Supergravity 1. Easter 2008

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1. In linearised supergravity the fields transform under a global supersymmetry transformation

$$h_{\mu\nu} \to h_{\mu\nu} - \frac{i}{2}\bar{\epsilon}(\gamma_\mu \psi_\nu + \gamma_\nu \psi_\mu)$$

$$\psi_\mu \to \psi_\mu - i\sigma^{\nu\rho}\partial_\nu h_{\rho\mu}\epsilon$$

Show that

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]h_{\mu\nu} = -i(\bar{\epsilon}_2\gamma^\rho\epsilon_1\partial_\mu h_{\rho\nu} + \bar{\epsilon}_2\gamma^\rho\epsilon_1\partial_\nu h_{\rho\mu} - 2\bar{\epsilon}_2\gamma^\rho\epsilon_1\partial_\rho h_{\mu\nu})$$

2. Use the above supersymmetry transformations but let $\epsilon(x)$, to show that the variation of the action for linearised gravity is

$$\delta S_{\text{linearised}} = \int d^4x\, j^\mu \partial_\mu \epsilon$$

where

$$j^\mu = \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\rho\gamma_5\gamma^\nu\sigma^{\lambda\tau}\partial_\lambda h_{\tau\sigma}$$

3. Explain why the Rarita-Schwinger equation describes a spin 3/2 field. Compute the number of (off-shell) degrees of freedom for the massless gravitino and compare this to the corresponding number for the massless graviton. Show that this leads to the inclusion of auxiliary fields.

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4. Compute the number of on-shell degrees of freedom for a massless gravitino and a massless graviton in a D-dimensional spacetime, where D is even. When D=10 would you expect the supergravity multiplet to contain other dynamical fields?

5. The Wess-Zumino model is

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + \frac{i}{2}\bar{\psi}\gamma^\mu \partial_\mu \psi$$

where ψ is a Majorana spinor. The supersymmetry transformations are

$$\delta A = \bar{\epsilon}\psi$$
$$\delta B = i\bar{\epsilon}\gamma_5\psi$$
$$\delta\psi = -i\gamma^\mu \partial_\mu (A + i\gamma_5 B)\epsilon$$

where

$$\phi = \sqrt{\frac{1}{2}}(A + iB)$$

when we let ϵ be space-time dependent, i.e. $\epsilon(x)$, show that

$$\delta\mathcal{L} = \partial_\mu \bar{\epsilon} j^\mu$$

where

$$j^\mu = \partial\!\!\!/(A - i\gamma_5 B)\gamma^\mu \psi$$

6. (*) Show that if we add

$$\mathcal{L}_1 = \frac{-\kappa}{2}\bar{\psi}_\mu j^\mu$$

to the Lagrangian of question 5, then

$$\begin{aligned}\delta\mathcal{L}_1 \quad = \quad & -\partial_\mu \bar{\epsilon} j^\mu + i\kappa \bar{\psi}^\mu \gamma^\nu \epsilon T_{\mu\nu} \\ & + i\frac{\kappa}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_\nu \partial_\rho \epsilon A \overleftrightarrow{\partial}_\sigma B \\ & + i\frac{\kappa}{2}\varepsilon^{\mu\nu\rho\sigma}\partial_\rho \bar{\psi}_\mu \gamma_\nu \epsilon A \overleftrightarrow{\partial}_\sigma B\end{aligned}$$

where $T_{\mu\nu}$ is the energy-momentum tensor for $\mathcal{L}$ and

$$A \overleftrightarrow{\partial}_\sigma B = A\partial_\sigma B - B\partial_\sigma A$$

and we have used the leading order transformation

$$\psi_\mu \rightarrow \psi_\mu + \frac{2}{\kappa}\partial_\mu \epsilon$$