INTRODUCTION

0.1. Accretion

If a particle of mass m falls from infinity and comes to rest on the surface of a star of mass M and radius R_*, the energy released is

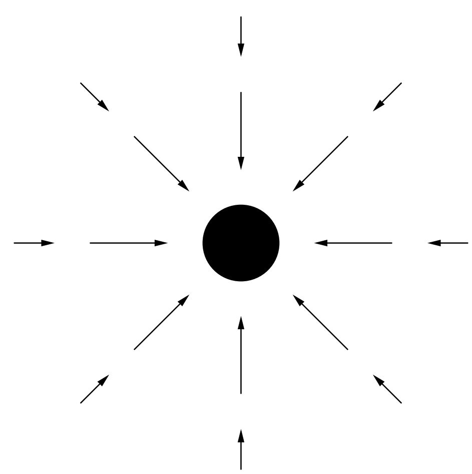
$$\frac{GMm}{R_*} = \left(\frac{R_\mathrm{S}}{2R_*}\right) mc^2,$$

where

$$R_\mathrm{S} = \frac{2GM}{c^2}$$

is the *Schwarzschild radius*. For a compact star such as a neutron star ($M \approx 3 \times 10^{33}\,\mathrm{g}$, $R_* \approx 10^6\,\mathrm{cm}$), the energy released is a significant fraction (about 20%) of the rest-mass energy of the particle, and accretion is an even more efficient source of energy than nuclear fusion.

A star placed in a static, uniform gaseous medium will accrete mass from its surroundings. This *spherical accretion* or *Bondi accretion* is the simplest type of accretion flow, but applies only when the gas has negligible angular momentum.

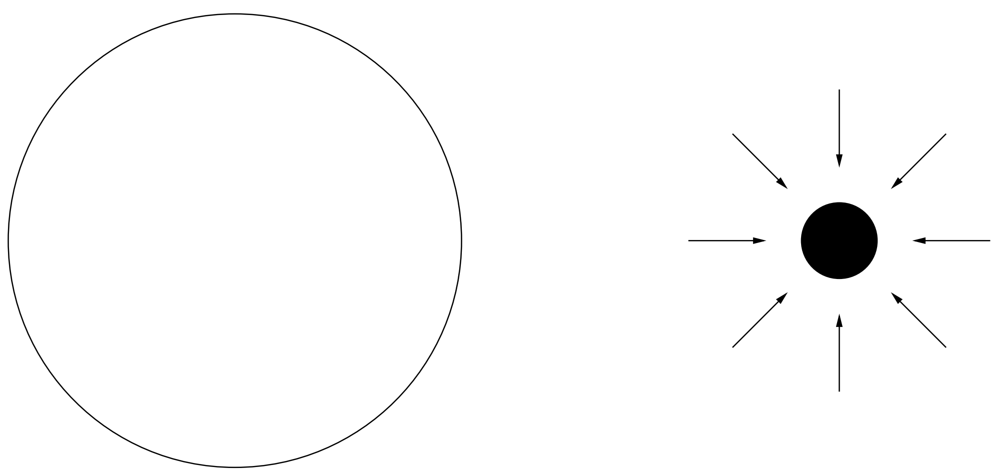


Spherical accretion

Consider instead a particle in a circular orbit around the star. If the orbit can be contracted from a large radius R to a much smaller radius $r \ll R$, the energy released is approximately equal to the binding energy of the smaller orbit, $GMm/2r$. However, in order to achieve this, almost all of the the angular momentum of the larger orbit, $(GMR)^{1/2}m$, must be removed.

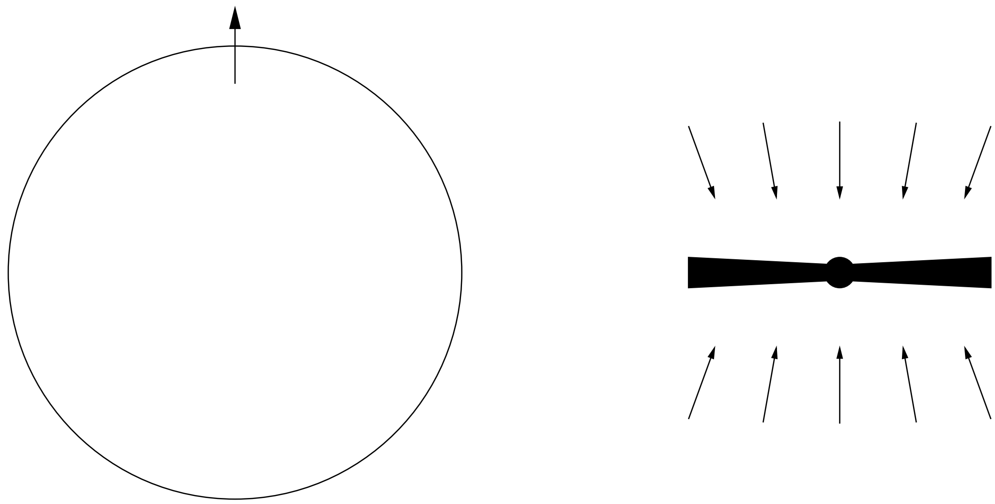
Most accretion flows in astrophysics are rapidly rotating, and one of the central problems is how to remove the angular momentum so that accretion can occur. While, in dissipative flows, energy can be converted into heat and then radiated away, angular momentum is more difficult to remove. An *accretion disc* is a flow that achieves this outward transport of angular momentum.

Although the Universe as a whole is expanding, most of the objects studied in astronomy have formed as a result of *gravitational collapse*. Consider a static, uniform, spherical cloud of gas that collapses under its own gravity. The collapse may be assumed to be spherically symmetric, and an object will be formed that is supported against gravity by pressure.



Collapse of a non-rotating cloud

If the cloud is initially in uniform rotation, however, the dynamics will be affected by the centrifugal force, which resists collapse in the plane perpendicular to the axis of rotation. Even if the centrifugal force is negligible in the initial state, it will become more important as the cloud collapses, conserving its angular momentum. A rapidly rotating disc will be formed around the central condensation, supported against gravity mainly by the centrifugal force.



Collapse of a rotating cloud

0.2. Discs in astrophysics

0.2.1. Protoplanetary discs

Since the Copernican revolution we have understood that the planets of the solar system all orbit the Sun in the same sense, and almost in the same plane. In the eighteenth century Kant and Laplace, recognizing that such a situation could not have arisen by chance, proposed that the planets condensed out of a flattened cloud of gas rotating around the Sun earlier in its life. Their models of the *solar nebula* introduced the concept of a *protoplanetary disc*.

Since 1995, the Hubble Space Telescope has provided images of such discs around young stars in the Orion Nebula, a nearby region of star formation. Whether or not they form planets, such discs are understood to be an essential part of the star-formation process (and are often called *protostellar discs*). They consist of relatively cool gas, mostly H_2 together with some dust, and are believed to survive for a few million years.

During that time planets are thought to form as the dust agglomerates into successively larger bodies, eventually forming the rocky cores of planets. To form a gas giant planet such as Jupiter, the core must subsequently accrete a substantial gaseous envelope from the surrounding disc. An alternative, less popular, theory is that gas giant planets form directly through a rapid gravitational instability of the disc.

Also since 1995, more than 100 planets have been discovered around nearby stars similar to the Sun. Because a star with a planet itself executes a much smaller orbit around the centre of mass of the system, its motion gives rise to a detectable periodic Doppler shifting of its spectral lines. In these systems, like the solar system, the protoplanetary disc has already dispersed. Observational constraints mean that planets cannot yet be detected around young stars.

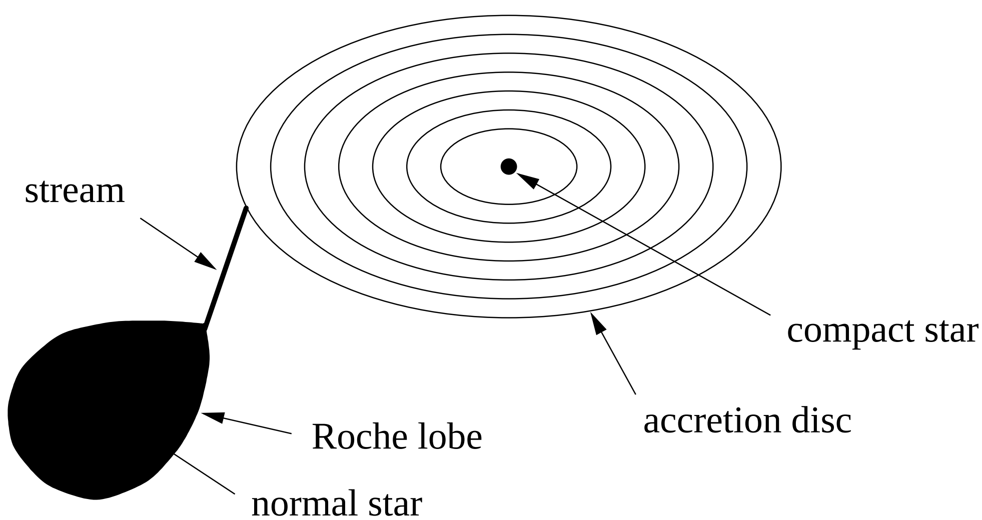
Useful data: solar mass $M_\odot = 1.99 \times 10^{33}\,\mathrm{g}$, solar radius $R_\odot = 6.96 \times 10^{10}\,\mathrm{cm}$, astronomical unit (mean distance of Earth from Sun) $1\,\mathrm{AU} = 1.50 \times 10^{13}\,\mathrm{cm}$.

Jupiter data: mass $M_\mathrm{J} = 1.90 \times 10^{30}\,\mathrm{g} \approx 0.001\,M_\odot$, radius $R_\mathrm{J} = 7.14 \times 10^9\,\mathrm{cm} \approx 0.1\,R_\odot$, orbital semi-major axis $5.20\,\mathrm{AU}$.

Typical parameters of protoplanetary discs: central mass $1\,M_\odot$, disc mass $\lesssim 0.1\,M_\odot$, outer radius $100 - 1000\,\mathrm{AU}$, angular semi-thickness $H/r \approx 0.05 - 0.1$, dimensionless viscosity parameter $\alpha \approx 0.001 - 0.01$, accretion rate $\dot{M} \approx 10^{-8}\,M_\odot\,\mathrm{yr}^{-1}$.

0.2.2. Interacting binary stars

Most stars are formed in binary systems. The more massive, 'primary' star evolves more rapidly and will reach the end of its life as a compact object: a white dwarf, a neutron star or a black hole. At this point the secondary star may still be on the main sequence of stellar evolution. If the binary orbit is sufficiently close, the secondary may at some stage overflow its critical equipotential surface or *Roche lobe*, and will then spill over towards its compact companion.



Accretion disc in an interacting binary star

Owing to its rotation in the binary orbit, the transferred gas has too much angular momentum to fall directly on to the surface of the primary. Instead, it forms an *accretion* *disc* around it. Under the action of 'viscous' torques within the disc, the gas gradually loses angular momentum and spirals inwards to be accreted by the central object. As the gas falls deeper into the potential well it liberates energy, making the disc luminous.

Systems with white dwarf primaries are known at *cataclysmic variables* because many of them exhibit dramatic outbursts. These include the *classical novae*, in which a layer accreted on the primary ignites in a thermonuclear runaway, and the *dwarf novae*, in which outbursts occur cyclically in the disc itself. Some of these objects have been known for over a century, although their physical nature was not understood.

Systems with neutron star or black hole primaries are known as *X-ray binaries*, as they are luminous in that part of the spectrum and were first discovered by X-ray satellites sent up in the 1960s. *Low-mass X-ray binaries* involving low-mass secondaries typically accrete by Roche-lobe overflow as described above. In *high-mass X-ray binaries* the accretion disc is typically formed from gas captured from the vigorous wind of the high-mass secondary. The discs usually consist mainly of hydrogen and helium in atomic or ionized form.

Typical parameters: white dwarf mass $0.5\,M_\odot$, radius 10^9 cm; neutron star mass $1.4\,M_\odot$, radius 10^6 cm; black hole mass $7\,M_\odot$, Schwarzschild radius 2×10^6 cm. Outer disc radius 10^{11} cm, angular semi-thickness $H/r \approx 0.01 - 0.05$, dimensionless viscosity parameter $\alpha \approx 0.01 - 0.1$, accretion rate $\dot{M} \approx 10^{-8}\,M_\odot\,\mathrm{yr}^{-1}$.

0.2.3. Active galactic nuclei

It is now generally accepted that most galaxies contain a *supermassive black hole* (of up to a few billion solar masses) at the centre. Some galaxies have *active galactic nuclei* (AGNs), which are luminous in all wavebands. The standard model for AGNs includes an accretion disc around the black hole, which provides the energy source and allows the hole to grow. The AGN phenomenon may represent a temporary phase in galactic evolution, that occurs when gas is made available to be accreted by the nucleus. The orbital motion of the gas in the accretion disc can be traced through the Doppler shift of spectral lines, and this allows a measurement of the mass of the hole, as in the case of M87. The physical conditions of accretion discs in AGNs are not very well constrained.

0.2.4. Other astrophysical discs

There are many other examples of discs in astrophysics. *Spiral galaxies* are discs composed of stars and gas. These differ from accretion discs in that they are strongly self-gravitating and not dominated by the central black hole. The stars have a long mean free path and constitute an almost collisionless system rather than a fluid. Also the time-scales corresponding to accretion processes would exceed the age of the Universe.

Planetary rings are very thin discs composed of solid bodies (ice and rock, up to several metres in size) that undergo frequent collisions. These share certain dynamical properties with gaseous accretion discs, but are significantly affected by the gravitational influence of the planet's moons, which 'shepherd' the rings and produce various types of fine structure in them.

Be stars, which are rapidly rotating high-mass stars, are surrounded by equatorial discs. These are understood to be *decretion discs*, formed when gas is expelled from the surface of the star by the action of a torque.

Some binary evolution scenarios lead to the formation of *compact binaries* involving, e.g., two white dwarfs, or a neutron star and a black hole, in a close orbit. The objects spiral inwards as energy and angular momentum are lost to gravitational radiation. When they approach, the lower-mass object is violently disrupted by tidal forces or instabilities, forms a disc around the higher-mass object, and is then rapidly accreted. Such systems (most likely involving a neutron star and a black hole) are thought to power the enormously energetic explosive fireballs of *gamma-ray bursts*.