3P1d **Quantum Field Theory: Example Sheet 4** Michaelmas 2018

Corrections and suggestions should be emailed to `B.C.Allanach@damtp.cam.ac.uk`*. Starred questions may be handed in to your supervisor for feedback prior to the class if you wish.*

1*. The Lagrangian density for a Yukawa theory is given by

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 + \bar\psi(i\,\partial\!\!\!/ - m)\psi - \lambda\phi\bar\psi\psi. \tag{1}$$

(a) Consider $\psi\bar\psi \to \psi\bar\psi$ scattering, with initial and final states given by

$$|i\rangle = \sqrt{4E_{\vec p}E_{\vec q}}\, b_{\vec p}^{s\,\dagger} c_{\vec q}^{r\,\dagger}\,|0\rangle \tag{2}$$

$$|f\rangle = \sqrt{4E_{\vec p}E_{\vec q}}\, b_{\vec p'}^{s'\,\dagger} c_{\vec q'}^{r'\,\dagger}\,|0\rangle\,. \tag{3}$$

Show that the amplitude is given by

$$\mathcal{A} = -(-i\lambda)^2\left(\frac{[\bar u^{s'}(\vec{p'})\cdot u^s(\vec p)][\bar v^r(\vec q)\cdot v^{r'}(\vec{q'})]}{t-\mu^2} - \frac{[\bar v^r(\vec q)\cdot u^s(\vec p)][\bar u^{s'}(\vec{p'})\cdot v^{r'}(\vec{q'})]}{s-\mu^2}\right). \tag{4}$$

where $t = (p-p')^2$ and $u = (p-q')^2$ are the usual Mandelstam variables. Draw the two Feynman diagrams that correspond to these two terms.

(b) Now consider $\psi(p,s)\phi(q) \to \psi(p',s')\phi(q')$ scattering. Show that the amplitude is given by

$$\mathcal{A} = (-i\lambda)^2\frac{[\bar u^{s'}(\vec{p'})\gamma^\mu\left((p_\mu - q'_\mu) + m\right)u^s(\vec p)]}{u-m^2} \tag{5}$$

Draw any Feynman diagrams that contribute.

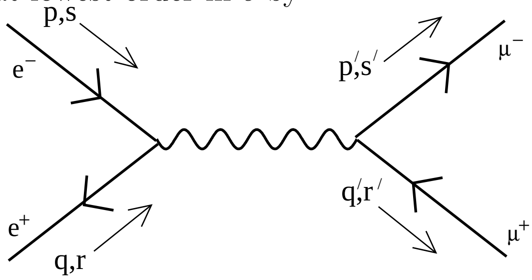
2. The Lagrangian density for a pseudoscalar Yukawa interaction is given by

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 + \bar\psi(i\,\partial\!\!\!/ - m)\psi - \lambda\phi\bar\psi\gamma^5\psi\,. \tag{6}$$

Write down the Feynman rules for this theory. Use these to write down the amplitude at order λ^2 for $\psi\psi \to \psi\psi$ scattering and $\psi\bar\psi \to \psi\bar\psi$ scattering.

3. Take QED coupled to a charge 1 complex scalar ϕ. Write down the Lagrangian, and hence derive the Feynman rules for any interactions between ϕ, $\phi^\dagger$ and the photon A^μ.

4. Use the Feynman rules to show that the QED amplitude for $e^+e^- \to \mu^+\mu^-$ is given at lowest order in e by



$$= (-ie)^2\frac{[\bar v_e^r(\vec q)\gamma_\mu u_e^s(\vec p)][\bar u_m^{s'}(\vec{p'})\gamma^\mu v_m^{r'}(\vec{q'})]}{s}, \tag{7}$$

where the subscripts e and m denote whether the spinors satisfy the Dirac equation for electrons or for muons, respectively and s is the usual Mandelstam variable.

5*. Calculate the total massless fermion spin-averaged cross-section at leading order for the process $e^+e^- \to \mu^+\mu^-$,

$$\sigma_{QED} = \frac{4\pi\alpha^2}{3s}$$

where s is the usual Mandelstam variable, fermion masses have been neglected and $\alpha = e^2/(4\pi)$. This agrees with experimental data:

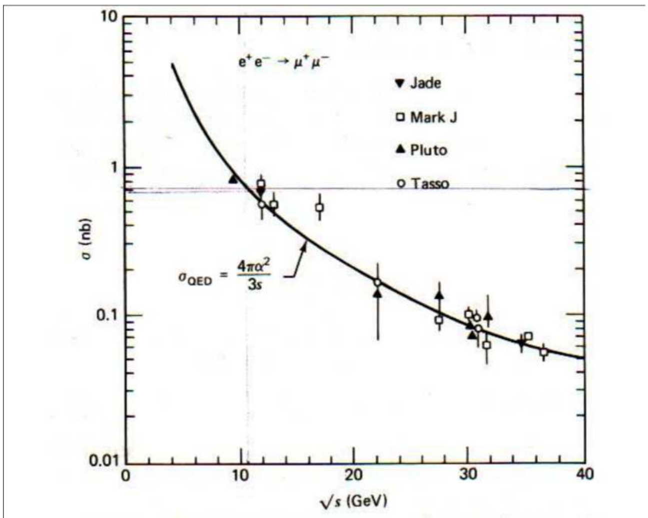


Fig. 6.6 **The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.**

Draw a Feynman diagram at the next higher order than the one depicted in question 4. If this diagram were included in the calculation, the expression for σ would be corrected to which order in $\alpha \approx 1/137$? How much would this change the curve?