3P7c **Supersymmetry: Example Sheet 3** Lent 2012

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1. Consider a renormalisable $N = 1$ supersymmetric Lagrangian for chiral superfields with F-term supersymmetry breaking. By analysing the mass matrix for scalars and fermions show that

$$\mathrm{STr}M^2 \equiv \sum_j (-1)^{2j+1} (2j+1)\, m_j^2 \;=\; 0$$

where j represents the total spin of the particles. Verify that this relation holds for the O'Raifertaigh model. What implication could this result have for the MSSM?

2. Consider a chiral superfield Φ of charge q coupled to an Abelian vector superfield V. Show that a nonvanishing vacuum expectation value of D, the auxiliary field of V, can break supersymmetry. Identify the corresponding goldstino field.

   Write down the D-term part of the scalar potential and find the condition that the Fayet-Iliopoulos term and the charge q have to satisfy for supersymmetry to be broken.

   Find the spectrum of this model after supersymmetry is broken. What is the mass splitting in the multiplet?

3. Consider a renormalisable $N = 1$ supersymmetric theory with chiral superfields $\Phi_i = (\varphi_i, \psi_i, F_i)$ and vector superfields $V_a = (\lambda_a, A_a^\mu, D_a)$ with both D and F term supersymmetry breaking ($F_i \neq 0,\ D_a \neq 0$). Show that

$$\frac{\partial V}{\partial \varphi^i} = -F^j \frac{\partial^2 W}{\partial \varphi^i \varphi^j} + D^a \varphi_j^\dagger \,(T^a)_i^j = 0,$$

in the vacuum. Here T^a refer to the generators of gauge group a. Also, since the superpotential W is gauge invariant, the gauge variation of W is

$$\delta^{(a)}_{gauge} W = \frac{\partial W}{\partial \varphi^i} \delta^{(a)}_{gauge} \varphi^i = F_i^\dagger \,(T^a)_j^i \,\varphi^j.$$

Write these two conditions in the form of a matrix M acting on a 'two-vector' with components $\langle F^j \rangle$ and $\langle D^a \rangle$. Identify this matrix and show that it is the same as the fermion mass matrix. Argue that it has one zero eigenvalue which can be identified with the Goldstone fermion.

4. Consider an $SU(2)$, $N = 1$ supersymmetric theory with three chiral superfields in the adjoint representation Φ_1, Φ_2, Φ_3 with superpotential:

$$W \;=\; \epsilon_{ijk}\, \mathrm{Tr}(\Phi_i [\Phi_j, \Phi_k])/3!.$$

Assuming minimal renormalizable Kähler potential, write down the scalar potential and look for the general solutions that have vanishing F and D fields. Is supersymmetry broken? Find the corresponding flat directions for the scalar fields (moduli space). Is the supersymmetric version of the Higgs mechanism at work for values of the scalar fields that break the gauge symmetry?

The field content and superpotential above actually describe $N = 4$ super Yang-Mills in $N = 1$ language. Adding extra terms to W will break explictly some of the supersymmetries down to $N = 2$, or $N = 1$. If we add

$$\Delta W \;=\; (m_1 \,\mathrm{Tr}\Phi_1^2 + \; m_2 \,\mathrm{Tr}\Phi_2^2 \; + m_3 \,\mathrm{Tr}\Phi_3^2)/4$$

to W, the theory will be $N = 1$ supersymmetric if all of the masses are different from zero. Show that the field equations now become:

$$[\Phi_i, \Phi_j] = i\epsilon_{ijk}\, m_k\, \Phi_k.$$

Which (matrices) Φ_i satisfy this equation?

5. Consider the $R-$parity violating superpotential of the MSSM. Show that combining two of the baryon/lepton number violating terms can induce proton decay: $p \rightarrow e^+ + \pi^0$. Estimate the rough order of magnitude of the decay rate of the proton via this channel, based on dimensional grounds.

   The experimental lower bound on the proton lifetime is approximately:

   $$\tau_{proton} \;>\; 5.5 \times 10^{32} yrs \;=\; 1.6 \times 10^{40} s \;=\; 2.4 \times 10^{64} GeV^{-1}$$

   Use this to determine an upper bound on the product of the two 'Yukawa' couplings that give rise to proton decay above.

   Verify that R-parity defined in the lectures precisely forbids all baryon/lepton number violating terms while preserving the fermion mass terms. Does the remaining scalar potential have a quartic term?

6. The Polonyi model of $N = 1$ supergravity has a single chiral superfield z, a superpotential

   $$W = m^2(z + \beta),$$

   where β is a real constant, and Kähler potential

   $$K = |z|^2.$$

   Calculate whether the model breaks or preseves supersymmetry. Calculate the scalar potential of z, $V_F(z)$. Imposing the observational constraint of a zero cosmological constant $V_F(\langle z \rangle) = 0$ and assuming that $\beta > 0$, find β and $\langle z \rangle$ in Planck units.

7. For non-abelian groups, the scalar $D-$terms of the potential for each gauge group are given by $V = \frac{1}{2}\sum_i g_i^2 D_i^a D_i^a$, where g_i is the gauge coupling of group i, and $D_i^a \equiv \sum_l \phi_l^*(T_{i,l}^a)\phi_l$, where $T_{i,l}^a$ are the generators of the group in the representation relevant for each scalar component of a chiral superfield ϕ_l. Using this, show that the $D-$terms of the MSSM Higgs potential are:

   $$V_D = \frac{g^2 + g'^2}{8}(|H_2^0|^2 + |H_2^+|^2 - |H_1^0|^2 - |H_1^-|^2)^2 + \frac{g^2}{2}|H_2^+ H_1^{0*} + H_2^0 H_1^{-*}|^2.$$

   Add these to the SUSY breaking terms

   $$V_{SUSYB} = B\mu(H_2^+ H_1^- - H_2^0 H_1^0) + H.c. + m_{H_1}^2(|H_1^0|^2 + |H_1^-|^2) + m_{H_2}^2(|H_2^0|^2 + |H_2^+|^2)$$

   and the pure-Higgs F-terms to obtain the full MSSM Higgs potential in terms of components. Calculate the mass squared eigenvalues of the neutral Higgs fields. What is the condition for one of them to be negative, as required by the Higgs mechanism?