

Microfluidics of cytoplasmic streaming and its implications for intracellular transport

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Found in many large eukaryotic cells, particularly in plants, cytoplasmic streaming is the circulation of their contents driven by fluid entrainment from particles carried by molecular motors at the cell periphery. In the more than two centuries since its discovery, streaming has frequently been conjectured to aid in transport and mixing of molecular species in the cytoplasm and, by implication, in cellular homeostasis, yet no theoretical analysis has been presented to quantify these processes. We show by a solution to the coupled dynamics of fluid flow and diffusion appropriate to the archetypal “rotational streaming” of algal species such as *Chara* and *Nitella* that internal mixing and the transient dynamical response to changing external conditions can indeed be enhanced by streaming, but to an extent that depends strongly on the pitch of the helical flow. The possibility that this may have a developmental consequence is illustrated by the coincidence of the exponential growth phase of *Nitella* and the point of maximum enhancement of those processes.

advection–diffusion | algae | cyclosis | *Chara* | metabolism

Since Bonaventura Corti’s discovery (1) in 1774 of the persistent circulation of the cytoplasm of plant cells, the phenomenon now known as cytoplasmic streaming or cyclosis has been conjectured to play an important role in metabolism (2). It occurs in organisms as diverse as amoebae (3), algae and terrestrial plants (4, 5), and fungi (6). In plants (4, 5, 7) it is driven by multitudes of the motor protein myosin moving along bundled actin at the boundary of the cytoplasm, carrying microscopic particles or organelles (8, 9), and entraining fluid. The motion of protoplasmic granules entrained in the flow includes unidirectional streaming, “fountain streaming” (in which the motion near the central axis of the cell is opposite to that near the periphery), and spiral “rotational streaming.”

Plant myosins can move at tens of micrometers per second (10–13), considerably faster than most animal myosins (although some can reach $\approx 30 \mu\text{m/s}$), and the speed U of cyclosis can reach $\approx 100 \mu\text{m/s}$ in cells whose radius R can reach 0.5 mm. These speeds greatly outpace diffusion, as measured by the Péclet number $Pe = UR/D$, where D is a molecular diffusion constant. For the smallest molecules, with $D \sim 10^{-5} \text{cm}^2/\text{s}$, we see that $Pe \sim 50$, and it can easily reach 500–1,000 for larger proteins. The fact that transport by fluid motion becomes necessary to outrun the slow pace of diffusion in larger organisms, as emphasized in the celebrated essay by Haldane on size in biology (14), has been a theme in discussions of cytoplasmic streaming for many years. Yet, there has been little theoretical work and fewer experiments that have quantified the full implications of cytoplasmic streaming for transport and mixing. Only recently has it been recognized (2, 15) that the large Péclet numbers found *in vivo* could enhance metabolite exchange with organelles such as chloroplasts. Still, a range of basic questions has remained unanswered (16): What is the role of cytoplasmic streaming in homeostasis? How does streaming affect metabolic rates? Why has nature chosen the often complex flow geometries seen in plants?

Here, we propose answers to these questions by examining the most basic aspects of streaming flows, motivated by the phe-

nomenology of *Chara corallina* (Fig. 1), which historically has been an organism of choice for studies of streaming (4). We develop the simplest hydrodynamic model of the rotational streaming flow of *Chara* and *Nitella*, and demonstrate very strong enhancement of mixing within the cell and of nutrient uptake from the environment, both correlated directly with the helical geometry of flow. Such flows constitute a previously uncharacterized solution found by evolution to transport on the microscale, complementary to the great variety of mechanisms proposed to enhance mixing in microfluidic devices (17).

Background

Chara corallina is an algal weed inhabiting ponds, consisting of a slender stem interrupted by nodes, from each of which sprout several branches. The cylindrical branches that connect the nodes are exceptionally large single cells 1 mm in diameter and up to 10 cm long (Fig. 1). These multinucleated “internodal” cells have chloroplasts along the inside of the cell wall, arranged in rows that are organized in a helical manner. The surface of the cell is divided into two helical bands with wavelength $\lambda \sim 1 \text{cm}$. These two domains are separated by two “indifferent zones,” which can be identified by missing rows of chloroplasts and are visible as a pair of diametrically opposite light lines spiralling along the internodal cell. Bundles of actin filaments line the inside of the chloroplast rows. Filamentary actin is a polar polymer, and its orientation determines the direction of motion of myosin. The two spiralling bands have opposite polarity, resulting in upward streaming along one band and downward streaming along the other. The result is aptly called the “barber pole” flow (Fig. 2), and the indifferent zones are therefore regions of high shear. Actin is localized in the cytoplasm, a region some 10–20 μm thick that is separated from the larger interior vacuole by the tonoplast, a membrane within which is a complex set of ion channels and pumps that tightly control numerous metabolic functions by keeping the cytosolic concentrations of ions and metabolites optimal, by storing nutrients, and by dynamically responding to external environmental conditions (18–20).

There is a hierarchy of questions one can ask about the fluid dynamics of streaming: the force–velocity relationship of individual motor proteins [a well studied problem (13)], the collective dynamics of interacting motors (21), the generation of shear by motors ferrying cargo, and ultimately the large-scale flow driven by that shear at the cell wall. Some aspects of the latter problem in fluid dynamics were studied long ago (22), with attention restricted to the radial velocity profile at a given cross-section, and for nonspiralling flows (23).

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the flux normalized to that in the absence of flow for the case $\lambda/R = 12$ and for various Péclet numbers. The difference in uptake rate between the maximum and minimum can be as large as the mean value.

Conclusions

The analysis presented here immediately suggests a number of specific experimental investigations. Chief among them are detailed studies of the geometry of flow, using appropriate tracer particles, to examine the asymmetry between the indifferent zones and their importance in mixing. The predicted variations in uptake rate across the cell surface suggest the possibility that photosynthetic activity itself might be spatially varying around the periphery of the organism. This can be tested with fluorescence methods (37) used in the study of photosynthetic activity in leaves. In such an investigation, it would be important to separate out the effects of variations in chloroplast size and intrinsic activity from effects of nutrient exchange. Finally, the connection between helical streaming in internodal cells and the fast circulatory streaming in the nodes remains to be elucidated. Taken together with recent work (35, 38) on the dynamics of advection by flagellated multicellular algae (e.g., *Volvox*), and collective flows in concentrated bacterial suspensions (39), the present analysis serves to highlight the unusual features of “life at high Péclet numbers,” in which advection dominates diffusion.

Materials and Methods

Stokes flow was solved as a mode expansion in cylindrical coordinates (31). Adopting a radial coordinate rescaled by the cell radius, the velocity field takes the form

$$\begin{aligned} u_r(r, \varphi) &= \sum_{n \text{ odd}} u_r^n(r) \cos(n\varphi) \\ u_\theta(r, \varphi) &= \sum_{n \text{ odd}} u_\theta^n(r) \sin(n\varphi) \\ u_z(r, \varphi) &= \sum_{n \text{ odd}} u_z^n(r) \sin(n\varphi) \\ p(r, \varphi) &= \sum_{n \text{ odd}} p^n(r) \cos(n\varphi). \end{aligned} \quad [4]$$

We impose the velocity $\tilde{u}(1, \varphi)$ at the boundary. The radial component $u_r(1, \varphi) = 0$ and the remaining two components are decomposed into Fourier modes.

The radial modes are most readily solved by the substitutions $u_r^n = -(a^n + b^n)/2$ and $u_\theta^n = -(a^n - b^n)/2$, after which one finds simple combinations of modified Bessel functions:

$$\begin{aligned} a_r^n(r) &= \frac{1}{I_n(n\kappa)} \left[A^n I_{n+1}(n\kappa r) + P^n \frac{\kappa r}{2\pi} I'_{n+1}(n\kappa r) \right] \\ b_r^n(r) &= \frac{1}{I_n(n\kappa)} \left[B^n I_{n-1}(n\kappa r) + P^n \frac{\kappa r}{2\pi} I'_{n-1}(n\kappa r) \right] \\ u_z^n(r) &= -\frac{1}{I_n(n\kappa)} \left[E^n I_n(n\kappa r) + P^n \frac{\kappa r}{2\pi} I'_n(n\kappa r) \right] \\ p_r^n(r) &= -\frac{1}{I_n(n\kappa)} \left[P^n \frac{\kappa}{\pi} I_n(n\kappa r) \right]. \end{aligned} \quad [5]$$

The pressure coefficients may now be eliminated using the incompressibility condition, which reduces to $P^n = -n\pi(A^n + B^n - 2E^n)$. Substitution of $r = 1$ into the above yields a linear system of equations, from which A^n , B^n , and E^n can be determined in terms of the Fourier decomposition of $\tilde{u}(1, \varphi)$. Computations used a smooth transition across the indifferent zone, with a width of $\pi/16$.

Numerical solution of the advection diffusion equation was done by substitution of helically symmetric modes for the concentration:

$$C(r, t) = \sum_n C^n(r, t) \cos(n\varphi). \quad [6]$$

The ODEs for the temporal evolution of the radial modes are discretized in r and integrated by using the Fortran-based LSODE solver (40), with 24 density modes, each with 100 radial grid points.

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1. Corti B (1774) *Osservazione Microscopiche sulla Tremella e sulla Circolazione del Fluido in Una Planto Acquaguola* (Appresso Giuseppe Rocchi, Lucca, Italy).
2. Pickard WF (2003) *Plant Cell Environ* 26:1–15.
3. Allen RD, Allen NS (1978) *Annu Rev Biophys Bioeng* 7:469–495.
4. Shimmen T (2007) *J Plant Res* 120:31–43.
5. Allen NS, Allen RD (1978) *Annu Rev Biophys Bioeng* 7:497–526.
6. Cole L, Orlovich DA, Ashford AE (1998) *Fungal Genet Biol* 24:86–100.
7. Kamiya N, Kuroda K (1956) *Bot Mag Tokyo* 69:544–554.
8. Kachar B (1985) *Science* 227:1355–1357.
9. Kachar B, Reese TS (1988) *J Cell Biol* 106:1545–1552.
10. Chaen S, Inoue J, Sugi H (1995) *J Exp Biol* 198:1021–1027.
11. Yamamoto K, Hamada S, Kashiyama T (1999) *Cell Mol Life Sci* 56:227–232.
12. Sugi H, Chaen S (2003) *J Exp Biol* 206:1971–1976.
13. Shimmen T, Yokota E (2004) *Curr Opin Cell Biol* 16:68–72.
14. Haldane JBS (1985) *On the Importance of Being the Right Size* (Oxford Univ Press, Oxford).
15. Pickard WF (2006) *J Theor Biol* 240:288–301.
16. Hochachka PW (1999) *Proc Natl Acad Sci USA* 96:12233–12239.
17. Squires TM, Quake SR (2005) *Rev Mod Phys* 77:977–1026.
18. Reisen D, Marty F, Leborgne-Castel N (2005) *BMC Plant Biol* 5:13.
19. Wink M (1993) *J Exp Bot* 44:231–246.
20. Martinoa E, Maeshima M, Neuhaus HE (2007) *J Exp Bot* 58:83–102.
21. Houtman D, Pagonabarraga I, Lowe CP, Esseling-Ozdoba A, Emons AMC, Eiser E (2007) *Europhys Lett* 78:18001.
22. Nothnagel EA, Webb WW (1982) *J Cell Biol* 94:444–454.
23. Pickard WF (1972) *Can J Bot* 50:703–711.
24. Noguchi H, Gompper G (2004) *Phys Rev Lett* 93:258102.
25. Cutler SR, Ehrhardt DW, Griffiths JS, Somerville CR (2000) *Proc Natl Acad Sci USA* 97:3718–3723.
26. Yoneda A, Kutsuna N, Higaki T, Oda Y, Sano T, Hasezawa S (2007) *Protoplasma* 230:129–139.
27. Staves MP (1997) *Planta* 203:579–584.
28. Pickard WF (1974) *Protoplasma* 82:321–339.
29. Zabielski L, Mestel AJ (1998) *J Fluid Mech* 370:297–320.
30. Childress S, Landman M, Strauss H (1989) in *Proceedings of the IUTAM Symposium on Topological Fluid Mechanics*, eds Moffatt HK, Tsinober A (Cambridge Univ Press, Cambridge, UK), pp 216–224.
31. Meleshko VV, Malyuga VS, Gomilko AM (2000) *Proc R Soc London Ser A* 456:1741–1758.
32. Mustacich RV, Ware BR (1977) *Biophys J* 17:229–241.
33. Acrivos A, Taylor TD (1962) *Phys Fluids* 5:387–394.
34. Magar V, Goto T, Pedley TJ (2003) *Q J Mech Appl Math* 56:65–91.
35. Short MB, Solari CA, Ganguly S, Powers TR, Kessler JO, Goldstein RE (2006) *Proc Natl Acad Sci USA* 103:8315–8319.
36. Green PB (1954) *Am J Bot* 41:403–409.
37. Daley PF, Raschke K, Ball JT, Berry JA (1989) *Plant Physiol* 90:1233–1238.
38. Solari CA, Ganguly S, Kessler JO, Michod RE, Goldstein RE (2006) *Proc Natl Acad Sci USA* 103:1353–1358.
39. Dombrowski C, Cisneros L, Chatkaew S, Kessler JO, Goldstein RE (2004) *Phys Rev Lett* 93:098103.
40. Hindmarsh AC (1983) in *HIMACS Transactions on Scientific Computation*, eds Stepleman RS, Carver M, Peskin R, Ames WF, Vichnesvetsky R (North-Holland, Amsterdam), Vol 1, pp 55–64.