Summary of Topics Covered in Symmetries and Particles Lectures – Michaelmas 2011

The following is a list of topics covered in the lectures, which is a subset of the material in the distributed lecture notes.

1) General overview of group theory in particle physics.

2) Overview of discrete groups and their reducible and irreducible matrix representations.

3) Continuous groups. Introduction to matrix Lie groups.

4) The rotation group $SO(3)$ and its double cover $SU(2)$.

5) Infinitesimal transformations and Lie algebras. The angular momentum algebra and its reducible and irreducible representations. Integer and half-integer angular momenta. Spinors of $SU(2)$. Combining representations. Tensors of $SO(3)$ and $SU(2)$. Irreducible representations of the rotation group. Unitary representations. Tensor operators. Casimir operators.

6) Isospin, the Wigner-Eckard theorem and application to strong interaction processes.

7) The Lorentz group $SO(3,1)$ and its double cover $SL(2,C)$. Infinitesimal transformations and the Lorentz algebra. Dotted and undotted spinors representations of $SL(2,C)$. $SL(2,C)$ tensors.

8) The Poincaré group $ISO(3,1)$ and its irreducible representations for (a) $(\text{mass})^2 > 0$ and (b) $(\text{mass})^2 = 0$. Casimir operators for the Poincaré group.

9) Geometry of continuous groups. Group manifolds, tangent spaces, vector fields and Lie algebras. Examples such as $SU(2)$. The exponential map between algebras and groups including the Baker-Cambell-Hausdorff formula. Integration over Lie groups. Compact and non-compact groups. Examples.

10) The adjoint representation. The Killing form and the conditions for semi-simple and simple Lie algebras.

11) $SU(3)$ and its representations. Expressing the algebra in its Cartan form – i.e., with generators H_1, $H_2 E_{i\pm}$ $(i = 1, 2, 3)$. Highest weight representations. Representations with triangular and hexagonal weight diagrams. The rôle of the three $SU(2)$ subalgebras. $SU(3)$ tensors and irreducible representations.

12) Recapitulation of some elementary applications to hadronic physics. Quark quantum numbers - $SU(2)$ of spin, $SU(3)$ of flavour (light quarks) and gauge $SU(3)$ of colour. Spectrum of baryons and mesons in QCD.

13) General structure of the Lie algebra of a rank-r group. Root vectors and generators of the Cartan subalgebra. The embeddings of $SU(2)$ subalgebras and root strings. Restrictions on possible lengths of roots and angles between roots. Simple roots and Dynkin diagrams. Simple rank-2 Lie algebras. Classification of all simple Lie algebras – $SU(r+1) \equiv A_r$, $SO(2r) \equiv D_r$, $SO(2r+1) \equiv B_r$, $Sp(2r) \equiv C_r$, G_2, F_4, E_6, E_7, E_8.