3P1c **Quantum Field Theory: Example Sheet 4** Michaelmas 2023

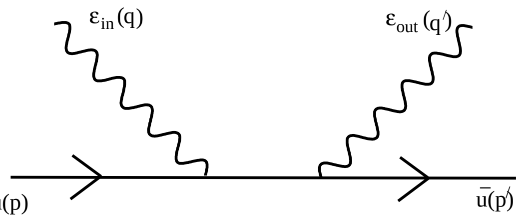
Corrections and suggestions should be emailed to `ac2553@cam.ac.uk`.

1. The Lagrangian density for a pseudoscalar Yukawa interaction is given by

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 + \bar{\psi}(i\,\not{\partial} - m)\psi - \lambda\phi\bar{\psi}\gamma^5\psi\,. \tag{1}$$

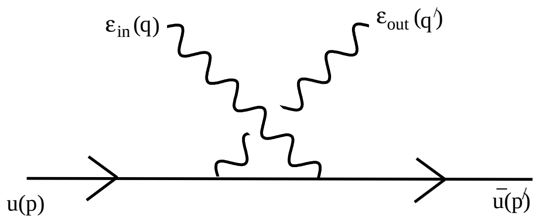
where $\gamma^5 := -i\gamma^0\gamma^1\gamma^2\gamma^3$. Evaluate the amplitude for $\psi\psi \to \psi\psi$ scattering at order λ^2 using Wick's theorem and write down Feynman rules for the theory which reproduce your answer. Use these Feynman rules to write down the amplitude for $\psi\bar{\psi} \to \psi\bar{\psi}$ scattering.

2. Consider Compton scattering in which a photon and an electron scatter off each other. Let the incoming photon have polarisation vector ϵ^μ_{in} and the outgoing photon have polarisation $\epsilon^\mu_{\text{out}}$. Use the Feynman rules to derive the following amplitude associated to the lowest order diagram,



$$= i(-ie)^2\bar{u}^{r'}(\vec{p}')\not{\epsilon}_{\text{out}}\frac{\not{p}+\not{q}+m}{s-m^2}\not{\epsilon}_{\text{in}}u^s(\vec{p}). \tag{2}$$

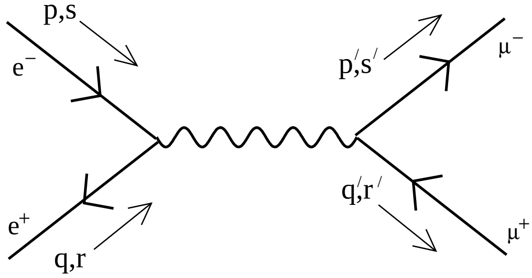
where $s = (p+q)^2$. Also, compute the contribution from the diagram



The complete amplitude at order e^2 is the sum of these two contributions. Show that the total amplitude vanishes if ϵ_{in} is replaced by the incoming photon momentum q. Check that the same holds true if ϵ_{out} is replaced by q'.

3. By defining an appropriate covariant derivative write down a gauge invariant Lagrangian for a complex scalar field coupled to the electromagnetic field (scalar QED). Deduce Feynman rules for the interaction vertices of this theory.

4. Use the Feynman rules to show that the reduced QED amplitude $\mathcal{M}$ for $e^+e^- \to \mu^+\mu^-$ is given at lowest order in e by



$$= (-ie)^2 \frac{[\bar{v}_e^r(\vec{q})\gamma_\mu u_e^s(\vec{p})][\bar{u}_m^{s'}(\vec{p'})\gamma^\mu v_m^{r'}(\vec{q'})]}{(p+q)^2}, \tag{3}$$

where the subscripts e and m denote whether the spinors satisfy the Dirac equation for electrons or for muons, respectively. Compute the spin-summed/averaged squared matrix element,

$$\mathcal{P} := \frac{1}{4} \sum_{s,r,s',r'=1}^{2} |\mathcal{M}|^2$$

Working in the center of momentum frame and in the approximation $m_e = 0$ show that,

$$\mathcal{P} = e^4 \left[\left(1 + \frac{m_\mu^2}{E^2}\right) + \left(1 - \frac{m_\mu^2}{E^2}\right) \cos^2\theta \right]$$

where E is the energy of each incident particle and θ is the scattering angle.

5. (*optional*) Calculate the total massless-fermion spin-averaged cross-section at leading order for the process $e^+e^- \to \mu^+\mu^-$,

$$\sigma_{QED} = \frac{4\pi\alpha^2}{3s}$$

where $s = (p+q)^2$, fermion masses have been neglected (set $m_e = m_\mu = 0$) and $\alpha = e^2/(4\pi)$. This agrees with experimental data:

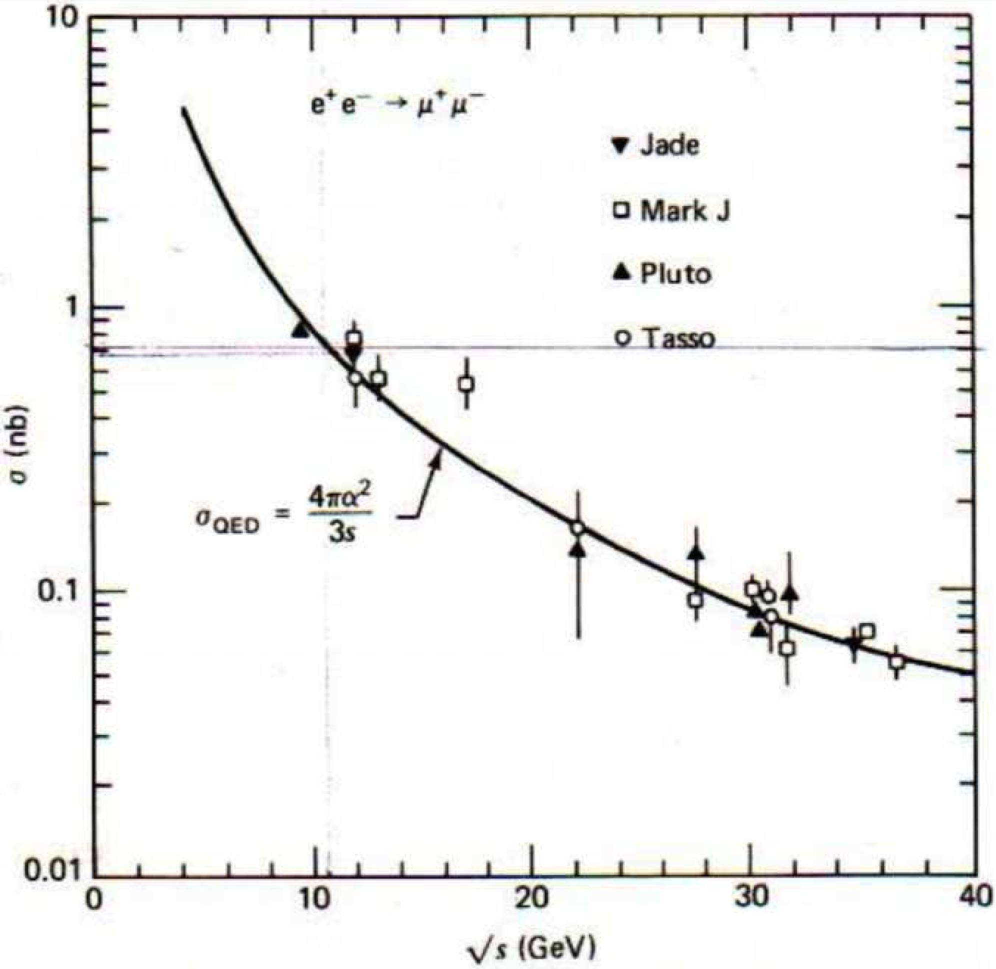


Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.