonic structures to form after recombination?
   The Jeans relation describes baryonic structures whose self-gravity is supported by internal pressure.
   Structures that are Jeans stable do not collapse and structure is suppressed on scales smaller than the Jeans scale. Consider a virialised cloud of particles at the stability limit, i.e.

\[
\frac{3}{2}kT = \frac{GM}{R_J^2}
\]

where subscript J indicates the Jeans limit.

Employing the relations \( N = M_J/m_p \), and \( M_J = 4\pi R_J^3 \rho / 3 \) one obtains the Jeans length

\[
R_J = \left( \frac{3kT}{\mu m_p (G\rho)} \right)^{1/2} = \frac{c_s}{\sqrt{G\rho}}
\]

Jeans length in early Universe

- The Jeans length is therefore proportional to the sound speed of baryons at each epoch.
- Prior to decoupling the baryonic sound speed was determined by the properties of the relativistic photon baryon fluid, i.e. \( c_s = c/\sqrt{3} = 0.58c \).
- The corresponding value of the Jeans length may be computed as

\[
R_J = 0.58c \times \sqrt{\frac{\pi}{6.67 \times 10^{-11} \, \text{Nm}^{-2}\text{m}^2 \times 5 \times 10^{-18} \, \text{km}^{-2}}} = 5.3 \times 10^{22} \, \text{m} \approx 1.6 \, \text{Mpc}
\]

- where we have taken the physical baryon density at the epoch of decoupling.
Jean's length in early Universe

- The associated Jean's mass is of order $10^{20} \, M_\odot$ which is several orders of magnitude larger than the most massive galaxy cluster known.
- Immediately after decoupling the sound speed is given by the expression:
  \[ \frac{kT}{m_p c} = \frac{0.26 \, eV}{563 \, MeV} = 2 \times 10^{-7} \, c. \]
- The corresponding Jean's length and mass are respectively:
  \[ R_J = 1.84 \times 10^{18} \, m \approx 50 \, pc \] and
  \[ M_J \approx 4 \times 10^6 \, M_\odot \]
- This limit is close to the lower mass limits of globular clusters and dwarf galaxies.
- Therefore, the epoch of decoupling marks an important transition in the development of baryonic structure.
- Prior to decoupling all fluctuations up to the horizon scale are effectively suppressed.
- After decoupling the Jean's scale is dramatically reduced and baryonic structures can begin to grow.

Structure formation

Define the density contrast: $\delta = (\rho - \rho_0)/\rho_0$

- density perturbations may be modified by:
  - amplification due to gravitational instability
  - pressure
  - dissipation
- growth of inhomogeneities
  - $\delta \ll 1$: linear theory
  - $\delta \sim 1$: need specific assumptions (i.e., spherical symmetry)
  - $\delta >> 1$ non-linear regime. solve numerically (or higher order perturbation theory)
- in general, Universe is lumpy on small scales and smoother on large scales.
- consider inhomogeneities as a perturbation to the homogeneous solution

Structure Formation schematically

\[ \frac{d^2 \delta}{dt^2} + [\text{Pressure} - \text{Gravity}] \cdot \delta = 0 \]

- (from continuity, Euler, Poisson eqns)
- in the absence of an expanding universe
  - if pressure is low, $\delta$ grows exponentially
  - if pressure is large, $\delta$ oscillates with time.
- the relevant scale here is the Jean's length (fluctuations grow if perturbations are longer than $R_J =$ sound speed/ dynamical time)
- now solve in an expanding universe

Structure formation

- for cold dark matter $P=0$, and
  \[ \frac{\dot{\delta}}{3} + \frac{2 \ddot{\delta}}{3} = 4\pi G \rho_\delta \]
- can write Friedmann eqn as $dH/dt + H^2 = -4\pi G/3 \rho_w$
  Since $\rho_w = \rho_0 a^{-3}$, differentiating wrt $t$...
  means this is like the equation of motion for a mini FRW Universe
- 2nd term is "Hubble drag" which suppresses perturbation growth due to expansion of the universe
- can be solved parametrically in the same way the Friedmann eqn can be solved
- General solutions with growing and decaying modes:
  \[ \delta(x) = f_1(x)D_1(t) + f_2(x)D_2(t) \]
  \[ D(t) = \delta(t_0) D_1(t)/D_1(t_0) + D_1(t) \cdot a(t) \]
Structure Formation

in a lambda dominated Universe

\[ H^2 = \frac{\Lambda c^2 \pi G}{3} \]

\[ \ddot{\delta} + 2H\dot{\delta} = 0 \]

\[ \delta = A(x) + B(x)e^{-2Ht} \]

frozen fluctuations

- solution for the matter dominated case
  \[ \delta = \delta_0(x)a \]
  linear growth

- solution for the lambda dominated case
  \[ \delta = A(x) + B(x)e^{-2Ht} \]
  frozen fluctuations

- general case
  \[ \delta = \delta_0(x)\gamma(a, \Omega_m) \]
  \( g \) is constant at early times and scales as \( 1/a \) at late times for our cosmology, the action ended around \( z = 0.5 \)

Linear Growth of Fluctuations

fluctuations grow more slowly in low \( \Omega_m \)

Universe

Growth of Structure

- CMB indicates that fluctuations in the baryon distribution at recombination had an amplitude less than \( 10^{-4} \)

- The existence of non-linear structures today implies that the growth of fluctuations must have been driven by non-baryonic dark matter which was not relativistic at recombination.

- After recombination, baryons decouple and fall into dark potential wells.

Growth of Structure

- after \( \delta > 1 \), non-linear regime

- matter accumulates in dense regions

- hard to approximate without numerical simulations

- random motions of particles halts the growth

- non-linear collapse on small scales doesn’t change the linear evolution of the large-scale perturbations (they don’t care whether the small scale power is lumpy – Gauss’s law!)

Linear Power Spectrum = Primordial Power Spectrum * Transfer Function * Growth Function

- Primordial Power Spectrum:

  \[ P_k \sim k^n \]

  \( n < 1 \): 'blue tilt', less power on small scales

  \( n = 1 \): 'scale-free', Harrison-Zeldovich-Peebles

  \( n > 1 \): 'red tilt', more power on small scales

  \( n(k) \): running scale index

  current constraints from WMAP+LSS: \( n = 0.95 \)
Transfer function

- the primordial power spectrum is 'processed' during the early Universe
- this is called the transfer function $T(k)$ (e.g. Bardeen et al. 1986; BBKS; Eisenstein & Hu 1999)

(depends on $\Omega_m$, $\Omega_b$, $H_0$, $\Omega_c$ type of dark matter particle...)

In pure CDM cosmology

- in the matter dominated era all scales grow equally
- in radiation-dominated era, pressure is important.
- scales smaller than the horizon: growth is stalled by the presence of radiation pressure. growth slows as $a^2$, and small scale modes are damped as $k^2 \rightarrow$ power spectrum suppressed by $k^4$.
- Universe is expanding too quickly for the dark matter to collapse
- scales above the horizon continue to collapse
- scales that enter horizon during the radiation-dominated era grow more slowly than those that enter during matter domination.

Power spectrum

- $P \sim k$ on large scales $P \sim k^3$ on small scales
- power below the Jeans length is suppressed
- the Jeans scale of the total system grows to the size of the horizon at matter-radiation equality and then shrinks to zero -- the transition scale marks the scale of the horizon at matter-radiation equality

Power spectrum from SDSS

![Power spectrum from SDSS](image)
Galaxies
Lecture 23: halos

- Spherical collapse
- Defining a halo
- Analytical representations
- Press-Schechter
- Structure of halos

Structure Formation

- we discussed the generation of fluctuations
- the evolution of the power spectrum under linear theory, applies when the fluctuations are small.
- this likely doesn’t work for big fluctuations; these fluctuations become non-linear
  - roughly, non linear is when the density fluctuations are ~ 1
    \[ \delta(x) = \frac{(\rho(x)-\rho_0)}{\rho_0} \]
- most of the field of structure formation and galaxy formation concerns understanding the non-linear regime, and collapsed objects in the density field

simulating the Universe

- choose a cosmological model \((\Omega_m, \Omega_\Lambda, \Omega_b, h, n, DM)\)
- choose a computational set up (box size, dynamic range, what physics to include)
- find the linear \(P(k)\) as per Lecture 22
- set up a random realization of \(P(k)\) in the linear regime \((200<z<30)\) in the chosen box
- follow the evolution of dark matter using particle N-body methods
- optionally, follow the evolution of the gas by numerically solving gas dynamic equations
- optionally, add sink and source terms to hydro equations, modeling heating and cooling of the gas, star formation, etc..
- evolve to the redshift of interest

A challenging problem …

- the scales of interest for dark matter structures are ~ pc to hundreds of \(\text{Mpc}\).
- would like to simulate a large volume, but also want to small scales & resolve collapsed objects with many particles
- your computer & the amount of time you have determines \(N\).
- Although not the only variable: refinement schemes, and the amount of clustering in the box (the more non-linear, the harder the problem) all play a role in computing time for a given \(N\)
- for a given particle number, always a trade-off between volume and mass resolution:
  - miss large scale modes and rare objects with small volume;
  - miss small objects and physics in dense regions with low particle number

Simulations
Basic things to study:
• how many collapsed things of various masses?
• how are they distributed in space?
• what internal properties do they have? (internal density distribution? internal angular distribution? shapes? internal substructure?)
• can we understand the collapsed objects (halo formation) analytically?

how to characterize this mess?
structure on a wide range of scales
• look at the evolution of $P(k)$ into the non-linear regime
• look at the distribution of density peaks

evolution of the matter power spectrum

\[ \Delta^2(k) = \frac{k^3 P(k)}{2\pi^2} \]

largest scales are still in the linear regime
finite volume box (large modes have noise)

Springel et al 2005

how to characterize this mess?
• clearly, it’s quite clumpy.
• look at the overdense regions.
• “dark matter halos”
• schematically: halos are self-gravitating systems in the universe
• or: halos are non-linear peaks in the dark matter density field whose self-gravity has overcome the Hubble expansion

evolution of matter clustering

\[ \langle r \rangle = \frac{\sqrt{V}}{2\pi} \int P(k) \sin(kr) dk \]

Collin et al 1999

equation of motion for the perturbation
(same as for a closed universe) (with a cosmological constant)

integrating

\[ \frac{1}{2} \dot{r}^2 - \frac{GM}{r} = E = \text{constant} \]

at turnaround,

\[ E = \frac{GM}{r_{\text{max}}} \]

the perturbation will be in virial equilibrium when

\[ 2K_{\text{vir}} + W_{\text{vir}} = 0 \]

\[ E = K_{\text{vir}} + W_{\text{vir}} = -W_{\text{vir}}/2 \approx \frac{GM}{2r_{\text{vir}}} = \frac{GM}{r_{\text{max}}} \]

which implies

\[ r_{\text{vir}} = r_{\text{max}}/2 \]

ie, 8 times denser than at turnaround
The “top-hat model”
Spherical Collapse

- consider a uniform, spherical perturbation
  \[ \delta_i = \frac{\rho(\tau_i)}{\rho_b(\tau_i)} - 1 \]
  \[ M = \rho_b(4\pi r_i^3/3)(1 + \delta_i) \]

now let’s count them

- but first, we need to define halo.
- schematically: halos are self-gravitating systems in the universe
- or: halos are non-linear peaks in the dark matter density field whose self-gravity has overcome the Hubble expansion
- operationally: a halo is a non-linear peak in the dark matter density field with its boundary defined by a given density contrast.
- perhaps we can use the model just described to define it. \[ IE: R_{180} = R_{vir} \]

sadly, halos don’t look like this:

halos look like this:

halo finder is typically looking for a non-linear peak in the dark matter density field, defined by some density contrast, such as a halo finder must choose a definition of mass and of boundary.

types of halo finders

- spherical overdensity
  - defines a halo as matter within a sphere, centered on a density peak, and enclosing a certain overdensity (for example, the "virial overdensity" we just defined)
- simple (one parameter \( \Delta_{vir} \)), requires spherical, doesn’t remove unbound particles
- friends-of-friends
  - all particles within a linking length are connected in a group. halo is connected region bounded by a density isosurface
  - simple (one parameter \( b \)) and accounts for non-spherical objects, doesn’t remove unbound particles.
- density maxima (DenMax, SKID, HOP)
  - define halos as connected regions above a certain overdensity, break neighboring peaks at saddle points.
The virial radius is the virial radius?

- the spherical collapse model is a reasonable approximation, but life is more complicated.
- stuff happens around $R_{\text{vir}}$, but really, nothing unique about $R_{\text{vir}}$ (or at $R_{180}$ or $R_{200}$)
- you thus see many choices, depending on the problem (or just depending on author’s whim)
- different definitions will result in different mass functions, bias relations, profiles, etc., and halo finders are also often different than analytic models (mostly at the few-10 percent level)
- if you know the density profiles of halos, then you can transform between them.
- “Let’s define the virial radius as the virial radius” — James Bullock

The halo mass function

**Press-Schechter theory in a nutshell**

- ansatz: the fraction of mass in halos more massive that $M$ is related to the fraction of the volume in which the smoothed initial density field is above some density threshold.
- this threshold depends on the window function, but for a spherical tophat in real space it is 1.69 (the extrapolated linear overdensity at which a spherical top-hat perturbation would collapse)

\[ f(\sigma, PS) = \sqrt{\frac{2}{\pi}} \frac{\delta_c}{\delta} \exp\left( -\frac{\delta^2}{2\delta_c^2} \right) \]

this can be modified for “ellipsoidal collapse” to produce something that matches the simulation data better (Sheth & Tormen). however, this is really just a fitting formula.
how can we characterize this?

- parameterize in terms of fluctuations in the mass field:
  \[ \sigma^2(M, z) = \frac{D^2}{2\pi^2} \int k^2 P(k) W^2(k; M) dk \]
- define the mass function:
  \[ f(\sigma, z; X) = \frac{M}{\mu} \frac{dN}{d\ln M} \]

Then the mass function is roughly "universal" (i.e., this formula works to ~15% for a wide range of cosmologies and redshifts)

\[ f(M) = 0.315 \exp(-1) + 0.61(0.38) \]

Jenkins et al. 2001

Halos also have substructure!

what do we know about dark matter halos?

why study halos?

- halos are the peaks in the density distribution; their properties (mass function, clustering, density profiles, substructures, etc) are sensitive to cosmological parameters
- halos are the sites of galaxy formation
  - galaxies form by gas cooling in a virialized dark matter halo (e.g., White & Rees)
  - astrophysics that determines galaxy properties mostly determined by the mass of the host halo (local environment) and not by surrounding structure (larger scale environment)
  - dark matter halos are comparatively easy: nonlinear, but primarily gravitational physics