Galaxies

Lectures 22-23: seeds of galaxy formation

- Goal: to fill in the early era of galaxy formation up to the point of gas cooling in DM halos
- Growth of structure
- Spherical collapse
- Press-Schechter

Cosmology

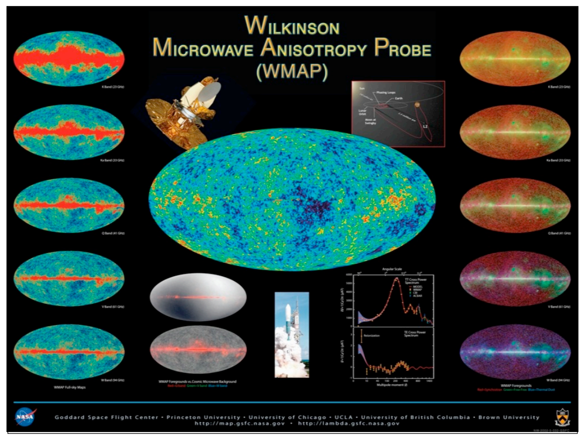
- Universe described by Friedmann equation -> particularly relevant as it is the same equation that describes growth of **dark matter overdensities**
- **Review Preliminaries:**
  – General relativity describes the Universe as a four dimensional spacetime.
  – The relationship between matter/energy and geometry is provided by the Einstein field equation and the metric tensor,
  where $T_{\mu\nu}$ is the stress-energy tensor (i.e. matter and energy) and G is Newton's constant in relativistic (c = 1) units. We realize Λ has to come into this in a proper treatment.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu},$$

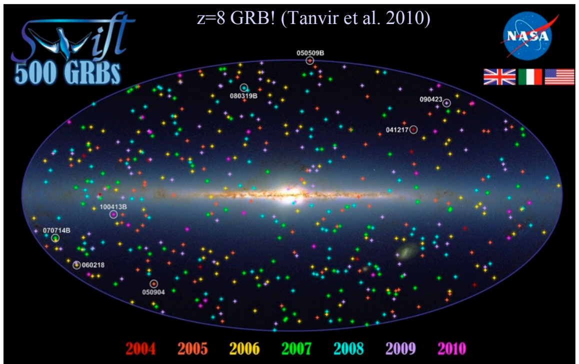
Cosmology

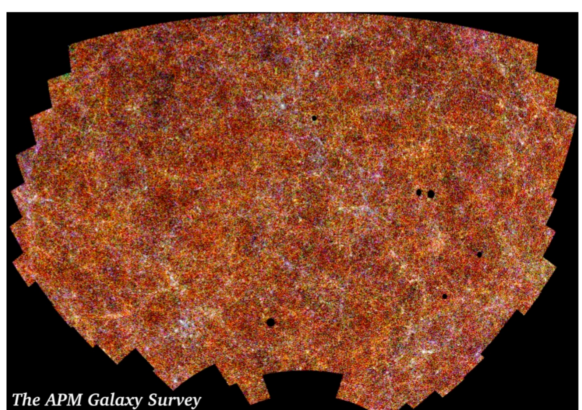
- The most basic geometrical operation is to measure the distance between nearby points via the line element
- Within the spacetime defined by the Riemann tensor, the line element, ds, describing the infinitesimal distance between two nearby points is $ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$.
- The Universe is isotropic on the largest detectable scales. Evidence for isotropy comes from the cosmic microwave background (CMB), they sky distribution of distant sources such as gamma ray bursts (GRBs) and, in a statistical sense, from the large scale distribution of galaxies.
- The implication of observed isotropy and the cosmological principle is that we exist in a homogeneous Universe.
- The simplest metric describing a homogeneous and isotropic spacetime is the Robertson-Walker (RW) metric

$$ds^2 = c^2\, dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$



z=8 GRB! (Tanvir et al. 2010)





The Friedmann equation

- The Freidmann equation describes the time variation of the scale factor a(t)
- It can be derived by substituting the metric terms described by the RW line element into the Einstein field equation.
- However, we will review the Friedmann equation using a Newtonian analogue:
  - Consider a universe with coordinate distances defined by the RW line element.
  - The universe is populated with galaxies with a space density ρ and pressure $P = 0$ (i.e. the galaxies do not interact with each other).
  - Consider a spherical volume of the universe of radius l and mass M .
  - We further consider the dynamical behaviour of a test particle (a single galaxy if you like) of mass m located on the surface of this spherical shell.
  - Birkhoff's theorem states that the mass within the sphere will act upon the test particle as if the entire mass M were concentrated at the centre of the sphere.

The Friedmann equation

- From the Newtonian equation of motion of the test particle we therefore obtain $$m\,\frac{d^2l}{dt^2} = -\frac{GMm}{l^2}.$$
- Multiplying the equation by *dl/dt:* $$\frac{d}{dt}\frac{\dot{l}^2}{2} = \frac{d}{dt}\frac{GM}{l}.$$
- Integrating yields $$\frac{\dot{l}^2}{2} - \frac{GM}{l} = E,$$

  where the integration constant has units of energy.
- Note: this equation may be interpreted as a basic energy relation of the form,
  Kinetic + Potential = Total.

The Friedmann equation

- The RW line element indicates that $l(t) = l_0\, a(t)$, where l_0 is independent of time and a(t) is the time-varying, universal scale factor. Therefore, we may write

$$\frac{\dot{l}^2}{2} - \left(\frac{G}{l}\right)\left(\frac{4\pi l^3 \rho(t)}{3}\right) = E$$

$$\frac{\dot{a}^2}{2} - \frac{4\pi G \rho(t) a^2(t)}{3} = \frac{E}{l_0^2},$$

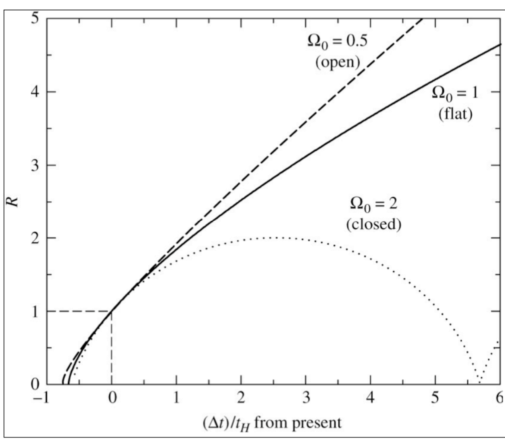
$$\dot{a}^2 - \frac{8\pi G}{3}\,\rho(t)\, a^2(t) = \frac{2E}{l_0^2}.$$

- The result is one form of Friedmann's equation and it has three general solutions:

The Friedmann equation

- **E < 0** : The potential term is proportional to *1/a(t)* and always dominates. Hence *a(t)* cannot increase without limit, instead it must reach some maximum value (at which point $d^2a/dt^2 < 0$) and decrease.
- **E = 0** : a(t) increases throughout time, tending toward (but never reaching) an asymptotic maxi- mum scale as $t \to \infty$. The critical density value corresponding to this case is $$\rho_c(t) = \frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2 = \frac{3H^2(t)}{8\pi G}.$$

  (Recall this result we have appealed to several times in past lectures)
  - The current value of the Hubble parameter is $H_0 = H(t_0) = 70 \pm 7$ kms^{-1} Mpc^{-1} and the corresponding value of the critical density is $\rho_{c,0} = (9.2 \pm 1.8) \times 10^{-27}$ kg m^{-3} = $(1.4 \pm 0.3) \times 10^{11}$ M⊙ Mpc^{-3} .
  - Current estimates of the space density of galaxies are of order 1.4×10^{-2} Mpc^{-3}
  - If one assumes that the mass of a typical galaxy is 10^{11} M⊙, then one concludes that visible galaxies contribute about 1% of the value of ρ .
- **E > 0** : a(t) increases monotonically for all time. The universe expands forever.



The Friedmann equation

- Returning to the form of Friedmann's equation, one may redefine the constant of integration to be $$\frac{2E}{l_0^2} = -kc^2.$$
- Re-writing the Friedmann equation yields $$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho(t) = -\frac{kc^2}{a^2}.$$
- Note that this form of the equation was that obtained by Friedmann (and re-discovered by Lemaitre) by inserting the corresponding elements of the metric tensor applicable to the RW line element into the EFQ.
  The parameter k takes one of three values
  $$E < 0 \;\Rightarrow\; k = +1$$
  $$E = 0 \;\Rightarrow\; k = 0$$
  $$E > 0 \;\Rightarrow\; k = -1.$$
- We have now obtained the complete link between the time dependence of the universal scale factor a(t) and the curvature constant *k* first encountered in the RW line element.
- Proceeding further, one may redefine the Friedmann equation in terms of a new dimensionless density parameter $$\Omega(t) = \frac{\rho(t)}{\rho_c(t)}.$$

The Friedmann equation

- If we divide the Friedmann equation by $H(t)^2$ and use the above identity, we obtain

$$\frac{kc^2}{H^2a^2} = \Omega(t) - 1.$$

- This expression has the following consequences

$$\begin{aligned} k = +1 &\Rightarrow \Omega(t) > 1 \\ k = 0 &\Rightarrow \Omega(t) = 1 \quad \text{for all times,} \\ k = -1 &\Rightarrow \Omega(t) < 1 \end{aligned} \qquad \begin{aligned} \Omega(t) > 1 &\Rightarrow \text{a closed universe,} \\ \Omega(t) = 1 &\Rightarrow \text{a spatially flat universe,} \\ \Omega(t) < 1 &\Rightarrow \text{an open or hyperbolic universe.} \end{aligned}$$

- Therefore, the total matter/energy content of the universe determines the overall spatial geometry, the time variation of a(t) and the ultimate fate of the universe.
- Importantly, these equations indicate that the universe has a well-defined characteristic matter density that marks the limit of each of the above cases.
  - the determination of the total matter content of the universe became an immediate challenge for early observational cosmologists.
- However, to determine this critical density, one requires H(t) or H0 (cosmological distance scale - previous lecture)

The seeds of galaxy formation:
The cosmological horizon and Inflation

- The cosmological horizon may be defined as the maximum distance a photon could travel within the lifetime of the universe.
- It is a convenient definition of the largest region of the universe that could exist in causal contact at any particular epoch.
- The horizon is defined by considering a radial null geodesic within the RW line element, i.e.

$$c\,dt = \frac{a(t)\,dr}{\sqrt{1-kr^2}}$$

$$\int_0^{t_0} \frac{c\,dt}{a(t)} = \int_0^{r_H} \frac{dr}{\sqrt{1-kr^2}},$$

- Where t_0 indicates the current age of the universe and r_H is the horizon distance.

Seeds of Galaxy Formation

- For an EdS Universe (universe containing a non-interacting gas of galaxies) with k = 0 (flat) we may write

$$a\,\dot{a}^2 = \frac{8}{3}\pi G\rho\, a^3 \qquad a \propto t^{2/3}. \qquad t_0 = \frac{2}{3}\frac{1}{H_0}.$$

$$r_H = \int_0^{t_0} \frac{c\,dt}{a(t)} = \int_0^{t_0} \frac{c\,dt}{a_0}\frac{t_0^{2/3}}{t^{2/3}} = 3\,c\,t_0.$$

  - The combination of a finite age of the universe with a finite speed of light ensures that we can only observe photons from a finite region of the universe.
- However, computation of the cosmological horizon gives rise to the *horizon problem.*
- When we look at the universe at the cosmological horizon, it is isotropic, i.e. large scale structure and CMB temperatures are statistically identical even though separated by 180° on the sky.
- However, these two regions, though in casual contact with us, are themselves causally isolated.
  - *How could the CMB and LSS have developed in exactly the same manner?*

Structure Formation:

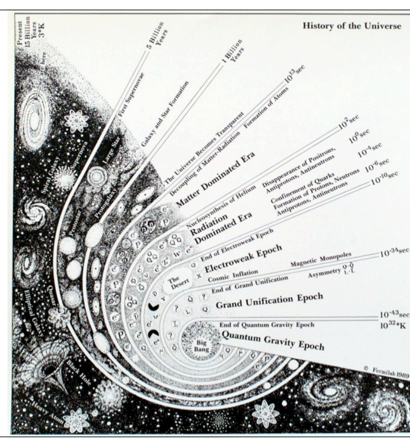
- **cosmic inflation** postulates that the universe underwent a rapid phase of expansion at early times.
  - Regions in the early universe were originally in causal contact and were subsequently inflated to scales much larger than the horizon during the inflationary epoch.
- and inflation must therefore produce initial density fluctuations

Define the density contrast: $\delta = (\rho - \rho_0)/\rho_0$

- density perturbations may be modified by
  - amplification due to gravitational instability
  - pressure
  - dissipation

Short history of early Universe

- The energy density in the early universe is dominated by radiation, $p = w\rho c^2$, $a(t) = a_0\, t^{2/[3(w+1)]}$
  Therefore,we may write
  $a(t) \propto t^{1/2}$; $T(t) \propto a(t)^{-1}$; $T(t) \propto t^{-1/2}$
- Using radiation-matter equality provides constant of proportionality.
  $T(t) \simeq 10^{10}\ \mathrm{K}\ (t/1\,\mathrm{sec})^{1/2}$
  $kT(t) \simeq 1\ \mathrm{MeV}\ (t/1\,\mathrm{sec})^{1/2}$
  $E_{mean} = 2.7kT(t) \simeq 3\ \mathrm{MeV}\ (t/1\,\mathrm{sec})^{1/2}$
- These formulae provide us with the energy scale as a function of epoch in the early universe.
- A comparison to the energy associated with particle interactions provides an understanding the physical conditions associated with each epoch:



Short history of early Universe

1) The hadron era: $t_{min} < t < \sim 10^{-5}$ sec.

- This marks the earliest Universal epoch where experimental physics can be applied with any confidence.
- Almost all matter, including electrons, protons, neutrons, neutrinos and their associated anti-particles are in thermal equilibrium with the photon radiation field.
- The disparity between particles and anti-particles is thought to be 1 part in $> \sim 10^{7}$ and is ultimately responsible for all matter in the present day universe.
- The exact physics (e.g. equation of state) of this epoch is not known.
- Thus the dependence of the scale factor a(t) and the temperature T (t) upon cosmic time is not known.

Short history of early Universe

2. The lepton era: 10^{-5} sec $< t < \sim 10$ sec.

- The temperature decreases such that kT is significantly lower than the rest mass energy of the proton (m_p = 938 MeV).
- Proton-antiproton pairs, in addition to other hadrons present, annihilate.
- The lepton era begins with photons in thermal equilibrium with electrons and positrons, muons, neutrinos and antineutrinos.
- The energy released by hadron annihilation is thus shared between all of these particle families.
- Each of the relativistic particles (photons, neutrinos and electrons – plus antiparticles) contributes an energy density $\propto T^4$.
- The lepton era ends when the radiation temperature drops significantly below $T \simeq 5 \times 10^{9}$ K (i.e. $kT \simeq m_e c^2$ = 511 keV).
- Electron-positron pairs annihilate and temperatures begin to decrease to levels where a protons fall out of equilibrium with neutrons.

Short history of early Universe

3. The plasma era: 10 sec $< \sim t < \sim 10^{13}$ sec.

- The universe consists of photons, neutrinos, electrons, protons and neutrons (the discussion assumes that at this stage Dark Matter particles do not interact).
- The early stages of the plasma era remain sufficiently hot and dense to produce light nuclear elements from hydrogen.
- Matter and radiation are coupled to form a photon-baryon fluid: photons are coupled to electrons via Thompson scattering, and electrons are coupled to protons via Coulomb interactions.
- Matter and radiation remain in thermal equilibrium until the photon temperature drops below $T \simeq 3 \times 10^{3}$ K where electrons combine with protons to form atomic hydrogen.
- The radiation field continues unimpeded to the present day where it is observed as the Cosmic Microwave Background.

Short history of early Universe

4. The post-recombination era: $t > \sim t_{rec} \simeq 10^{13}$ sec. Various astrophysical processes combine to produce the present day universe.

Understanding this cosmic history preceeding the formation of galaxies provides insight into two important questions.

I] Why do galaxies contain so much helium?

Gaseous nebulae in galaxies (HI and HII regions) contain some 25-27% of ^{4}He by mass.

Simple calculation indicates that this abundance could not have been created by stellar nucleosynthesis.

The helium is primordial in that it was created in the earliest (first few minutes) of the Universe's history.

Short history of early Universe

II] What were the first baryonic structures to form after recombination?

The Jeans relation describes baryonic structures whose self-gravity is supported by internal pressure.

Structures that are Jeans stable do not collapse and structure is suppressed on scales smaller than the Jeans scale. Consider a virialised cloud of particles at the stability limit, i.e.

$$3NkT = \frac{GM_J^2}{R_J},$$

where subscript J indicates the Jeans limit.

Employing the relations $N = M_J / \mu m_p$ and $M_J = 4\pi\rho R_J^3/3$ one obtains the Jeans length

$$R_J = \left(\frac{3kT}{\mu m_p}\cdot(G\rho)^{-1}\cdot\frac{3}{4\pi}\right)^{1/2} = c_s\sqrt{\frac{\pi}{G\rho}}.$$

Jeans length in early Universe

- The Jeans length is therefore proportional to the sound speed of baryons at each epoch.
- Prior to decoupling the baryonic sound speed was determined by the properties of the relativistic photon baryon fluid, i.e. $c_s = c/\sqrt{3} = 0.58c$.
- The corresponding value of the Jeans length may be computed as

$$R_J = 0.58c \times \sqrt{\frac{\pi}{6.67\times10^{-11}\ \mathrm{Nm^2kg^{-2}} \times 5\times10^{-19}\ \mathrm{kgm^{-3}}}}$$

$$= 5.3\times10^{22}\ \mathrm{m} \approx 1.6\ \mathrm{Mpc},$$

- where we have taken the physical baryon density at the epoch of decoupling.

Jeans length in early Universe

- The associated Jeans mass is of order 10^{20} M⊙ which is several orders of magnitude larger than the most massive galaxy cluster known.
- Immediately after decoupling the sound speed is given by the expression

$$c_s \approx \sqrt{\frac{kT}{\mu m_p}} = \sqrt{\frac{kT}{\mu m_p c^2}}c = \sqrt{\frac{0.26 \text{ eV}}{563 \text{ MeV}}}c = 2 \times 10^{-5}c.$$

- The corresponding Jeans length and mass are respectively $R_J = 1.84 \times 10^{18}$ m ≈ 50 pc and $M_J \approx 4 \times 10^6$ M⊙ which lies around the mass of a typical globular cluster or dwarf spheroidal galaxy.
- Therefore, the epoch of decoupling marks an important transition in the development of baryonic structure.
  - Prior to decoupling all fluctuation scales up to the horizon scale are effectively suppressed.
  - After decoupling the Jeans scale is dramatically reduced and baryonic structures can begin to grow.

Jeans length in early Universe

- Dark matter haloes exist on all scales down to some mass/length scale determined by the free streaming scale of the dark matter particle itself.
- If the mass of the dark matter particle is of order 1 MeV the mass associated with the free streaming scale is of order 10^6 M⊙.
- Therefore there appears to be a key mass scale of order 10^6 M⊙ imprinted on baryonic structures in the post-recombination Universe.
- This limit is close to the lower mass limits of globular clusters and dwarf galaxies.
- In the following slides we will explore the growth and structure of these overdensities and which of these early overdensities ends up associated in the present day with galaxies of different mass/type.

Structure formation

Define the density contrast: $\delta = (\rho-\rho_0)/\rho_0$

- density perturbations may be modified by
  - amplification due to gravitational instability
  - pressure
  - dissipation
- growth of inhomogenieties
  - δ << 1: linear theory
  - δ ~ 1: need specific assumptions (ie, spherical symmetry)
  - δ >> 1 non-linear regime. solve numerically (or higher order perturbation theory)
- in general, Universe is lumpy on small scales and smoother on large scales
- consider inhomogenieties as a perturbation to the homogeneous solution

Structure Formation

schematically

- $d^2\delta/dt^2$ + [Pressure - Gravity] + δ= 0
- (from continuity, Euler, Poisson eqns)
- in the absence of an expanding universe
  - if pressure is low, δ grows exponentially
  - if pressure is large , δ oscillates with time.
  - the relevant scale here is the Jeans length (fluctuations grow if perturbations are longer than R_J = sound speed/ dynamical time)
- now solve in an expanding universe

Structure formation

- for cold dark matter P=0, and $\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\bar{\rho}\delta$
- Can write Friedmann eqn as $dH/dt + H^2 = -4\pi G/3\ \rho_m$
  Since $\rho_m \sim a^{-3}$, differentiating wrt t ...

means this is **like the equation of motion for a mini FRW Universe**

- 2nd term is "hubble drag" which suppresses perturbation growth due to expansion of the universe
- can be solved parametrically in the same way the Friedmann eqn can be solved
- General solutions with growing and decaying modes: $\delta(x)=f_1(x)D_1(t) + f_2(x)D_2(t)$

$D(t) = \delta(t_0)\ D_1(t)/D_1(t_0)$ $D_1(t) \propto a(t)$

Structure Formation

in a matter dominated Universe $H = \frac{2}{3t}$

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

$$\delta = A(x)t^{2/3} + B(x)t^{-1}$$

linear growth

$$\delta = \delta_0(x)a$$

expansion of the universe slows growth from exponential to linear

Structure Formation

in a lambda dominated Universe $$H^2 = \frac{\Lambda c^2 \pi G}{3}$$

$$\ddot{\delta} + 2H\dot{\delta} = 0$$

$$\delta = A(x) + B(x)e^{-2Ht}$$

frozen fluctuations

Structure Formation

- solution for the matter dominated case

$$\delta = A(x)t^{2/3} + B(x)t^{-1}$$

$$\delta = \delta_0(x)a$$ linear growth

- solution for the lambda dominated case

$$\delta = A(x) + B(x)e^{-2Ht}$$ frozen fluctuations

- general case

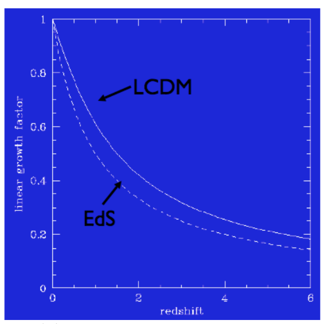
$$\delta = \delta_0(x)ag(a, \Omega_{m0})$$

g is constant at early times and scales as 1/a at late times

for our cosmology, the action ended around z=0.5

Linear Growth of Fluctuations

fluctuations grow more slowly in low Ω_m Universe



Growth of Structure

- CMB indicates that fluctuations in the baryon distribution at recombination had an amplitude less than 10^{-4}
- The existence of non-linear structures today implies that the growth of fluctuations must have been driven by non-baryonic dark matter which was not relativistic at recombination.
- After recombination, baryons decouple and fall into dm potential wells.

Growth of Structure

- after $\delta > 1$, non-linear regime
- matter accumulates in dense regions
- hard to approximate without numerical simulations
- random motions of particles halts the growth
- non-linear collapse on small scales doesn't change the linear evolution of the large-scale perturbations (they don't care whether the small scale power is lumpy -- Gauss's law!)

Growth of Structure

Linear Power Spectrum = Primordial Power Spectrum * Transfer Function * Growth Fucntion

- *Primordial Power Spectrum:*

$$P_k \sim k^n$$

n < 1 :'blue tilt', less power on small scales

n=1 :'scale-free', Harrison-Zeldovich-Peebles

n>1:'red tilt', more power on small scales

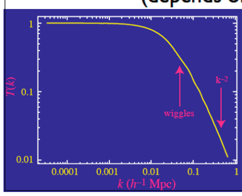
n(k): running scale index

current constraints from WMAP+LSS: n = 0.95

Transfer function

- the primordial power spectrum is 'processed' during the early Universe
  - this is called the transfer function T(k) (e.g. Bardeen et al. 1986; (BBKS); Eisenstein & Hu 1999)

(depends on Ω_m, Ω_Λ, H_0, Ω_b, type of dark matter particle...)



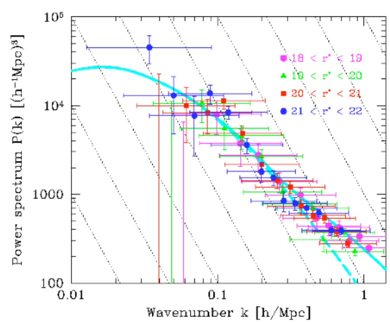
In pure CDM cosmology

- in the matter dominated era all scales grow equally
- in radiation-dominated era, pressure is important.
- scales smaller than the horizon: growth is stalled by the presence of radiation pressure. growth slows as a^2, and small scale modes are damped as k^{-2} --> power spectrum suppressed by k^4.
- Universe is expanding too quickly for the dark matter to collapse
- scales above the horizon continue to collapse
- scales that enter horizon during the radiation-dominated era grow more slowly than those that enter during matter domination.

Power spectrum

- P~ k on large scales P ~ k^{-3} on small scales
- power below the Jeans length is suppressed
- the Jeans scale of the total system grows to the size of the horizon at matter-radiation equality and then shrinks to zero -- the transition scale marks the scale of the horizon at matter-radiation equality

Power spectrum from SDSS



Galaxies

Lecture 23: halos

- Spherical collapse
- Defining a halo
- Analytical representations
- Press-Schechter
- Structure of halos

Structure Formation

- we discussed the generation of fluctuations
- the evolution of the power spectrum under linear theory, applies when the fluctuations are small.
- this likely doesn't work for big fluctuations; these fluctuations become non-linear
  - roughly, non linear is when the density fluctuations are ~ 1 $\delta(x) = (\rho(x)-\rho_0)/\rho_0$
- most of the field of structure formation and galaxy formation concerns understanding the non-linear regime, and collapsed objects in the density field

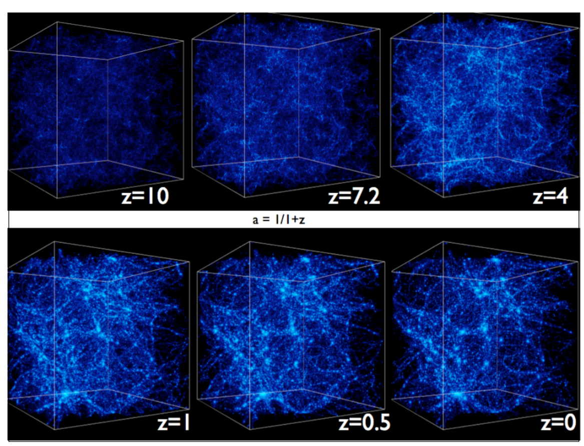
simulating the Universe

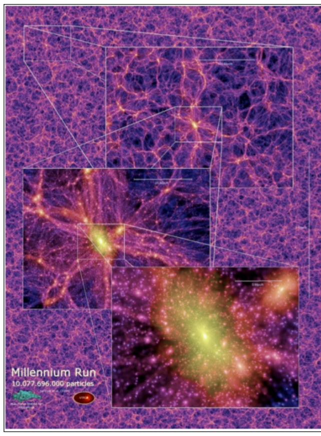
- choose a cosmological model (Ωm,$\Omega \Lambda$,Ωb,h,n, DM)
- choose a computational set up (box size, dynamic range, what physics to include)
- find the linear P(k) as per Lecture22
- set up a random realization of P(k) in the linear regime (200<z<30) in the chosen box
- follow the evolution of dark matter using particle N-body methods
- optionally, follow the evolution of the gas by numerically solving gas dynamic equations
- optionally, add sink and source terms to hydro equations, modeling heating and cooling of the gas, star formation, etc..
- evolve to the redshift of interest

A challenging problem …

- the scales of interest for dark matter structures are ~ pc to hundreds of Mpc.
- would like to simulate a large volume, but also want to small scales & resolve collapsed objects with many particles
- your computer & the amount of time you have determines N.
- Although not the only variable: refinement schemes, and the amount of clustering in the box (the more non-linear, the harder the problem) all play a role in computing time for a given N
- for a given particle number, always a trade-off between volume and mass resolution:
  - miss large scale modes and rare objects with small volume;
  - miss small objects and physics in dense regions with low particle number

Simulations





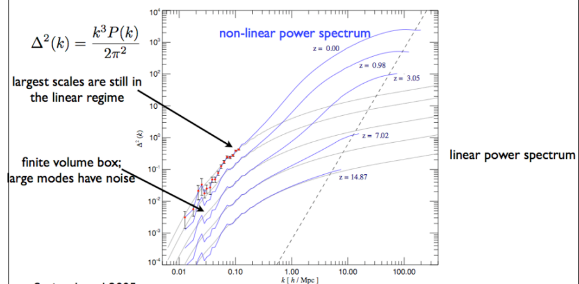
how to characterize this mess?

structure on a wide range of scales

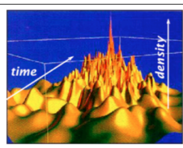
- look at the evolution of P(k) into the non-linear regime
- look at the distribution of density peaks

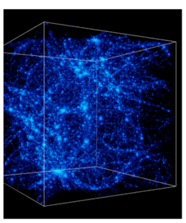
evolution of the matter power spectrum

$$\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$$



Springel et al 2005



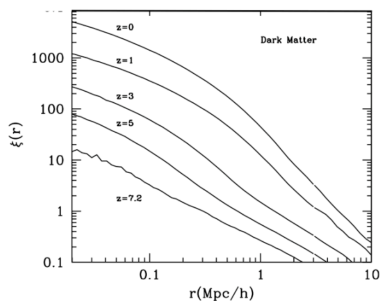


how to characterize this mess?

- clearly, it's quite clumpy.
- look at the overdense regions.
- "dark matter halos"
- schematically: halos are self-gravitating systems in the universe
- or: halos are non-linear peaks in the dark matter density field whose self-gravity has overcome the Hubble expansion

evolution of matter clustering

$$\xi(r) = \frac{V}{2\pi^2}\int P(k) sinkr/krk^2 dk$$



Colin et al 1999

Basic things to study:

- how many collapsed things of various masses?
- how are they distributed in space?
- what internal properties do they have? (internal density distribution? internal angular distribution? shapes? internal substructure?)
- can we understand the collapsed objects (halo formation) analytically?

equation of motion for the perturbation (same as for a closed universe)

$$\ddot{r} = -\frac{GM}{r^2}$$

integrating

$$\frac{1}{2}\dot{r}^2 - \frac{GM}{r} = E = \text{constant}$$

at turnaround,

$$E = -\frac{GM}{r_{\max}}$$

the perturbation will be in virial equilibrium when

$$2K_{vir} + W_{vir} = 0$$

$$E = K_{vir} + W_{vir} = -W_{vir}/2 \approx \frac{GM}{2r_{vir}} = -\frac{GM}{r_{\max}}$$

which implies

$r_{vir} = r_{\max}/2$ ie, 8 times denser than at turnaround

(with a cosmological constant)

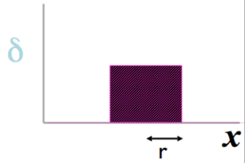
$$\ddot{r} = -\frac{GM}{r^2} + \frac{\Lambda}{3}r$$

for EdS, solving this equation gives the density at turnaround as

$$\frac{\rho_{\max}}{\bar{\rho}} \approx 5.5$$

The "top-hat model" Spherical Collapse

- consider a uniform, spherical perturbation

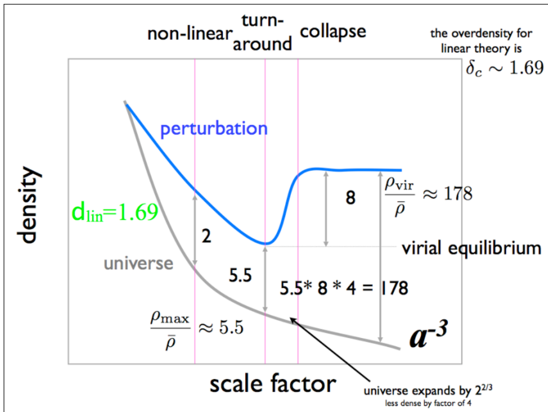


$\delta_i = \rho(t_i)/\rho_b(t_i)-1$

$M = \rho_b(4\pi r_i^3/3)(1+\delta_i)$

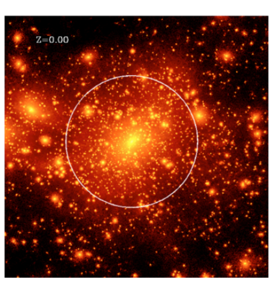
now let's count them

- but first we need to define halo.
- schematically: halos are self-gravitating systems in the universe
- or: halos are non-linear peaks in the dark matter density field whose self-gravity has overcome the Hubble expansion
- operationally: a halo is a non-linear peak in the dark matter density field with its boundary defined by a given density contrast.
- perhaps we can use the model just described to define it. *IE: R180 = Rvir*



sadly, halos don't look like this:

halos look like this:



halo finder is typically looking for a nonlinear peak in the dark matter density field, defined by some density contrast. each halo finder has to choose a definition of mass and of boundary.

types of halo finders

- spherical overdensity
  - defines a halo as matter within a sphere, centered on a density peak, and enclosing a certain overdensity (for example, the "virial overdensity we just defined)
  - simple (one parameter delta_vir), requires spherical. doesn't remove unbound particles.
- friends-of-friends
  - all particles within a linking length are connected in a group. halo = connected region bounded by a density isosurface
  - simple (one parameter b) and accounts for non-spherical objects. doesn't remove unbound particles.
- density maxima (DenMax, SKID, HOP)
  - define halos as connected regions above a certain overdensity, break neighboring peaks at saddle points

but let's assume they look like this anyways.

choose the overdensity

$$\Delta_c = 178\Omega_m(z)^{0.45}$$

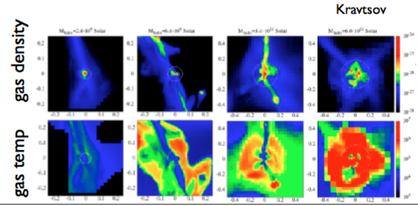
the virial mass is then

$$M = \frac{4\pi}{3}\Delta_{\rm vir}\bar{\rho}^2$$

is the virial radius the virial radius?

- The choice of density threshold or linking length is motivated by the spherical collapse model, but:
  - bound, virialized matter can exist outåside Rvir
  - density profile extends beyond Rvir
  - virial shocks around halos are found out to ~few Rvir

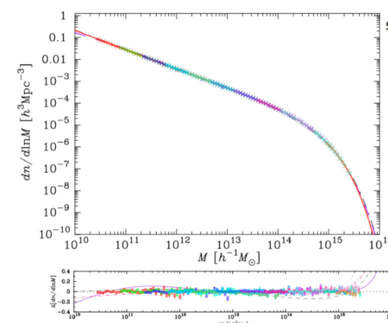
Kravtsov



is the virial radius the virial radius?

- the spherical collapse model is a reasonable approximation, but life is more complicated.
- stuff happens *around* Rvir, but really, nothing unique about Rvir. (or at R180 or R200)
- you thus see many choices, depending on the problem (or just depending on author's whim)
- different definitions will result in different mass functions, bias relations, profiles, etc., and halo finders are also often different than analytic models (mostly at the few-10 percent level)
- if you know the density profiles of halos, then you can transform between them.
- "Let's define the virial radius as the virial radius" -- James Bullock

The halo mass function



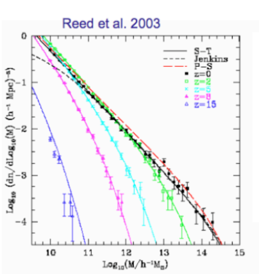
Press & Schechter 1974
Sheth & Tormen 1999, 2001
Jenkins et al 2001
Reed et al 2005
Warren at al 2005
...

note: this is for friends of friends halo finder, but similar results for other choices.

Warren et al 2004

for a specific cosmology, calibrated to ~5%

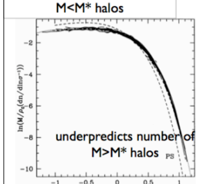
The halo mass function



the halo mass function evolves strongly with time.

Press-Schechter theory in a nutshell

PS overpredicts number of M<M* halos

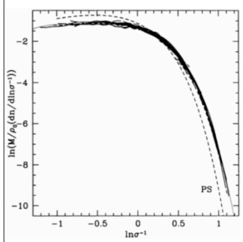


- ansatz: the fraction of mass in halos more massive that M is related to the fraction of the volume in which the smoothed initial density field is above some density threshold.
- this threshold depends on the window function, but for a spherical tophat in real space it is 1.69 (the extrapolated linear overdensity at which a spherical top-hat perturbation would collapse)

$$f(\sigma, PS) = \sqrt{\frac{2}{\pi}}\frac{\delta_c}{\sigma}\exp(-\frac{\delta_c^2}{2\sigma^2})$$

this can be modified for "ellipsoidal collapse" to produce something that matches the simulation data better (Sheth & Tormen). however, this is really just a fitting formula.

how can we characterize this?



- parameterize in terms of fluctuations in the mass field:

$$\sigma^2(M,z) = \frac{b^2(z)}{2\pi}\int k^2 P(k) W^2(k;M)dk$$

- define the mass function:

$$f(\sigma, z; X) \equiv \frac{M}{\bar{\rho}}\frac{dn_X(M,z)}{d\ln\sigma^{-1}}$$

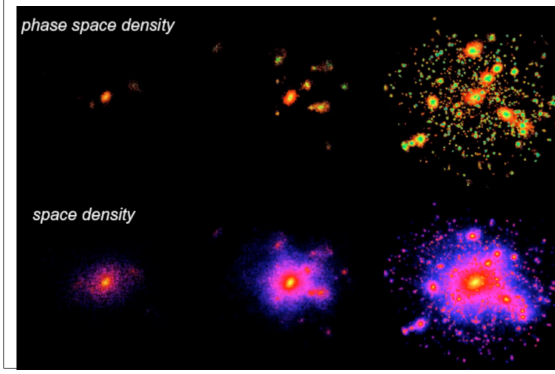
(where X = cosmology + halo finder)

Then the mass function is roughly "universal"
(ie, this formula works to ~15% for a wide range of cosmologies and redshifts)

$$f(M) = 0.315\exp(-|ln\sigma^{-1} + 0.61|^{0.38})$$

Jenkins et al. 2001

Halos also have substructure!



how to characterize this mess?

- Counting
  - collapsed lumps: the halo mass function
  - underdense lumps: void statistics
- Clustering
  - the non-linear matter power spectrum
  - galaxy clustering (or, clustering of the peaks)
- Evolution
  - mass assembly & merging history
- Structure
  - density profiles
  - angular momentum
  - substructure
  - shapes
- Velocity Statistics

+ correlations between these:
mass function & environment
density profiles & formation time
formation time & clustering
...

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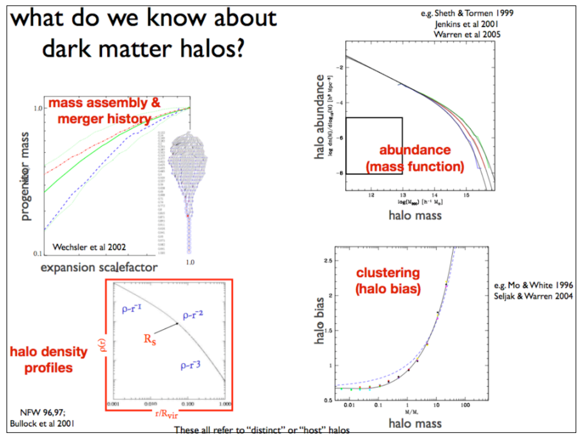
what do we know about dark matter halos?



why study halos?

- ☆ halos are the peaks in the density distribution; their properties (mass function, clustering, density profiles, substructures, etc) are sensitive to cosmological parameters
- ☆ halos are the sites of galaxy formation
  - galaxies form by gas cooling in a ~virialized dark matter halo (eg White & Rees)
  - gastrophyiscs that determines galaxy properties mostly determined by the mass of the host halo (local environment) and not by surrounding structure (larger scale environment)
  - dark matter halos are comparatively easy: nonlinear, but primarily gravitational physics

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