Example Sheet 0

This is a revision sheet. If you did NST Mathematics A or B last year you should be able to do the questions already (let me know if I have made an incorrect assumption here, especially if you did Course A). If you did the Mathematical Tripos last year you will have to read up on Fourier Series for question 3. Some of the material will be touched on in the first couple of lectures, so you might prefer to wait until then.

1. (a) Let h be a function of one variable. Working from first principles, differentiate with respect to x the function

$$I(x) = \int_a^x h(y)\,\mathrm{d}y\,,$$

where a is a constant.

(b) Let $f(x,y)$ be a function of two variables. Working from first principles, differentiate with respect to x the function

$$J(x) = \int_a^x f(x,y)\,\mathrm{d}y\,.$$

2. Let $g(x,y,z)$ be a function of three variables. Working to $O(\delta x, \delta y, \delta z)$, write down the Taylor expansion of $g(x+\delta x, y+\delta y, z+\delta z)$.

3. (a) Let $f(y)$ be an odd periodic function of y with period 2, i.e. $f(-y) = -f(y)$ and $f(y+2) = f(y)$. Given that $f(y) = \frac{1}{2}y(y-1)$ for $0 \leqslant y \leqslant 1$, sketch the function $f(y)$ for $-2 \leqslant y \leqslant 2$, and find the Fourier (sine) series for f.

(b) Let $g(x)$ be an odd periodic function of x with period $2L$. Given that $g(x) = (x-L)/L$ for $0 \leqslant x \leqslant L$, find the Fourier series for g.

4. Show that, if $\boldsymbol{e}_i \cdot \boldsymbol{e}_j = \delta_{ij}$ $(i,j = 1,2,3)$, where δ_{ij} is the Kronecker delta, and $\boldsymbol{e}_1 \times \boldsymbol{e}_2 = \boldsymbol{e}_3$, then

$$\boldsymbol{e}_2 \times \boldsymbol{e}_3 = \boldsymbol{e}_1 \quad \text{and} \quad \boldsymbol{e}_3 \times \boldsymbol{e}_1 = \boldsymbol{e}_2\,.$$

5. Show that for

(a) vector components a_i $(i = 1,2,3)$,

$$\sum_{j=1}^{3} a_j \delta_{ij} = a_i\,;$$

(b) independent variables q_i $(i = 1,2,3)$,

$$\frac{\partial q_i}{\partial q_j} = \delta_{ij}\,.$$

6. Show that, if $\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$, where $\boldsymbol{\omega}$ is a constant vector and $\boldsymbol{r}$ is the position vector, then $\boldsymbol{\nabla} \times \boldsymbol{v} = 2\boldsymbol{\omega}$. Comment.

7. Evaluate $\boldsymbol{\nabla} \cdot \boldsymbol{r}$ and $\boldsymbol{\nabla} \times \boldsymbol{r}$, where $\boldsymbol{r}$ is the position vector.

8. Let $\psi(\boldsymbol{r})$ and $\chi(\boldsymbol{r})$ be scalar fields, and $\boldsymbol{u}(\boldsymbol{r})$ and $\boldsymbol{v}(\boldsymbol{r})$ vector fields. Show that

   (a)
   $$\boldsymbol{\nabla} \times (\psi \boldsymbol{v}) = (\boldsymbol{\nabla}\psi) \times \boldsymbol{v} + \psi(\boldsymbol{\nabla} \times \boldsymbol{v}),$$

   (b)
   $$\boldsymbol{\nabla} \cdot (\boldsymbol{u} \times \boldsymbol{v}) = \boldsymbol{v} \cdot (\boldsymbol{\nabla} \times \boldsymbol{u}) - \boldsymbol{u} \cdot (\boldsymbol{\nabla} \times \boldsymbol{v}),$$

   and hence that
   $$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}\psi \times \boldsymbol{\nabla}\chi) = 0.$$

This example sheet is available at `http://www.damtp.cam.ac.uk/user/examples/`
Please send any comments and corrections to `gio10@cam.ac.uk`