Part III
Geological Fluid Mechanics
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SHEET 2

GRAVITY CURRENTS

Many of the following problems involve generalising the two-dimensional concepts in the lectures to axisymmetric situations. To save writing, and for extra clarity, CAC stands for 'consider an axisymmetric gravity current propagating over a rigid, horizontal surface'.

1. CAC. Assuming that the volume of the current $V = Qt^\alpha$, where t is time and Q and α are constants, determine estimates of the inertial, buoyancy and viscous forces, F_i, F_g and F_v respectively. By balancing F_i with F_g determine the rate of propagation for $Re >> 1$. Determine the corresponding rate of propagation for $Re << 1$. For what range of times are each of these relationships correct? Evaluate an appropriate Froude number for each of these currents.

2. CAC. Write down the lubrication equations valid for $Re << 1$. Derive the similarity variables for these equations on the assumption that the volume of the current $V = Qt^\alpha$, where t is time and Q and α are constants. Determine to within a multiplicative constant the rate of propagation of the current. Derive the full governing equations in these similarity variables. Integrate the equations analytically for $\alpha = 0$.

3. CAC. Assuming that the current results from the instantaneous release of a finite volume of fluid V at the centre of the current, and that the relevant Reynolds number is large, derive a simple box model of the flow. In this way determine the radius of the current as a funciton of time. Derive the appropriate 'shallow-water' equations and determine their (similarity form of) solution. Compare both these results with each other and with the results of question 1.

4. From time $t = 0$ hot lava is extruded from a very long fissure at a volume flow rate of $Q(t)$ per unit length. The lava then flows down a uniform slope at angle α to the horizontal. It cools as it flows and the kinematic viscosity of the flow, which is represented by $\nu(x)$, where x is the coordinate down the slope (with $x = 0$ at the fissure), increases.

   Derive a partial differential equation and appropriate boundary conditions for $h(x,t)$, the free-surface height of the flow, stating clearly any assumptions you make.

   Setting $h(x,t) = \nu^{\frac{1}{3}}(x) f(x,t)$, or otherwise, determine the solution of the differential system.

   Evaluate the asymptotic rate of propagation of the front of the flow for $Q(t) = Q_o t^{\frac{3}{2}}$ and $\nu(x) = \nu_o e^{3\beta x}$, where Q_o, ν_o and β are positive constants, and write down a condition in the form $t >> t_1$ for which your asymptotic result is valid.

   Briefly comment on how realistic you think the model under consideration is for naturally-occurring lava flows.

5. A short lived but violent eruption from a circular vent in flat countryside results in a hot and heavy ash-laden particulate flow. Write down the appropriate shallow water equations and boundary conditions governing the propagation of the flow assuming that all the ash particles are of one size, and neglecting any effects due to entrainment of the (cold) atmosphere into the flow. State clearly any assumptions you make.

   Show how to reduce the complexity of these equations by assuming that the flow is horizontally uniform (i.e. by making a box model assumption). State clearly any further assumptions you have made.

Neglecting any effects due to the difference in density between the hot *air* in the flow and the atmosphere, determine the furthest point attained by ash. Evaluate also the time taken to reach this point and the resultant density of deposit of the ash.

Comment briefly on how you think the results would change if the ash in the flow was considered to be made up of: (i) two widely spaced size distributions; or (ii) a continuous size distribution of ash particle.