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Mathematical Tripos Part IIA/B Prof G.W. Gibbons

GENERAL RELATIVITY (16 lectures) Lent 2004

Pre-Requistes:
Part IB Methods and Special Relativity are essential and IB Principles of Dynamics is desirable. You are particularly advised to revise Cartesian tensors, the Einstein summation convention and the practice of "index-shuffling".

The material will be lectured in the order given in the schedules, which read as follows.

The Schedules
Curved and Riemannian spaces. Special relativity and gravitation, the Pound-Rebka experiment. Introduction to general relativity: interpretation of the metric, clock hypothesis, geodesics, equivalence principles. Static spacetimes, Newtonian limit. [4]

Covariant and contravariant tensors, tensor manipulation, partial derivatives of tensors. Metric tensor, magnitudes, angles, duration of curve, geodesics. Connection, Christoffel symbols, absolute and covariant derivatives, parallel transport, autoparallels as geodesics. Curvature. Riemann and Ricci tensors, geodesic deviation. [5]

Vacuum field equations. Spherically symmetric spacetimes, the Schwarzschild solution. Rays and orbits, gravitational red-shift, light deflection, perihelion advance. Event horizon, gravitational collapse, black holes. [5]

Equivalence principles, minimal coupling, non-localisability of gravitational field energy. Bianchi identities. Field equations in the presence of matter, equations of motion. [2]

There will be three example sheets.

Units
In order not to clutter up formulae, for the most part, units will be used in which the velocity of light, c, and Newton's constant of Gravitation G are set to unity. When required they may, and will, be restored using elementary dimensional analysis.

Signature Convention
Beginners often find remembering various notational conventions which abound in the subject confusing. The best strategy is to cultivate the ability to switch as desired, specially as no physical statement can depend on such arbitrary conventions. Some hints to facilitate changing conventions are given below, however you are advised not to do so in the middle of a formula. The signature convention I will use is (+ + + -) and spacetime indices (which run over 4 values) will be denoted by lower case latin letters (rather than say greek, cyrillic or hebrew) usually taken from the beginning of the alphabet. Space indices will be denoted by i,j,k and will take values from 1 to 3. If you need to change the signature conventions (for example to consult a textbook which uses the opposite one), it suffices to replace the metric g_{ab} by $-g_{ab}$.

Curvature Conventions
The curvature and Ricci tensor conventions (whose meaning will be explained later in the course) are $\nabla_a \nabla_b V^c - \nabla_b \nabla_a V^c = R^c{}_{dab} V^d$, $R_{db} = R^c{}_{dcb}$. If the signature convention is switched to the opposite one, keeping the curvature and Ricci tensor conventions unchanged then $\Gamma_a{}^b{}_c$, $R^c{}_{dab} \to R^c{}_{dab}$ and $R_{db} \to R_{db}$. The Ricci-scaler $R = g^{ab} R_{ab}$ and R_{abcd} change sign.

Other miscellaneous conventions
A comma followed by a subscript $,_a$ after a tensor means the same as ∂_a in front of the tensor and denotes partial derivative. A semi-colon followed by a subscript after a tensor $;_a$ or ∇_a in front of a tensor denotes covariant derivative. The symbol $\pm(a \leftrightarrow b)$ after a tensorial expression containing the index pair ab means add or subtract the same expression with a and b interchanged. Round brackets will be used to denote symmetrization and square brackets to denote anti-symmetrization thus $S_{(ab)} = \frac{1}{2}\big(S_{ab} + S_{ab}\big)$ and $A_{[ab]} = \frac{1}{2}\big(A_{ab} - A_{ab}\big)$.

Appropriate books

The following are listed in the schedules:
— C. Clarke Elementary General Relativity. Edward Arnold 1979 (out of print)

— L.P. Hughston and K.P. Tod An Introduction to General Relativity. London Mathematical Society Student Texts no. 5, Cambridge University Press 1990 (+ - - -)(L)

— † R. d'Inverno Introducing Einstein's Relativity. Clarendon Press 1992 (+ - - -)(R)

— W. Rindler Relativity: Special, General and Cosmological Oxford University Press 2001

— B.F. Schutz A First Course in General Relativity. Cambridge University Press 1985

— H. Stephani General Relativity 2nd edition. Cambridge University Press 1990 (+ + + -)(R).

In addition, the following more advanced books contain much useful material at about the level of the present course. They should all be available in college libraries.

— C. W. Misner, K.S. Thorne and J.A. Wheller, Gravitation. W.H. Freeman (- + + +)(G)

— S. Weinberg, Gravitation and Cosmology (Wiley) (- + + +)(G)

— L.D. Landau and E.M. Lifshitz, The Classical Theory of Fields (Pergamon)(+ - - -)(R)

— J.M. Stewart, Advanced General Relativity (Cambridge University Press)(+ - - -)(G)

— R.M. Wald, General Relativity (Chicago University Press)(- + + +)(R)

The third covers both Electrodynamics at a level suitable for the Part II course and then develops General Relativity. The first book contains a useful summary of the various conventions used in some of the better known textbooks. (R) means spacetime indices are roman, (G) means that they are greek.