Concepts in Theoretical Physics

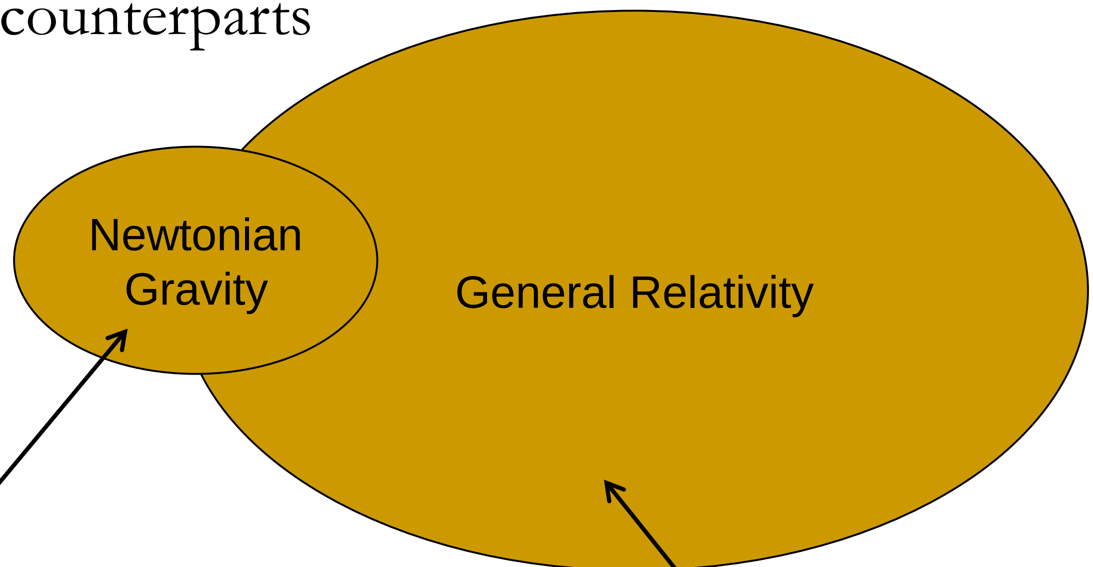
Lecture 7: The Maths of Whole Universes

John D Barrow

'I am very interested in the Universe. I am specialising in the Universe and all that surrounds it'
Peter Cook

GR $\to$ NG for weak gravity + slow motion

But not all solutions have counterparts in both theories



$V \to \infty$, $F \to \propto$
since
$F \propto 1/r^2 \to \infty$ as $r \to 0$
1 potential, Φ, 1 field eqn $\nabla^2\Phi = 4\pi G\rho$

$V \leq c$, $F \leq c^4/4G$
10 potentials, g_{ab}
10 field eqns, $G_{ab} = \kappa T_{ab}$

Every solution of GR is an entire Universe

Einstein's 10 partial differential eqns
Spacetime geometry = matter

$$G_{ab} \equiv R_{ab} - \tfrac{1}{2}Rg_{ab} = \kappa T_{ab} + \Lambda g_{ab}$$

Every solution is an entire universe

A New Piece of Gravity

$$\text{Force} \propto -GM/r^2 + \Lambda c^2 r/3$$

Newton Einstein

Λ - the 'cosmological constant' - does it exist?

$$\Phi(r) = Ar^2 + Br^{-1}$$

is the most general gravitational potential

with Newton's spherical property

Time-dependent solutions of Einstein's eqns

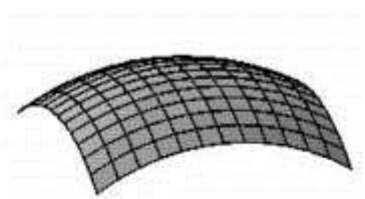
$r \rightarrow a(t)r$ expand the space isotropically and homogeneously

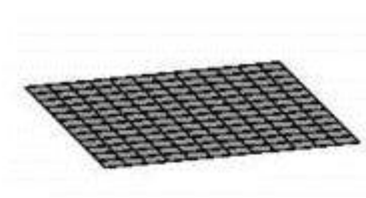
$$ds^2 = c^2dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

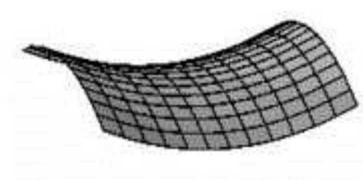
$$\rightarrow\ ds^2 = c^2dt^2 - a^2(t)\{ dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\}$$

Most generally: the 3 spaces of constant curvature ($k = 0, >0, <0$)

$$ds^2 = c^2dt^2 - a^2(t)\{dr^2/(1\text{-}kr^2) + r^2(d\theta^2 + \sin^2\theta d\phi^2)\}$$







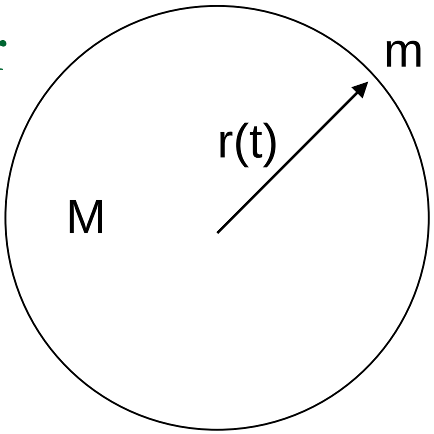
K > 0 K = 0 K < 0

Einstein's Equations for a uniform, isotropic universe: given $p(\rho)$, solve for $a(t)$

- $3a'^2/a^2 = 8\pi G\rho - 3kc^2/a^2 + \Lambda c^2$, (k = 0, +1, or -1)
- $3a''/a = -4\pi G(\rho + 3p/c^2) + \Lambda c^2$
- $\rho' + 3a'/a(\rho + p/c^2) = 0$

$' = d/dt$

$r(t) = a(t)r_0 \quad v = dr/dt = (a'/a)r$



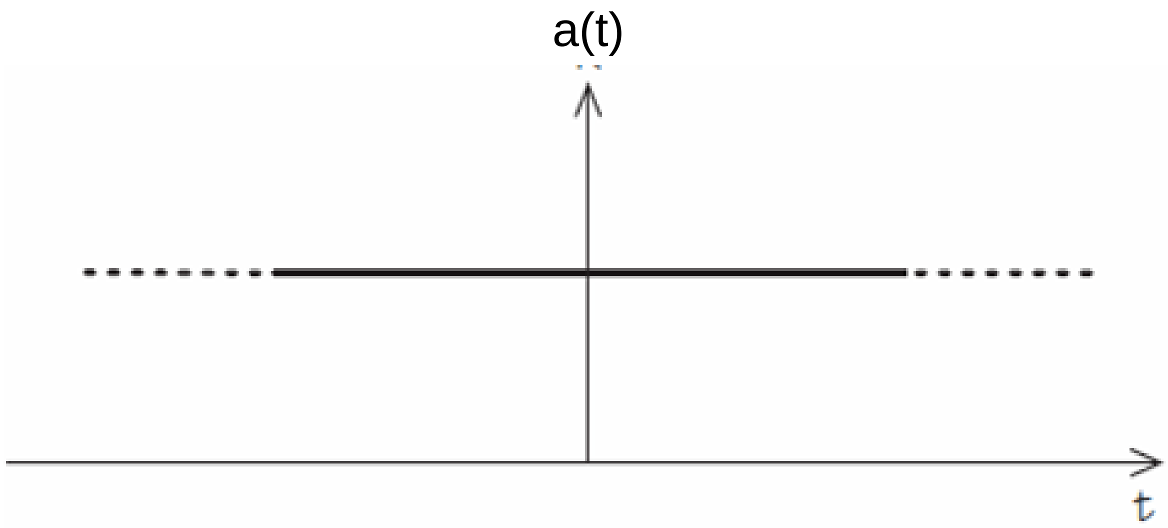
Energy/unit mass $= v^2/2 - GM/r =$ constant

$dE = -pdV$

$dE/dt = d(Mc^2)/dt - pdV/dt$

Volume, $V = 4\pi r^3/3$

Einstein's Static Universe (1917)



Non-Euclidean finite spherical geometry of space required

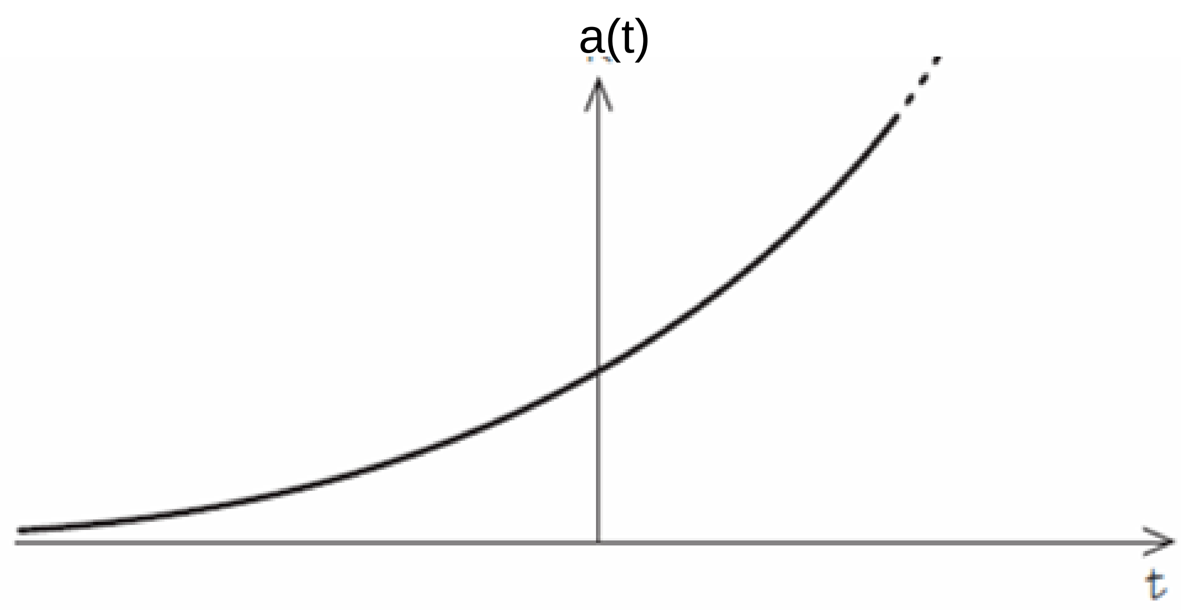
$3a'^2/a^2 = 8\pi G\rho - 3kc^2/a^2 + \Lambda c^2$, $3a''/a = -4\pi G\rho + \Lambda c^2$

so $a' = a'' = 0$ requires

$4\pi G\rho/c^2 = \Lambda = k/a^2 > 0$

UNSTABLE !

De Sitter's Accelerating Universe (1917)



Always expanding

exponential curve a = exp[ct√Λ/3]

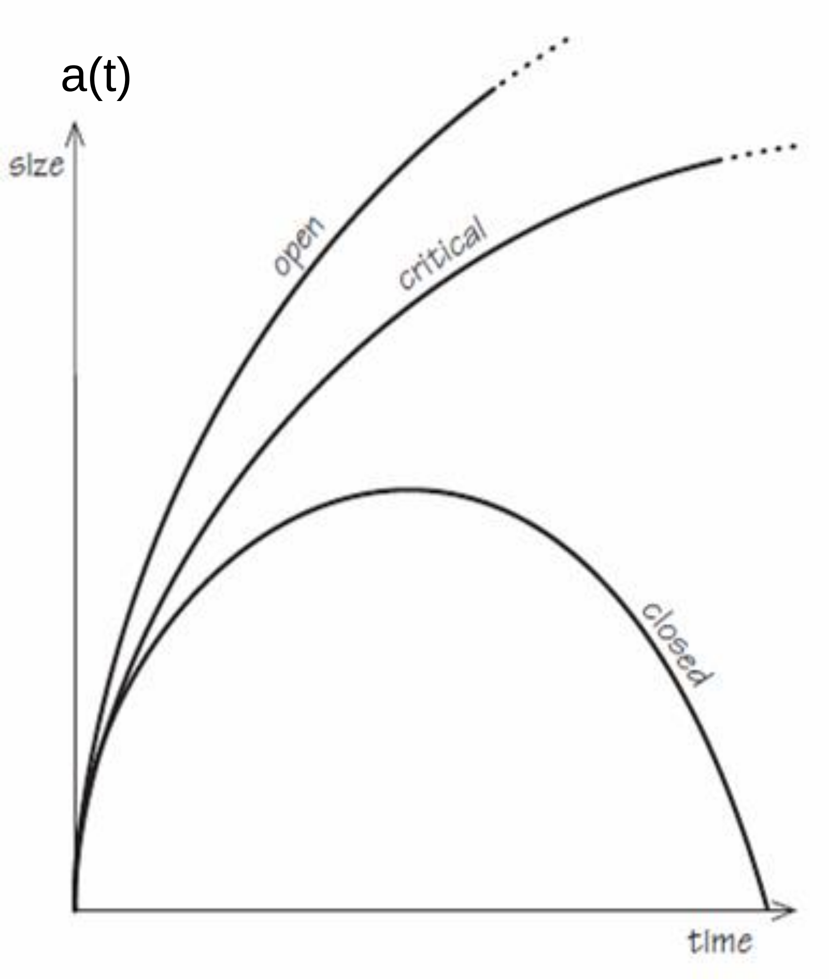
No matter – only Λ

It has no beginning and no end

Willem De Sitter (1872-1934)

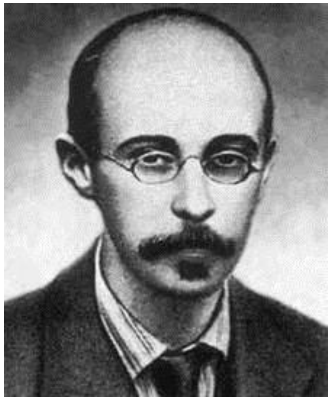
Alexander Friedmann's universes

a(t)



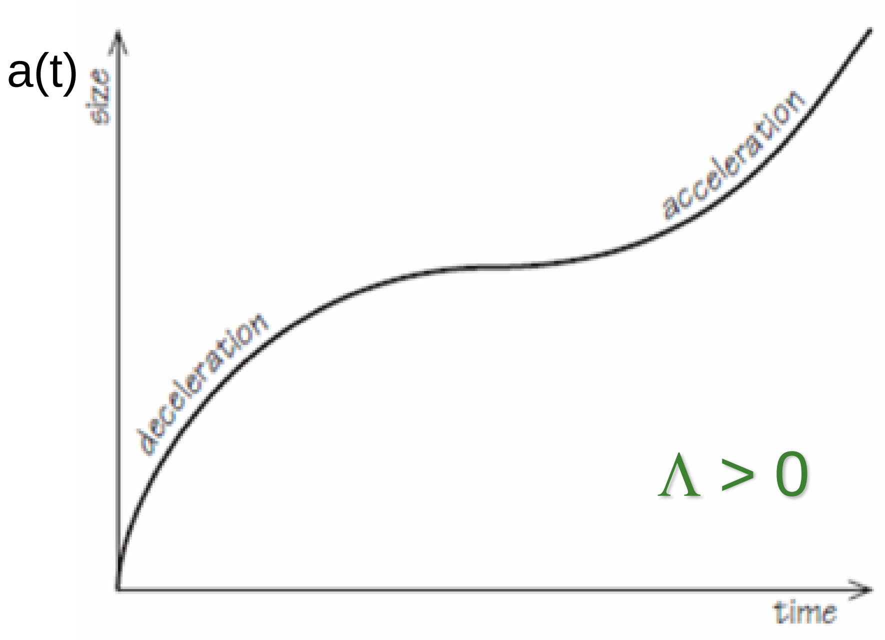
Λ=0

(1922,1924)



Alexander Friedmann,1888-1925

Georges Lemaître's Universe



1927

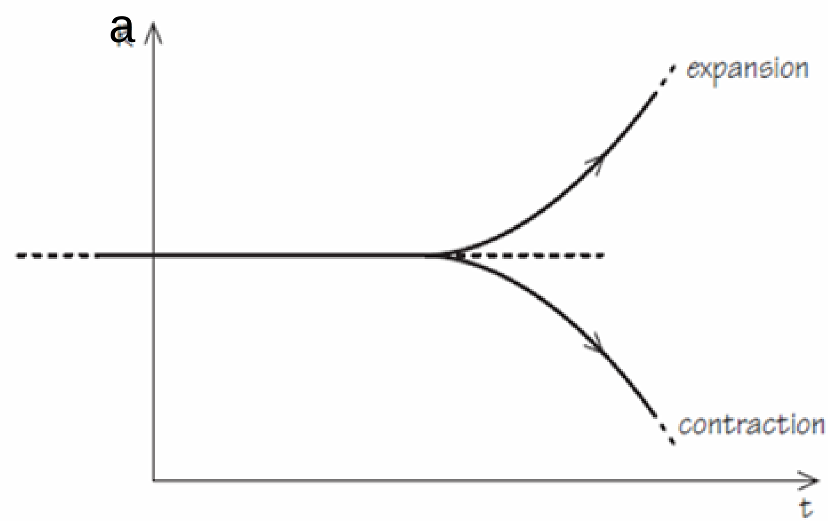
The best description of
the visible universe today

Georges Lemaître, 1894-1966

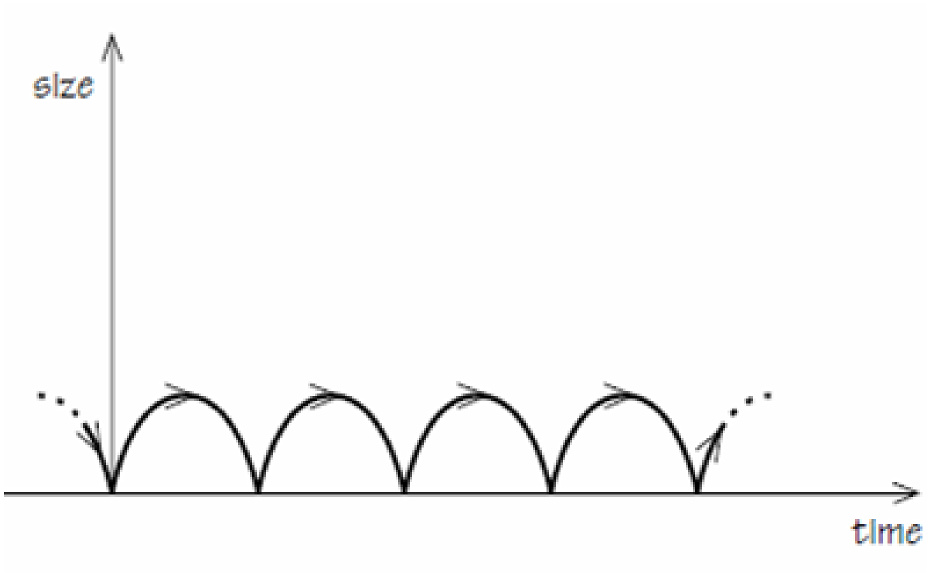
Eddington-Lemaître Universe (1930)

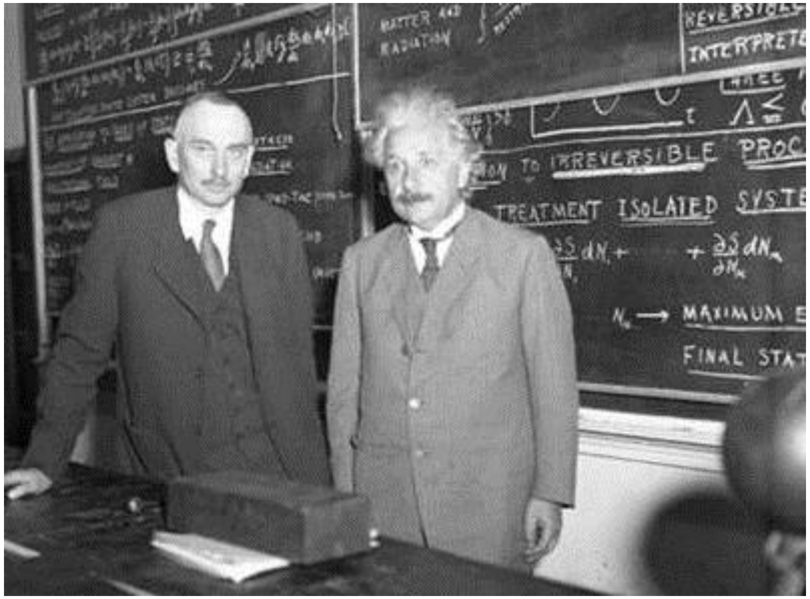
Albert Einstein and Arthur S Eddington
at the Cambridge University
Observatories, on Madingley Rd

Einstein's Static Universe
is unstable



Tolman's Oscillating Universe

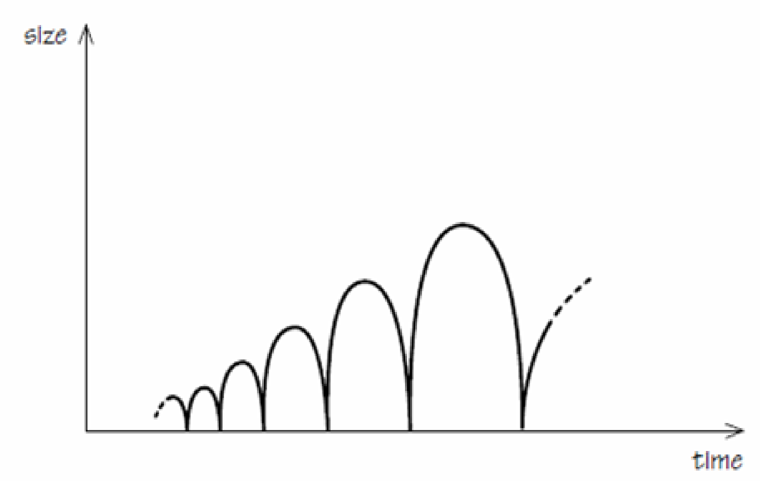




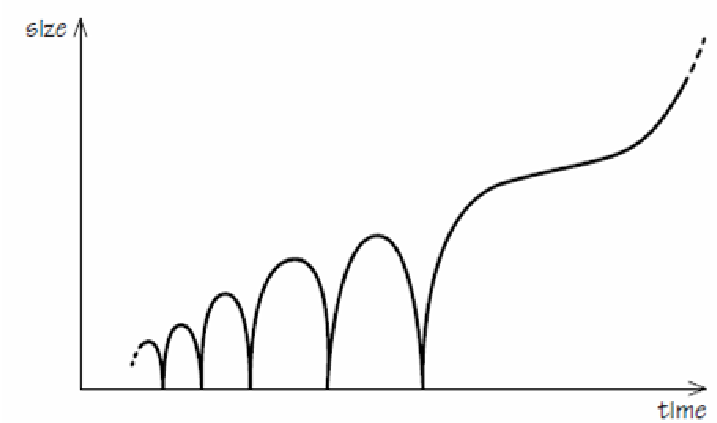
1932

Richard Tolman and Einstein at Cal Tech

Adding the 'Second Law' and $\Lambda > 0$

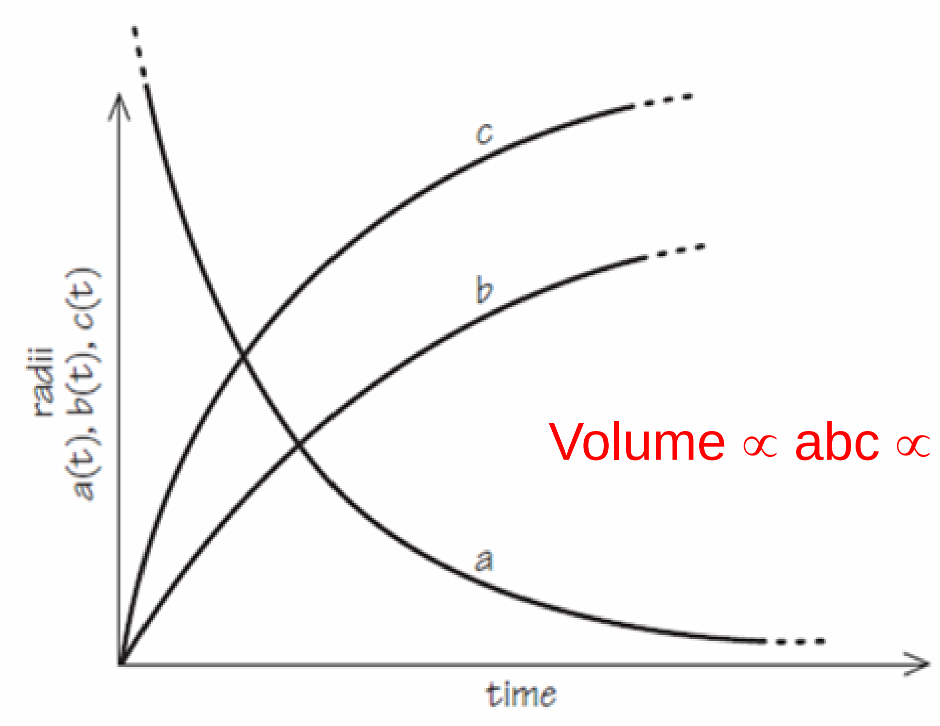


Tolman includes the
2nd Law of thermodynamics:
Oscillations grow
(1932)

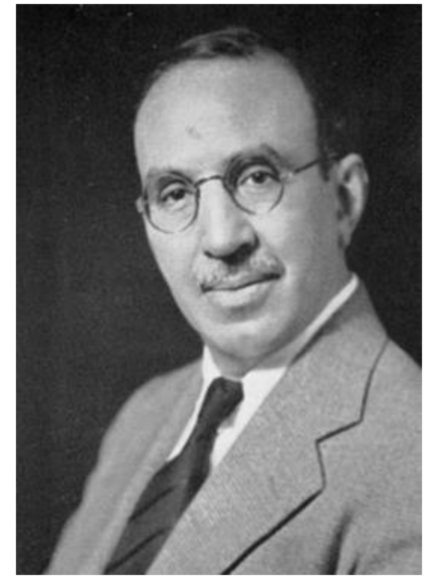


JDB + Dabrowski include the
cosmological constant:
Oscillations always end
(1995)

Edward Kasner's Anisotropic Universe



Volume $\propto$ abc $\propto$ t



Edward Kasner
(1878-1955)

$(a,b,c) = (t^p, t^q, t^r)$

$p + q + r = 1$ and $p^2 + q^2 + r^2 = 1$

$-1/3 \leq p \leq 0 \leq q \leq 2/3 \leq r \leq 1$

1921

Speed of light

$ds^2 = c^2\, dt^2 - t^{2p}\, dx^2 - t^{2q}\, dy^2 - t^{2r}\, dz^2$, and $T_{ab} = 0$

The Group Theory of Universes

The Ten Universes

Of

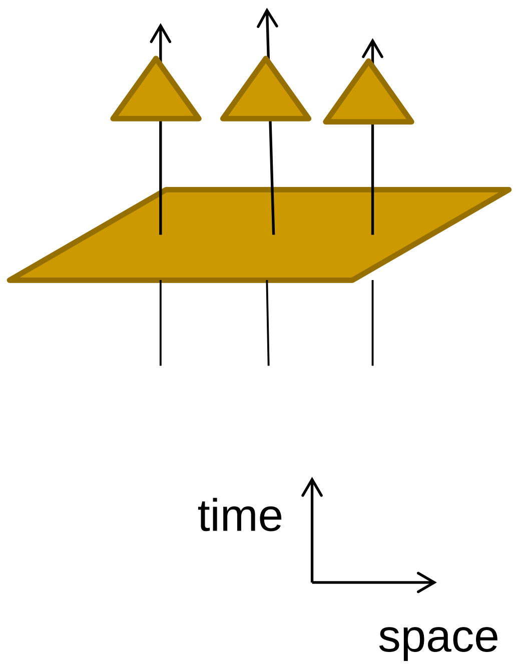
Bianchi and Taub

Shear Distortion

Rotation

Gravitational Waves

Anisotropic expansion



A. Taub,1951

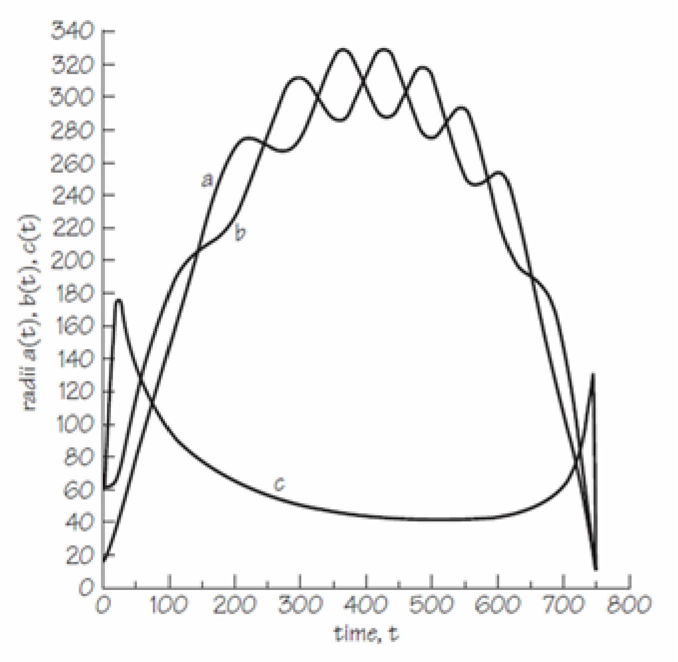
Everyone sees the same history

Spatially homogeneous

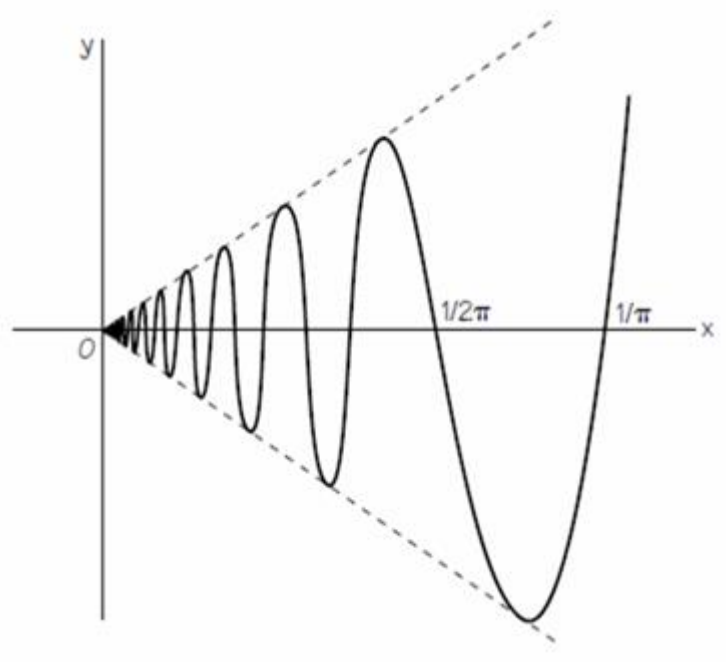
Ordinary Differential Equations

$\rightarrow$ 3-parameter invariance group

Misner's Chaotic Mixmaster Universe (1969)



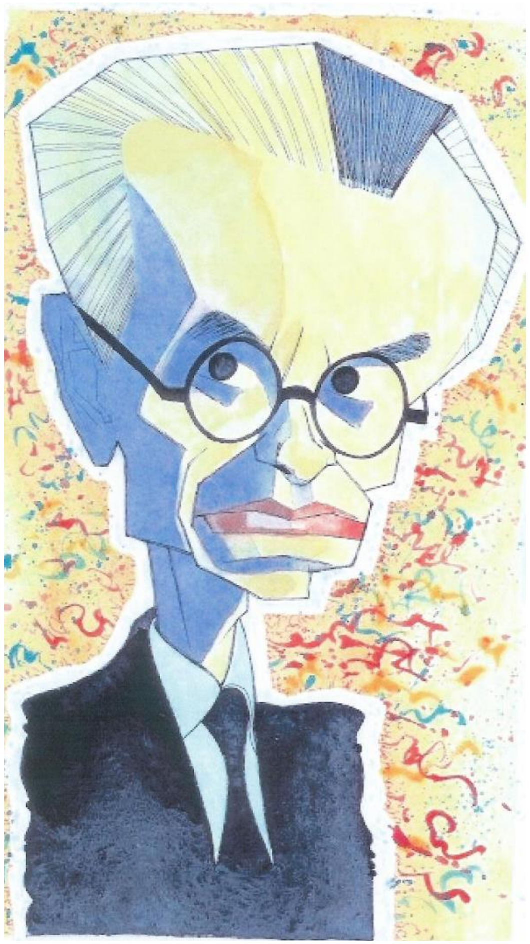
C. Misner



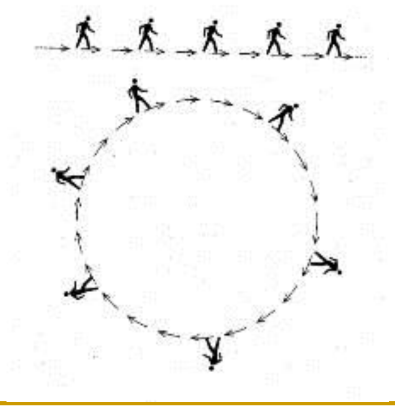
An infinite number of things happen in a finite time

$X_{n+1} = 1/X_n - [1/X_n]$ for $0 < X_n < 1$ continued fraction map see lecture 1

Kurt Gödel's Rotating Universe (1949)



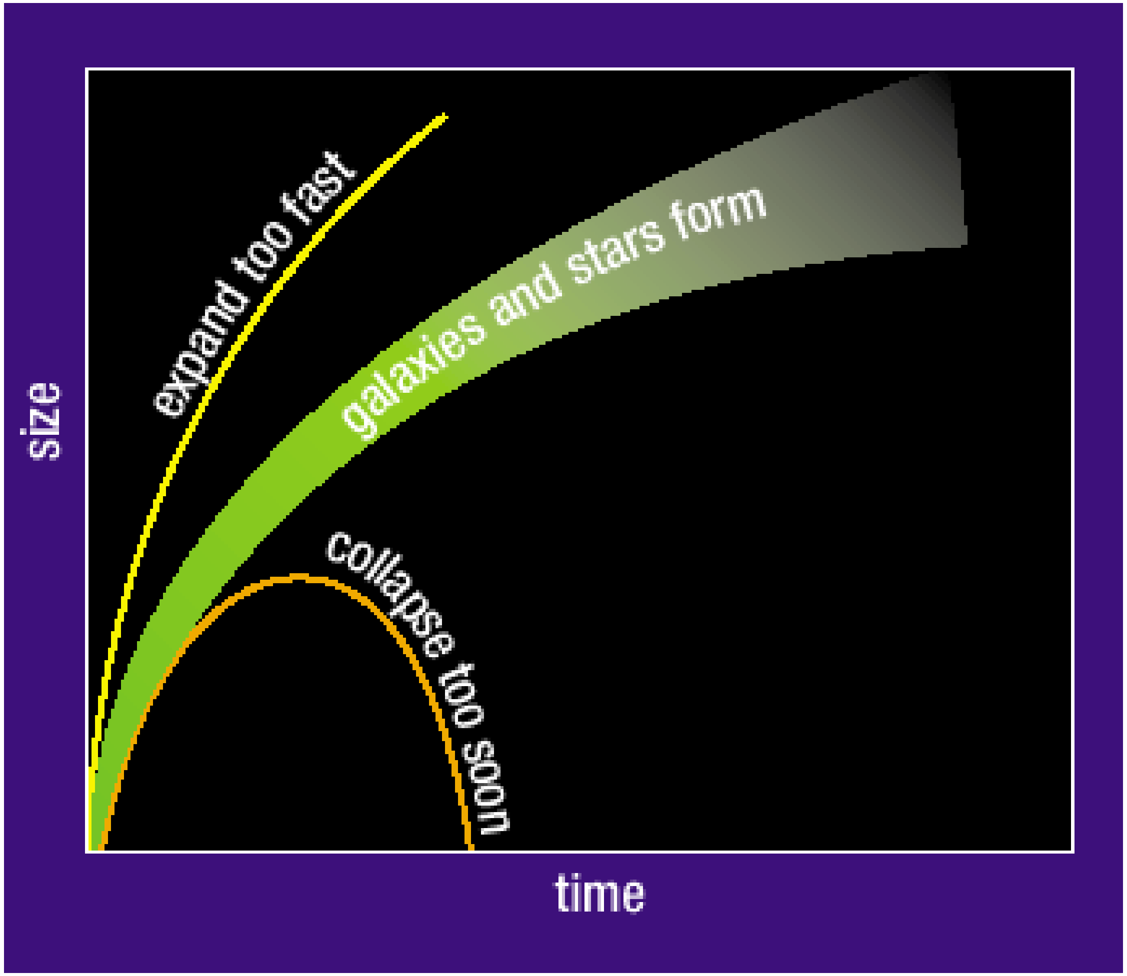
K. Gödel (1906-78)

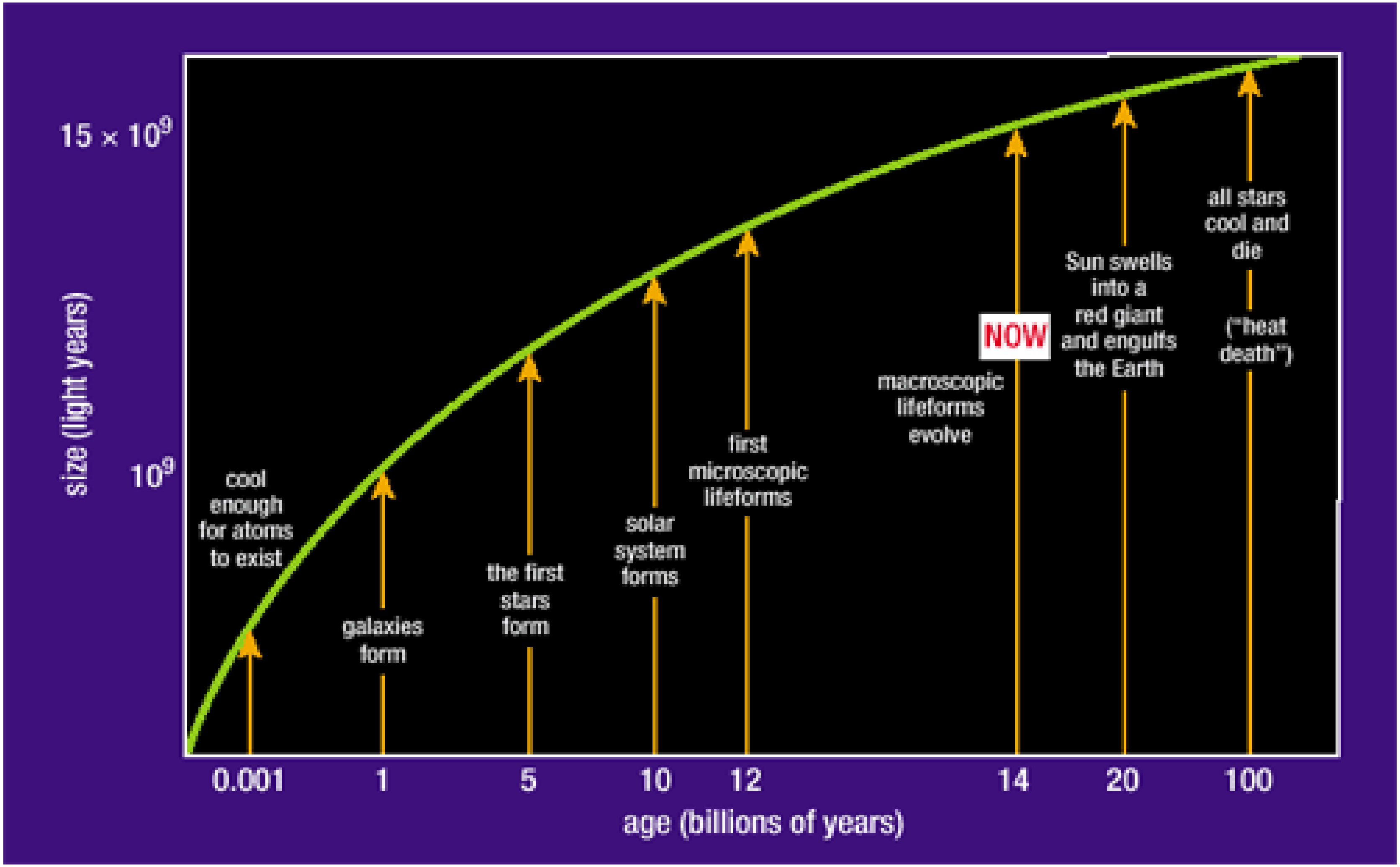


Allows time travel to occur

Doesn't expand

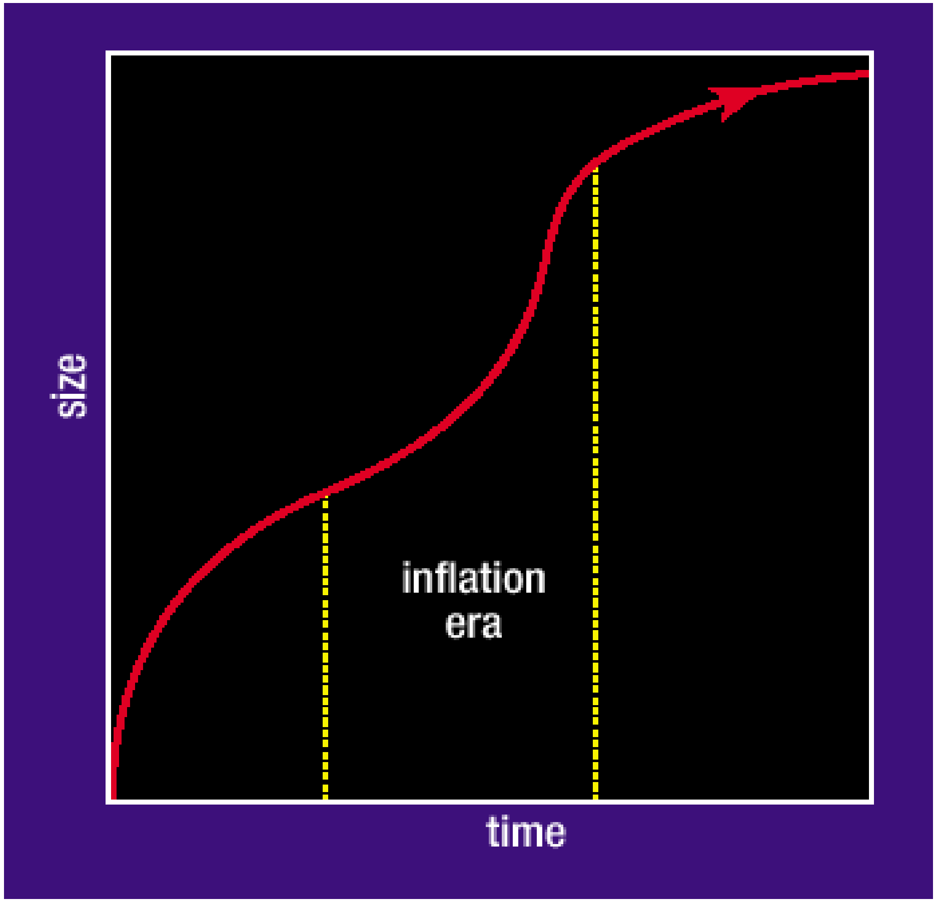
Big Bang Universes





15 × 10⁹
10⁹
size (light years)
cool enough for atoms to exist
galaxies form
the first stars form
solar system forms
first microscopic lifeforms
macroscopic lifeforms evolve
NOW
Sun swells into a red giant and engulfs the Earth
all stars cool and die
("heat death")
0.001
1
5
10
12
14
20
100
age (billions of years)

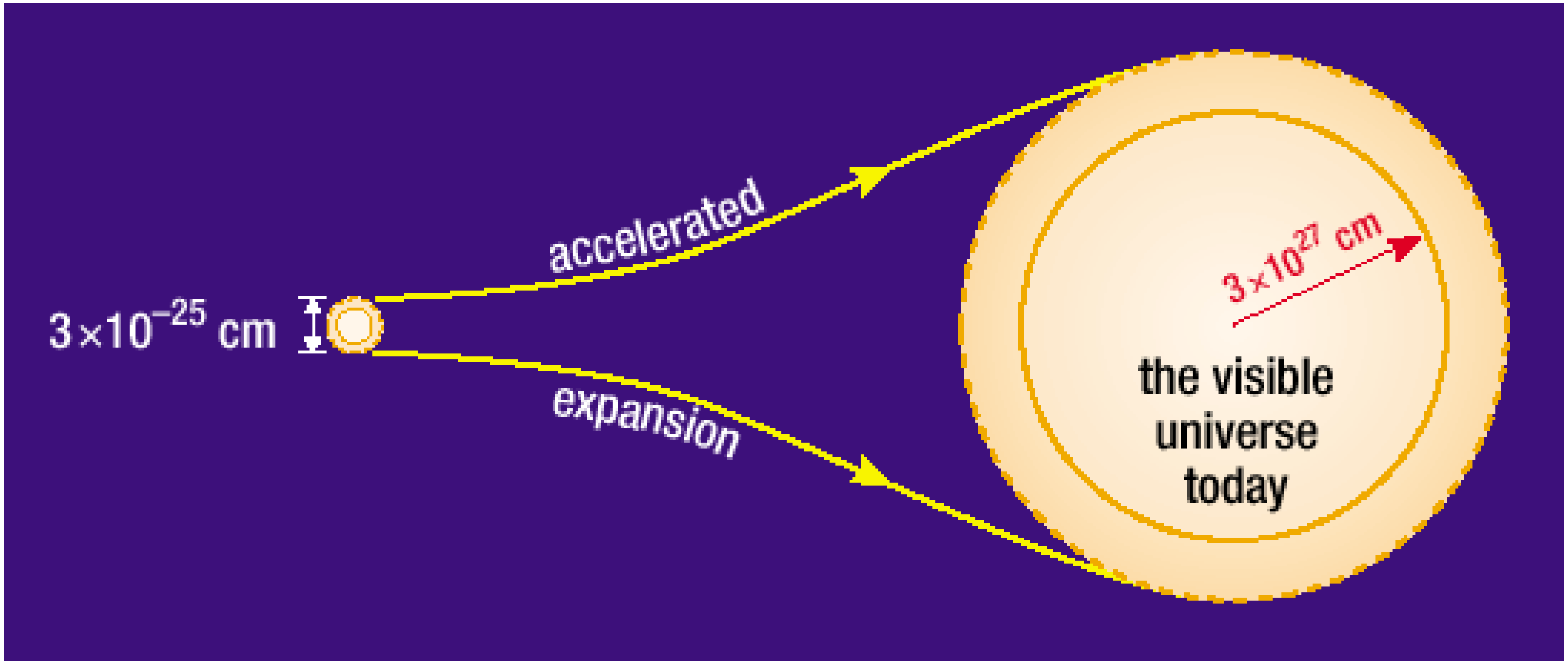
The Inflationary Universe



A. Guth, 1981

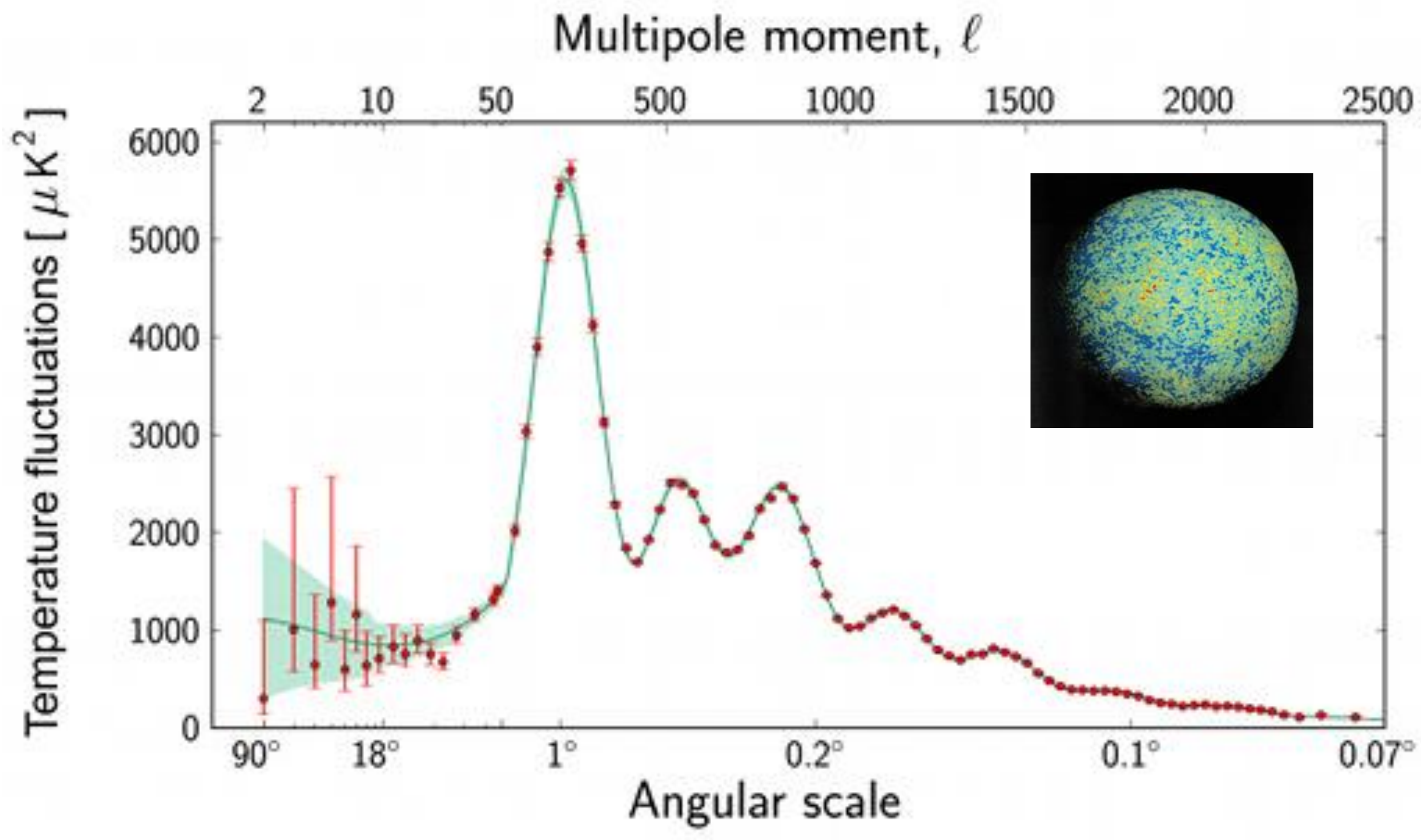
A temporary Λ causes acceleration at $t \approx 10^{-35}$ sec

Even Smaller Beginnings

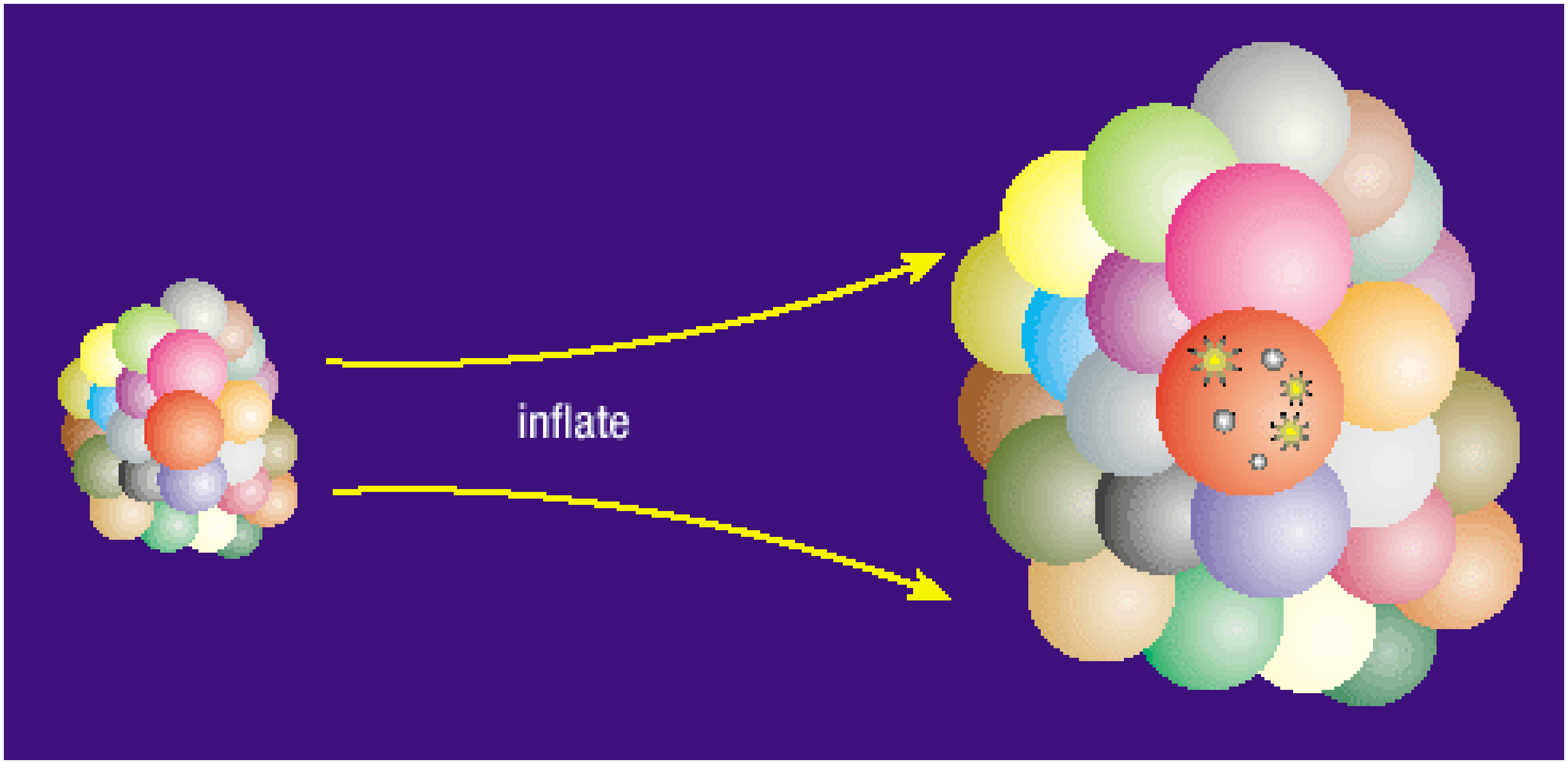


Solving the 'horizon problem' and creating density fluctuations that can seed galaxies

Observational tests: background radiation

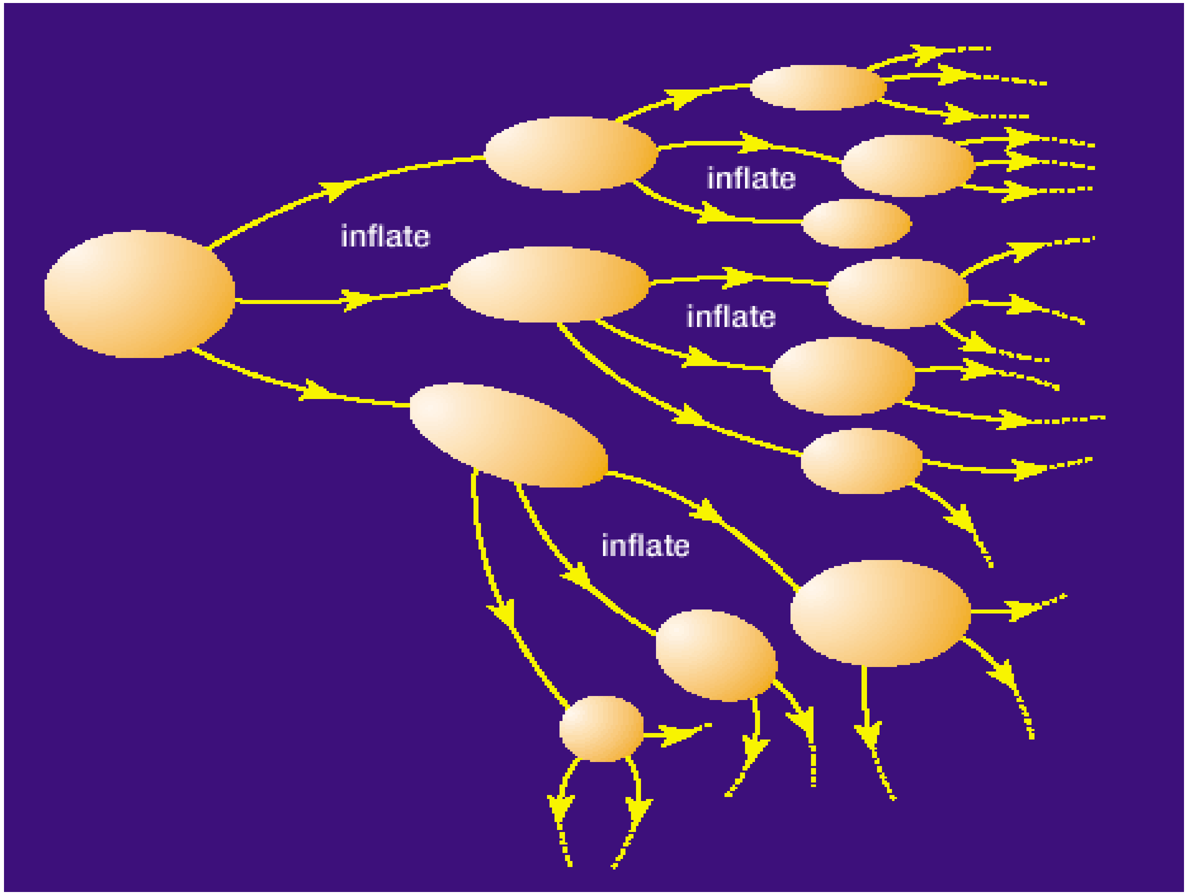


Chaotic Inflation



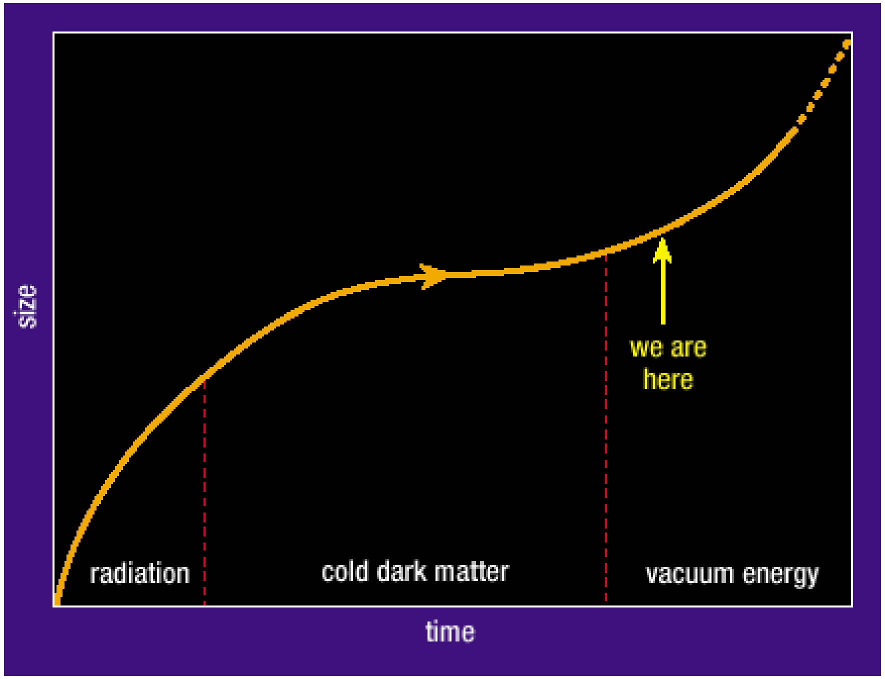
Expansion only approaches isotropy and homogeneity locally

Eternal Inflation



More than $10^{10^{77}}$ by-universes from our patch alone

The Universe is Accelerating Again Now



$\Lambda = 1.19 \times 10^{-52}\ \mathrm{m}^{-2} > 0$

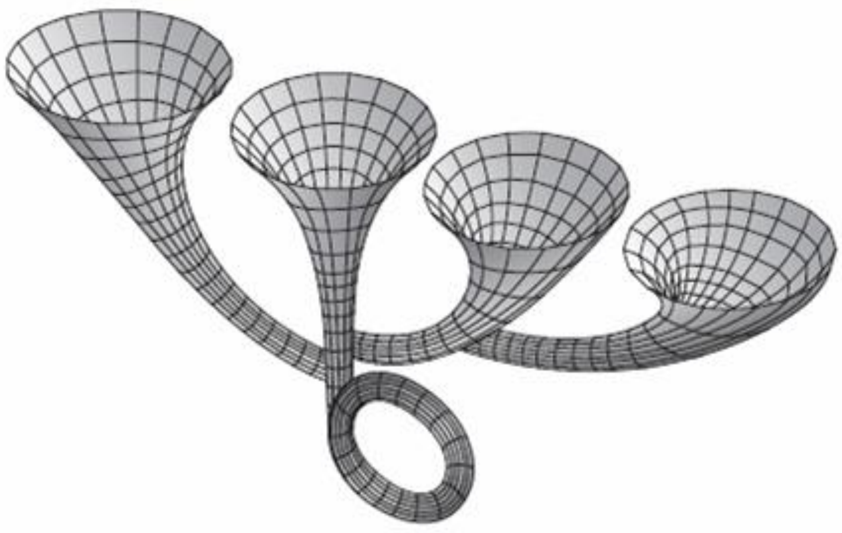
What is its origin?

A cosmic quantum vacuum energy?
It is a fluid with $p = -\rho c^2$ eqn of state

Lemaître's universe describes our visible universe to high precision

WHY?

Any Beginning in a Quantum Universe?



Lorentzian and Euclidean
Gott; Hartle & Hawking

Further reading

- J.D. Barrow, The Book of Universes, Bodley Head, (2011)
- S. Weinberg, The First Three Minutes, Basic Books (1993)
- A.H. Guth, The Inflationary Universe, 2nd edn.(Vintage 1998)
- S.W. Hawking and R. Penrose, The Nature of Space and Time, Cambridge UP, (1996)
- B. Ryden, Introduction to Cosmology, Cambridge UP, (2106)
- E.V. Linder, First Principles in Cosmology, 2nd edn, Addison Wesley, (1997)