Supergravity 2. Easter 2008

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1. Briefly discuss Noether's theorem. Sketch how it can be used in to construct a supergravity lagrangian. Use your notes to select a lagrangian you wish to discuss. You should outline the necessary steps, but are not expected to reproduce long calculations. Give an example of a lagrangian you can construct using this method, dicussing how the relevant terms arise.

2. Compute the number of degrees of freedom for the massless gravitino using the Rarita-Schwinger equation (get this from your notes). How does this change when there is a mass term in the Rarita-Schwinger equation. For the massless case compare your result to the corresponding number for the massless graviton. Show that this leads to the inclusion of auxiliary fields.

3. Discuss symmetry breaking in supergravity and show how this differs from supersymmetry. Consider the example of the superpotential

$$W = \mu(\Phi - m)^2 \tag{1}$$

with Kahler potential

$$K = \Phi^* \Phi \tag{2}$$

Calculate the scalar potential. Is supersymmetry broken in this model and what is the value of the potential? What is the mass of the gravitino?

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4. Discuss the superHiggs effect. When there is a D-term included show how the Goldstone fermion acquires a mass. You should use your notes to get the starting equations.

5. In N=1 supergravity with three chiral superfields, S, T and C and Kahler potential

$$K = -log(S + S^*) - 3log(T + T^* - C^*C) \tag{3}$$

and superpotential

$$W = C^3 + a\exp(-bS) + c \tag{4}$$

where a and c are arbitary complex numbers and b is positive. Compute the scalar potential. Find the auxiliary field for S,T and C and verify that supersymmetry is spontaneously broken. What is the value of the vacuum energy at the minimum? Are there any flat directions? What is the gravitino mass?

6. Consider the superpotential

$$W = \lambda\Phi_0\Phi_+\Phi_- \tag{5}$$

with simple Kahler potential and D-term

$$V_D = g(\phi_+^*\phi_+ - \phi_-^*\phi_- - \xi)^2 \tag{6}$$

By computing the full potential show that there are two minima; a metastable one where supersymmetry is broken and the true vacuum, supersymmetric vacuum. In each case give the VEVs of the fields. What is the gravitino mass in the former case? Discuss any differences between this case and the corresponding case in global supersymmetry.

7. Now consider the superpotential

$$W = \lambda\Phi_0[\Phi_+\Phi_- - \mu] \tag{7}$$

with the same Kahler potential as above and no D-term. Again compute the scalar potential and show that there are two minima; one where supersymmetry is broken and a supersymmetric, true minimum. Discuss any differences between the the supergravity and global supersymmetry case.

8. For a Polonyi potential

$$W = m^2(z + \beta) \tag{8}$$

where z is a chiral superfield with Kahler potential

$$K = z^* z \tag{9}$$

compute the scalar potential. For the choice

$$\beta = 2 - \sqrt{3} \tag{10}$$

show that your potential has a minimum with $V = 0$. What is the value of z at the minimum. Is supersymmetry broken? What is the gravitino mass?

9. For the Polonyi potential above, show that the A parameter of supersymmetry breaking, as defined in lectures, is $3 - \beta$.

10. Discuss symmetry breaking in supergravity, explaining the differences between the local and global case.

11. Consider extended supergravity theories. Verify that N=8 is the largest supergravity theory, unless one considered particles with spins greater than 2. In lectures you have a table of particle content for extended supergravity theories. By explicit calculation, verify this table.

12. In lectures we discussed the superHiggs mechanism. Verify that the particle η where

$$\eta = \kappa^{-1} G^i \psi_i \tag{11}$$

disappears from the physical lagrangian. You should take the lagrangian from notes.

***note the first 8 are revision style questions. ***