Part III: GALAXIES - 2011

Examples Sheet 3: Galaxy Interactions, The Milky Way Galaxy, Distance scales with SZ effect

Lent Term

1. THE OORT LIMIT AND THE MASS DENSITY OF THE GALACTIC DISC: A classic application of the Jeans equations is to "weigh" the mass surface density of the Galactic disc, using measurements of the velocity dispersion and vertical falloff in densities of low-mass stars above the Galactic plane. This was first analyzed by Jan Oort in 1932.

(a) Begin with the Jeans equation and Poisson's equation in cylindrical coordinates:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\nu(z)\sigma_z^2\right] = -\frac{\partial\Phi}{\partial z}\nu(z)\ . \tag{1}$$

$$\nabla^2\Phi\left(R,z\right) = 4\pi G\rho\left(R,z\right) = \frac{\partial^2\Phi}{\partial z^2} + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Phi}{\partial R}\right)\ . \tag{2}$$

Recall that ν(z) and ρ(z) are the number and mass densities of stars (per unit volume), respectively, and σ_z is the velocity dispersion as a function of height above the disc z.

Show that for an axisymmetric disc this can be expressed as:

$$4\pi G\rho\left(R,z\right) = \frac{\mathrm{d}}{\mathrm{d}z}\left\{-\frac{1}{\nu(z)}\frac{\mathrm{d}}{\mathrm{d}z}\left[\nu(z)\sigma_z^2\right]\right\} + \frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}R}\left[V^2\left(R\right)\right]\ . \tag{3}$$

(b) Show that in the limit of a flat uniform disc with a constant rotation velocity, and velocity dispersion that is independent of z-height, the projected surface density of the disc (volume mass density integrated over height) has an exponential falloff, with e-folding scale height

$$z_0 = \sigma_z^2/(2\pi G\Sigma) \tag{4}$$

(c) Derive an estimate for the mass surface density of the Galactic disc in the solar neighborhood, based on the observation that K-dwarf stars have a scale height of ~250 pc, and a vertical velocity dispersion of 20 km/sec. The observed surface densities of stars and gas are approximately 55–60 $\mathrm{M}_\odot/\mathrm{pc}^2$ in total. How does this compare to your result from stellar dynamics? What are the possible sources of any discrepancy?

2. GALAXY INTERACTIONS

Consider a dwarf companion galaxy in orbit around a giant parent galaxy. Each is a spheroidal galaxy, with a distribution of stars approximated by an isothermal sphere:

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

where σ is the (constant) velocity dispersion and r the radius from the centre of each galaxy.

(a) The density profile of the dwarf galaxy is truncated by the gravitational field of the giant galaxy. Derive an expression for the truncation radius in terms of the velocity dispersions and radial separation of the two galaxies.

(b) Using the result above, derive an expression for the mass of the dwarf spheroidal galaxy, again in terms of its velocity dispersion and separation from the parent galaxy.

(c) Derive the value of the radius and mass of the dwarf galaxy assuming its velocity dispersion is 10km s^{-1}, and it is orbiting at a distance of 50kpc from a giant galaxy with velocity dispersion 300 km s^{-1}. Calculations to one significant figure are sufficient (here and in what follows).

(d) Suppose that the optical (e.g., blue) luminosity of the giant galaxy is $10^{11} L_{\odot}$, which is a typical observed value for a system with $\sigma = 300$km s^{-1}. Estimate the luminosity of thedwarf galaxy (in solar units), if it lies on the main fundamental plane for elliptical galaxies (i.e., Faber-Jackson relation). Derive the mass/light ratio of the dwarf galaxy, and compare it with the values observed for (a) typical giant elliptical galaxies, and (b) dwarf spheroidal galaxies in the local Universe.

3. GETTING A FEEL FOR IMFs: The measured form of the stellar initial mass function is well approximated by a 2-segment power law, with slope $d\psi/dm = -2.35$ (Salpeter) for stars with masses of 1–100 $M_\odot$, and -1.0 for stars with masses 0.1–1 $M_\odot$.

(a) For this IMF derive the average stellar mass, meaning in this case the value dividing equal halves of the total mass of the formed stellar population. How does this compare to the value you derive if you assume a Salpeter IMF for all stellar masses (0.1–100 $M_\odot$)?

(b) Observations show that averaged over this entire stellar mass range the bolometric stellar luminosity scales roughly as M^3. For the two-segment IMF derive the stellar mass above which half of the total luminosity of the population is emitted. As before compare your result to the value you would derive for a purely Salpeter IMF. Why is the comparison so different in this case?

(c) Derive the bolometric mass/light (M/L) ratio for this population, for both IMFs, and compare the resulting values to those measured for galaxies. What factors account for the large differences?

(d) The Milky Way is observed to have a total present-day star formation rate of about 3 $M_\odot$/yr, and a core-collapse supernova rate of about one per century. Suppose we assume for a moment that all stars with an initial mass larger than X eventually explode as supernovae. Use the information above to derive an estimate for the threshold mass X, for the 2-segment and Salpeter IMFs. We actually think that stars with masses above ~30-40 $M_\odot$ may collapse directly to black holes, without producing visible supernovae. Show that this possibility has negligible effect on the derived value of X.

4. THE SUNYAEV-ZELDOVICH EFFECT:

The change in the inferred temperature of the CMB due to the scattering of CMB photons from electrons in an ionized plasma is given by

$$\frac{\Delta T}{T_{CMB}} = f \int \sigma_T (\frac{k_b T}{m_e c^2}) n_e d\ell$$

where f is a relativistic correction dependent on the frequency of the observation, σ_T is the Thompson cross section, T is the temperature of the gas, and n_e is the electron density. The X-ray surface brightness at a given point on the sky is given by

$$S = \frac{1}{4\pi(1+z)^4} \int \Lambda_c n_e n_H d\ell$$

where z is the redshift of the cluster, Λ_c is a temperature dependent cooling function, and n_H is the hydrogen density. Assume the density profile of a cluster of galaxies is given by a spherical isothermal model with $n_e = n_e(0)(1 + (r/r_c)^2)^{-3\beta/2}$ where r_c is the core radius. Show that the two expressions for observables given above, evaluated along the line of sight through the cluster center, can be used to determine the angular size distance (the ratio of the physical size of an object to its projected angular size) as:

$$D_A = \frac{(\Delta T)^2}{S} (\frac{m_e c^2}{k_B T})^2 \frac{\Lambda_c \mu_e / \mu_H}{4\pi^{3/2} f^2 T_{CMB}^2 \sigma_T^2 (1+z)^4} (1/\theta_c) \times (\frac{\Gamma(3\beta/2)}{\Gamma(3\beta/2 - 1/2)})^2 \frac{\Gamma(3\beta - 1/2)}{\Gamma(3\beta)}.$$

Hint: eliminate $n_e(0)$

END