

### Line, Area and Volume Elements in Cylindrical Polars

$$\mathbf{r}(\rho, \phi, z) = \rho \cos \phi \mathbf{i} + \rho \sin \phi \mathbf{j} + z \mathbf{k}$$

**Line element:**

$$d\mathbf{r} = h_\rho \mathbf{e}_\rho d\rho + h_\phi \mathbf{e}_\phi d\phi + h_z \mathbf{e}_z dz$$

$$h_\rho = 1 \quad \mathbf{e}_\rho = \cos \phi \mathbf{i} + \sin \phi \mathbf{j}$$

$$h_\phi = \rho \quad \mathbf{e}_\phi = -\sin \phi \mathbf{i} + \cos \phi \mathbf{j}$$

$$h_z = 1 \quad \mathbf{e}_z = \mathbf{k}$$

**Area elements:**

$$d\mathbf{S} = \begin{cases} \mathbf{e}_\rho \rho d\phi dz & (\rho \text{ const}) \\ \mathbf{e}_\phi d\rho dz & (\phi \text{ const}) \\ \mathbf{e}_z \rho d\rho d\phi & (z \text{ const}) \end{cases}$$

**Volume element:**

$$dV = \rho d\rho d\phi dz$$

### Differential Operators in Cylindrical Polars

$$\nabla = \frac{1}{h_\rho} \mathbf{e}_\rho \frac{\partial}{\partial \rho} + \frac{1}{h_\phi} \mathbf{e}_\phi \frac{\partial}{\partial \phi} + \frac{1}{h_z} \mathbf{e}_z \frac{\partial}{\partial z}$$

**Gradient:**

$$\nabla f = \frac{\partial f}{\partial \rho} \mathbf{e}_\rho + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \mathbf{e}_\phi + \frac{\partial f}{\partial z} \mathbf{e}_z$$

**Curl:**

$$\nabla \times \mathbf{F} = \frac{1}{\rho} \begin{vmatrix} \mathbf{e}_\rho & \rho \mathbf{e}_\phi & \mathbf{e}_z \\ \partial/\partial \rho & \partial/\partial \phi & \partial/\partial z \\ F_\rho & \rho F_\phi & F_z \end{vmatrix}$$

**Divergence:**

$$\nabla \cdot \mathbf{F} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho F_\rho) + \frac{1}{\rho} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

**Laplacian:**

$$\nabla^2 f = \frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial f}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

for a scalar field  $f(\mathbf{r})$  and a vector field  $\mathbf{F}(\mathbf{r}) = F_\rho \mathbf{e}_\rho + F_\phi \mathbf{e}_\phi + F_z \mathbf{e}_z$

**Line, Area and Volume Elements in Spherical Polars**

$$\mathbf{r}(r, \theta, \phi) = r \sin \theta \cos \phi \mathbf{i} + r \sin \theta \sin \phi \mathbf{j} + r \cos \theta \mathbf{k}$$

**Line element:**

$$d\mathbf{r} = h_r \mathbf{e}_r dr + h_\theta \mathbf{e}_\theta d\theta + h_\phi \mathbf{e}_\phi d\phi$$

$$h_r = 1 \quad \mathbf{e}_r = \sin \theta \cos \phi \mathbf{i} + \sin \theta \sin \phi \mathbf{j} + \cos \theta \mathbf{k}$$

$$h_\theta = r \quad \mathbf{e}_\theta = \cos \theta \cos \phi \mathbf{i} + \cos \theta \sin \phi \mathbf{j} - \sin \theta \mathbf{k}$$

$$h_\phi = r \sin \theta \quad \mathbf{e}_\phi = -\sin \phi \mathbf{i} + \cos \phi \mathbf{j}$$

**Area elements:**

$$d\mathbf{S} = \begin{cases} \mathbf{e}_r r^2 \sin \theta d\theta d\phi & (r \text{ const}) \\ \mathbf{e}_\theta r \sin \theta dr d\phi & (\theta \text{ const}) \\ \mathbf{e}_\phi r dr d\theta & (\phi \text{ const}) \end{cases}$$

**Volume element:**

$$dV = r^2 \sin \theta dr d\theta d\phi$$

**Differential Operators in Spherical Polars**

$$\nabla = \frac{1}{h_r} \mathbf{e}_r \frac{\partial}{\partial r} + \frac{1}{h_\theta} \mathbf{e}_\theta \frac{\partial}{\partial \theta} + \frac{1}{h_\phi} \mathbf{e}_\phi \frac{\partial}{\partial \phi}$$

**Gradient:**

$$\nabla f = \frac{\partial f}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \mathbf{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \mathbf{e}_\phi$$

**Curl:**

$$\nabla \times \mathbf{F} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \mathbf{e}_r & r \mathbf{e}_\theta & r \sin \theta \mathbf{e}_\phi \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial \phi \\ F_r & r F_\theta & r \sin \theta F_\phi \end{vmatrix}$$

**Divergence:**

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta F_\theta) + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi}$$

**Laplacian:**

$$\nabla^2 f = \frac{\partial^2 f}{\partial r^2} + \frac{2}{r} \frac{\partial f}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

for a scalar field  $f(\mathbf{r})$  and a vector field  $\mathbf{F}(\mathbf{r}) = F_r \mathbf{e}_r + F_\theta \mathbf{e}_\theta + F_\phi \mathbf{e}_\phi$