B12d

Fluid Dynamics IB
Lent 2014
Professor M G Worster

Tripos questions suitable for 1B Fluid Dynamics

Note that in the 1B Tripos papers up to and including 2011, all fluids were assumed to be inviscid. Note also that an 'ideal' fluid is an 'incompressible, inviscid fluid'. Questions from the 2012 Tripos papers are all relevant to the current syllabus.

Questions in the table below are given in the format paper:question number.

Starred questions indicate questions that are either not strictly on the new course but are accessible given the material lectured or contain some small part or rider that is not on the new course.

There are some useful questions for the material from the first three lectures on parallel viscous flow from Part II of the Tripos as indicated in the table.

There are no recent Tripos questions prior to 2012 corresponding to the last two lectures on rotating flows. Below are two questions adapted from an earlier Part II(A) course on Theoretical Geophysics, which contain suitable practice material.

Year	**Part 1B**	**Part II**
2011	1:5, 1:17, 2:7, 4:18	4:37 first paragraph only
2010	1:5, 1:17, 3:18, 4:18	1:37 omit first paragraph, rectilinear=parallel
2009	1:5, 1:17*, 2:8, 3:18*, 4:18	
2008	1:5, 1:17, 2:8, 3:18, 4:18	1:36 omit first paragraph
2007	1:5, 1:17, 2:8, 3:18, 4:18*	
2006		1:36

1. In a frame of reference rotating rapidly about a vertical axis with angular velocity $f/2$, the horizontal components of the momentum equation for a shallow layer of inviscid, incompressible fluid of uniform density ρ are

$$\frac{\partial u}{\partial t} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$
$$\frac{\partial v}{\partial t} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y},$$

where u and v are independent of the vertical coordinate z, and p is given by hydrostatic balance. State the equations for conservation of mass for such a flow in a layer occupying $0 < z < h(x, y, t)$.

By linearising the equations about a state of rest and uniform thickness H, show that small disturbances $\eta = h - H$, where $\eta \ll H$, to the height of the free surface obey

$$\frac{\partial^2\eta}{\partial t^2} - gH\left(\frac{\partial^2\eta}{\partial x^2} + \frac{\partial^2\eta}{\partial y^2}\right) + f^2\eta = f^2\eta_0 - fH\zeta_0,$$

where η_0 and ζ_0 are the values of η and the relative vorticity ζ at $t = 0$.

Explain what is meant by geostrophic balance. Find the long-time geostrophically balanced solution η_∞ and (u_∞, v_∞), that results from initial conditions $\eta_0 = A\text{sgn}(x)$ and $(u, v) = 0$.

2. In a frame of reference rotating about a vertical axis with angular frequency $f/2$, the horizontal components of the momentum equation for a shallow layer of inviscid, incompressible fluid of constant density ρ are

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial P}{\partial x}$$
$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial P}{\partial y},$$

where u, v and P are independent of the vertical coordinate z.

Define the Rossby number Ro for a flow with typical velocity U and lengthscale L. What is the approximate form of the above equations when $Ro \ll 1$. Show that the solution to the steady, approximate equations is given by a streamfunction ψ proportional to P.