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Soft Matter and Biological Physics

SOLUTIONS for Sheet 1

Version: November 24, 2011

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Michaelmas 2010

Question 1: Optical Tweezers calibration

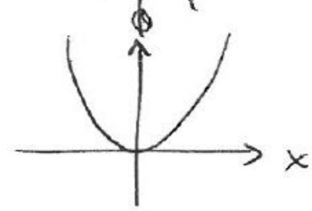
Solution:

Langevin eq.

$$m\ddot{x} + \beta\dot{x} + \kappa x = \mathcal{F}(t) \qquad (*)$$

Trap potential is harmonic for small Δx, $F = -\kappa \Delta x$

$$\phi = \frac{1}{2}\kappa x^2$$



Main forces due to laser

Scattering force, gradient force

other forces:

thermal motion, friction

$m\ddot{x}$ can be neglected at time scales longer than relaxation time of colloid which is fulfilled for optical traps (low Reynolds number)

Calculate Power spectrum

$$\beta\dot{x} + \kappa x = \mathcal{F}(t)$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} dt$$

$$\frac{dX(f)}{dt} = -2\pi i f \, X(f)$$

Fourie transform of (*)

$$-2\pi i f \beta X(f) + \kappa X(f) = \mathcal{F}(f)$$

$$2\pi\beta \left(\underbrace{\frac{\kappa}{2\pi\beta}}_{f_c} - i f \right) X(f) = \mathcal{F}(f)$$

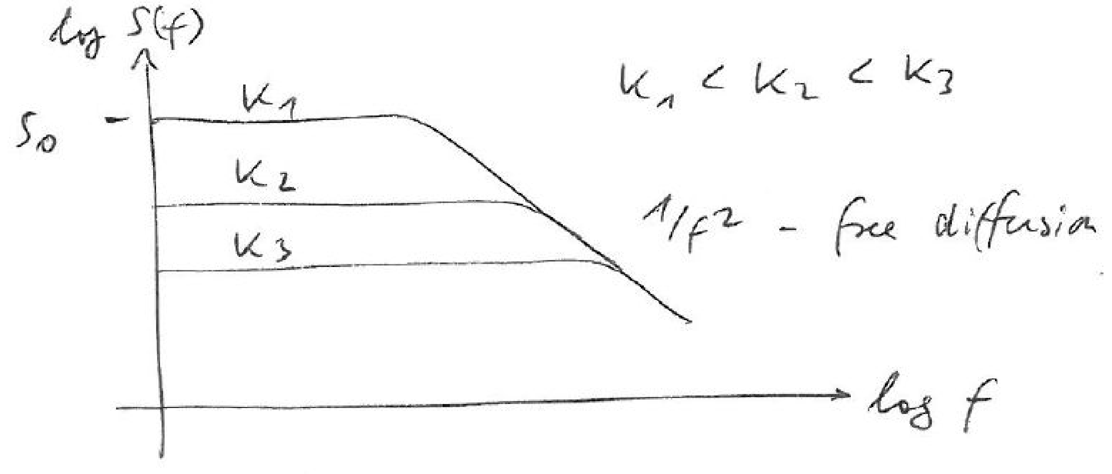
take $| \ |^2$ of both sides

$$4\pi\beta^2 \left(f_c^2 + f^2 \right) S_x(f) = S_{\mathcal{F}}(f)$$

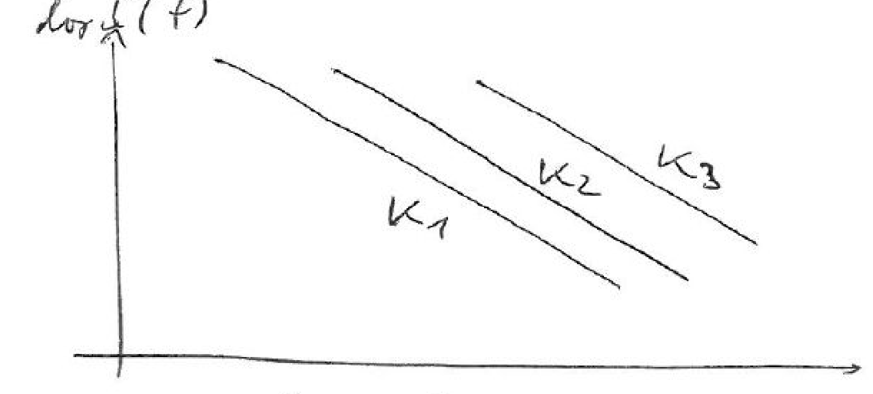
with $S_{\mathcal{F}}(f) = 4\beta k_B T$ we get

$$S_x(f) = \frac{k_B T}{\beta \pi^2 \left(f_c^2 + f^2 \right)}$$

Sketch $S_x(f)$



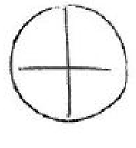
$f > f_c$



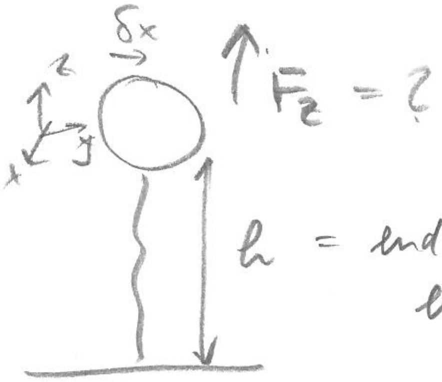
high frequency part

due to laser heating in trap at IR wavelength. $\eta(T)$ viscosity of T.

Two techniques for detection

QPD  and video detection by correlation function

Q2 Sheet 3 2012



h = end-to-end distance of entropic spring

inverted pendulum, restoring force given by entropic force $\frac{F}{h}\delta x$

DNA stiffness $k_x = \frac{F}{h} \Leftrightarrow F = k_x \cdot h$

Equipartition $\frac{1}{2} k_x \langle \delta x^2 \rangle = \frac{1}{2} k_B T$

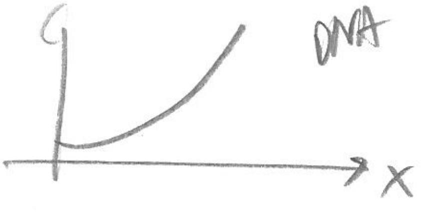
$$k_x = \frac{k_B T}{\langle \delta x^2 \rangle}$$

$\Rightarrow$ Force is $F = \dfrac{k_B T h}{\langle \delta x^2 \rangle}$

Measure fluctuations and extract force from fluctuations

stiffer DNA $\Leftrightarrow$ smaller fluctuations

Protocol measure fluctuations at range of positions $\Rightarrow$ Force distance relationship!



DNA

WLC ...

Q 13

(i) from lecture (Nelson)

$$E(R) = \left(\frac{k_B T A}{2}\right)\left(\frac{\pi}{2R}\right) \qquad 90° \text{ bend}$$

complete loop

$$E(R) = \frac{k_B T A \pi}{R} \qquad 360° \text{ bend}$$

applied force f

$$\Rightarrow \quad E(R) = \frac{k_B T A \pi}{R} + \underbrace{2\pi R f}_{\text{work against outside force}}$$

(ii)

$$\frac{dE(R)}{dR} \overset{!}{=} 0 = -\frac{k_B T A \pi}{R^2} + 2\pi f$$

$$\Rightarrow \quad R^* = \sqrt{k_B T A / 2f}$$

$f = 1\,pN$, $A = 50\,nm$, $T = 300K \Rightarrow$

$$2\pi R \approx 60\,nm$$

(iii) Torsional energy

$$E_T = \frac{1}{2}\underbrace{k_B T C \theta^2}_{L} = \underbrace{\frac{1}{2} k_B T C (2\pi n)^2}_{L}$$

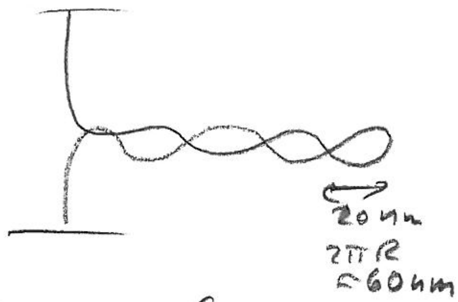
$$\Delta E_T = \frac{1}{2} k_B T C\, 4\pi^2 \left((n+1)^2 - n^2\right) / L$$

for going from $n \to n+1$ turns

$$\approx 2\pi^2 k_B T C (2n) / L \text{ for large } n$$

$$\Rightarrow \Delta E_T \simeq 4\pi^2 k_B T C n / L$$

what makes sense



if $\Delta E_T > E(R^*)$ plectoneme forms

$$\frac{4\pi^2 k_B T C n_{buck}}{L} = \frac{k_B T A \pi}{R^*} + 2\pi R^* f$$

(iv) critical torque is

$$2\pi \Gamma_{buck} = \frac{k_B T A \pi}{R^*} + 2\pi R^* f$$

$$\Gamma_{buck} = \frac{k_B T A}{2 R^*} + R^* f$$

$R^* = 10\,nm$, $f = 1\,pN$, $A = 50\,nm$

$$\Gamma_{buck} \simeq 2 \cdot 10^{-20}\ Nm$$

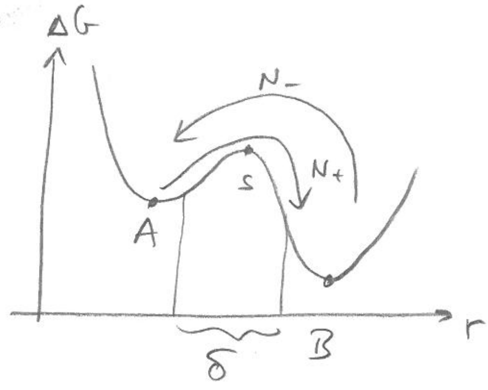
at $\Gamma > 3.4 \cdot 10^{-20}\ nm$ p-DNA can be formed!

Q4, Sheet 3, 2012

N_+ $\rightarrow$ molecules move towards B

N_- $\rightarrow$ towards A

total concentrations [A]



total number of molecules in δ

$$N_g = N_+ + N_- = k\,[A]$$

assumption: no trajectory changes direction in δ

equilibrium: net flux vanishes

$$N_+ - N_- = 0 \Rightarrow \underline{\underline{N_+}} = \underline{N_-} = \frac{1}{2} N_g = \underline{\underline{\frac{1}{2} k [A]}}$$

in equilibrium

$$\frac{dN}{dt} = \frac{[\text{No. of reactions}]}{\text{volume time}} = N_+ \frac{\bar{v}}{\delta}$$

$\bar{v}$ is drift velocity in δ $\Rightarrow$ $\dfrac{dN}{dt} = \dfrac{1}{2} k [A] \dfrac{\bar{v}}{\delta}$

---

now investigate μ:

$$Z = \prod_i \frac{z_i^{N_i}}{N_i!}$$

$\uparrow$ molecular species "i"

use Stirling approximation (as usual) to get μ_A

$$\ln \frac{z_A^{N_A}}{N_A!} = N_A \ln z_A - \ln N_A!$$
$$= N_A \ln z_A - N_A \ln N_A - N_A$$
$$\Rightarrow \frac{\partial}{\partial N_A}\left(\frac{z_A^{N_A}}{N_A!}\right) \approx \ln \frac{z_A}{N_A}$$

$$\mu_A = \mu_s \Rightarrow \mu_j = -k_B T \ln\left(\frac{z_j \exp\left(-\frac{E_j}{k_B T}\right)}{N_j}\right)$$

$$\ln\left(\frac{Z_A \exp\left\{-\frac{E_A}{k_BT}\right\}}{N_A}\right) = \ln\left(\frac{Z_S \exp\left\{-\frac{E_S}{k_BT}\right\}}{N_S}\right)$$

$$k = \frac{N_S}{N_A} = \frac{Z_S}{Z_A}\exp\left\{\frac{-E_S+E_A}{k_BT}\right\} = \frac{Z_S}{Z_A} e^{-E_0/k_BT}$$

rate depends on relative occupation in N_S, N_A

energy difference between A, S

now we need $Z_S = Z_* Z_r$

simply integrate

$$Z_r = \frac{1}{h}\int_{-\delta/2}^{\delta/2} dx \underbrace{\int_{-\infty}^{+\infty} dp \exp\left(\frac{-p^2}{2m_r k_BT}\right)}$$

$$Z_r = \frac{\delta}{h}\sqrt{\pi}\sqrt{2m_r k_BT}$$

$$\Rightarrow \frac{dN}{dt} = \frac{1}{2}\frac{Z_* Z_r}{Z_A} e^{-E_0/k_BT}[A]$$

$$= \frac{1\cancel{\delta}}{2h}\frac{Z_*}{Z_A}\sqrt{\cancel{2\pi m_r} k_BT}\, e^{-E_0/k_BT}[A]\frac{v}{\cancel{\delta}}$$

$$v = \sqrt{\frac{2k_BT}{\pi m_r}}$$

$$= \underbrace{\frac{k_BT}{h}\frac{Z_*}{Z_A} e^{-E_0/k_BT}}_{k}[A]$$

$$v_0 = \frac{k_BT}{h}\frac{Z_*}{Z_A} \quad \square$$

Electro osmotic flow

for $r \gg \lambda$ one can write Poisson-Boltzmann and velocity

$$\eta \frac{d^2 v}{dx^2} = \varepsilon_0 \varepsilon_r \frac{d^2 \psi}{dx^2} E$$

ψ surface potential, thin double layer

$\Leftrightarrow$ η ε_r E are constant

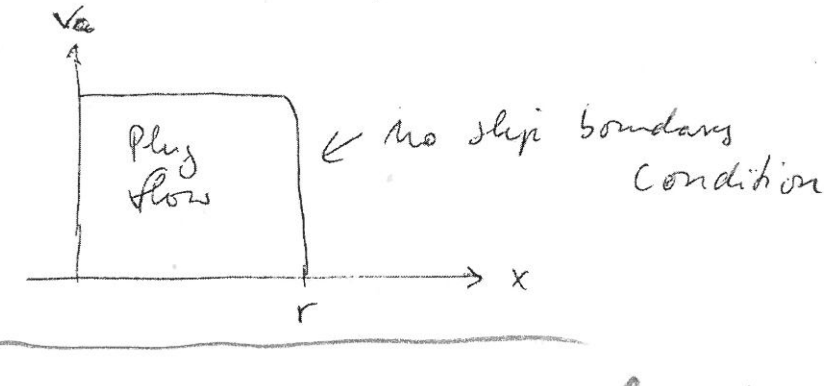
$$\eta \frac{dv}{dx} = \varepsilon_0 \varepsilon_r \frac{d\psi}{dx} E + A$$

$$\eta \left[v \right]_0^{v_e} = \varepsilon_0 \varepsilon_r \left[\psi \right]_\zeta^{\psi_e} + A x$$

$$\eta v_c = \varepsilon_0 \varepsilon_r (\psi_e - \zeta) + A_1 x$$

A_1 has to ~~be~~ 0 otherwise pressure driven flow, $\psi_c = 0$ outside of double layer

$$\Rightarrow \quad v_c = -\frac{\varepsilon_0 \varepsilon_r}{\eta} \zeta \quad \square$$



if $r \gtrsim \lambda_{DH}$ $\Leftrightarrow$ $v_0 \neq$: Helmholtz Smoluchowski

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Soft Matter and Biological Physics

Question Sheet 3

Michaelmas 2010

Version: November 14, 2011

Question 9: Gel Electrophoresis

Solution:

Three hopefully correct and readable scans.

Q 11

(i) Stokes - Einstein

$$\frac{k_B T}{\beta} = D \text{ with } \beta = \eta N b$$

we get

$$D = \frac{k_B T}{\eta N b}$$

and since we can ignore the interaction with the gel we get for the time τ to diffuse Nb

$$N^2 b^2 = D \tau$$

$$\Rightarrow \tau = \frac{N^2 b^2}{D} = \frac{\eta N b}{k_B T} N^2 b^2 = \eta \frac{N^3 b^3}{k_B T} \quad \square$$

put in numbers $L = 30{,}000 \cdot 0.34\,\text{nm} \approx 10\,\mu\text{m}$

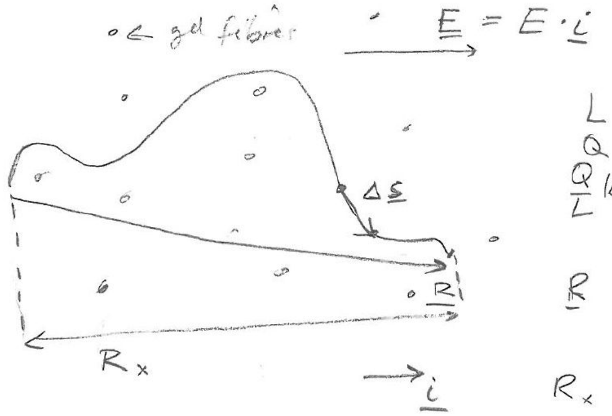
$b = 100\,\text{nm}$ (Kuhn segment)

$$\Rightarrow 100 \cdot 100\,\text{nm} = N \cdot b = L$$

$$\Rightarrow \tau \approx 240\,\text{s}$$

Q11

(ii)



$L = Nb$ contour length

Q total charge

$\frac{Q}{L}|\Delta \underline{s}|$ charge on segment $\Delta \underline{s}$ $= \frac{Qb}{L}$

$\underline{R}$ end-to-end distance

R_x in x direction

Force on one segment $\underline{F}_1 = \frac{Q}{L} \Delta s \underline{E}$

Chain can only move along its axis and mainly in x-direction $\Rightarrow$ project $\underline{\Delta s} \cdot \underline{E}$ $\Rightarrow$

$$F_N = \frac{Q}{L} \sum_1^N \underline{E} \cdot \Delta \underline{s}_j = \frac{QE}{L} \sum \underline{i} \cdot \Delta \underline{s}$$

$$\Rightarrow F_{N,x} = \frac{QE}{L} R_x = \beta_{DNA} v = \beta_{DNA} \dot{s}$$

$F_{N,x}$ is the total axial force on the chain in direction of $\underline{E}$. So the velocity of the chain along its contour is

$$\dot{s} = \frac{QER_x}{\beta L} \quad (*)$$

We need now the movement of the centre of mass:

$$M \underline{R}_{cm} = \sum_{i=1}^N m_i \underline{r}_i$$

With mass per unit length $m_i = M \frac{\Delta s_i}{L}$ and using that ith segment is $\dot{\underline{r}}_i = \dot{s} \underline{t}_i$

↖ tangent vector $|\underline{t}_i| = 1$

$$\cancel{M} \dot{\underline{R}}_{cm} = \sum_{i=1}^N \left(\cancel{M} \frac{\Delta s_i}{L} \dot{s} \underline{t}_i \right)$$

$$\dot{\underline{R}}_{cm} = \frac{\dot{s}}{L} \sum \underline{t}_i \cdot \Delta s_i = \frac{\dot{s}}{L} \underline{R}$$

we just want the movement in x

$\Rightarrow \quad \dot{x}_{cm} = \frac{\dot{s}}{L} R_x$

to get v_{drift} we average and use (*) (around)

$$\dot{x}_{cm} = \frac{\dot{s} R_x}{L} \qquad \dot{S} = \frac{QE\,R_x}{\beta L}$$

$$= \frac{QER_x^2}{\beta L^2} \Rightarrow \langle \dot{x}_{cm} \rangle = \frac{QE \langle R_x^2 \rangle}{\beta L^2}$$

$$v_{drift} = \langle \dot{X}_{cm} \rangle = \frac{Q \cdot \cancel{L} \cdot b}{\beta L^{\cancel{2}}} \cdot E = \frac{Q \cdot b}{L} \cdot E \frac{1}{\beta}$$

$\langle R_x^2 \rangle \approx N b^2$ ignoring pre-factors

force on one monomer $f = \frac{Q \cdot b}{L} \cdot E$ $\overset{!}{=} \frac{F}{\beta}$ $\square$

$F = Nf$

$\beta = \eta N b$

=

$v_{drift} = \frac{10\,cm}{1\,hour}$

$\Rightarrow \quad E \approx 3000\ V/m$ for 30,000 bp DNA

at same voltage:

$v_{drift} = \frac{F}{Nb\eta} \approx 3.33 \cdot 10^{-5} \frac{m}{s}$

$L = 25{,}000$ bp $\hat{\approx} 8.5\,\mu m$

$\Rightarrow L_{25kb} \approx 12$ cm $\square$ in 1 hour

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Question 7: Polymers in Confinement

Solution:

(i) The electrostatic potential in the channel is $V(x) = -Ex$ so we can easily write down the energy for the molecule when entering the channel is

$$\Delta U(x) = -\rho E \int_0^x x dx = -\frac{1}{2}\rho E x^2$$

i believe that we do not have to mention that $U(x = 0) = 0$ as a boundary condition. This is of course a major simplification as there will be a finite electric field outside of the channel due to the access resistance.

(ii) First explain the sign: the DNA has to be straight inside the channel so fewer configurations are available so entropy is lower in this state with $\Delta S < 0$.
Dependence on x: entropy is an extensive quantity and therefore proportional to the length L of the strand. Since the DNA in the cavity has a fixed configuration (i.e. no entropy), we have $S = S(L)(1 - x/L$ so obviously $\Delta S \propto -x$. Well in principle this explains also the sign. I believe it is good to split this into two discussions - but do as you see fit.
Remark: In the event that there are questions about entropy as extensive quantity for WLC polymers ... The number of possible configurations will be

$$Z_N = \text{(No. of configs/link1)(No. of configs/link2)...(No. of configs/linkN)} = (\text{const})^N$$

So we get for the entropy

$$S = k_B \ln(Z_N) = N k_B \ln(\text{const}) \Leftrightarrow S \propto N$$

(iii) Now we can write down ΔG as was asked in the question as

$$\Delta G = -\frac{1}{2}\rho E x^2 + \gamma T x$$

A sketch will look like the plot shown below. Of course the position of the barrier will depend on all parameters as expected from the question. Calculate the stationary points of ΔG:

$$\frac{d\Delta G}{dx} = -\rho E x_c = \gamma T = 0$$

$$x_c = \frac{\gamma T}{\rho E}$$

This is expected since at higher fields the 'barrier'/maximum gets closer and closer to the channel entrance - and smaller of course. No we can get the height:

$$\Delta G^* = \Delta G(x_c) = \frac{\gamma^2 T^2}{2\rho E}$$

(iv) As T is increased the entropic energy cost for reaching the transition state increases while the electrostatic energy remains unchanged. In other words, the entropic chain gets stiffer with increasing temperature resisting the pulling force.

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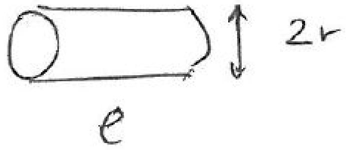
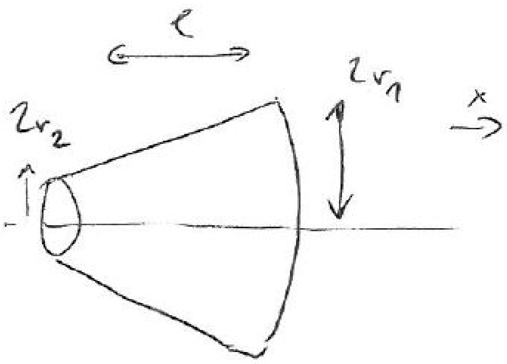
Question 12: Nanopores

This is the solution for (i) and (iii), (ii) is just putting in numbers, (iv) is described in Smeets et al. Nano Letters 2006 http://www.bss.phy.cam.ac.uk/ ufk20/teaching/pdf/smeets.pdf:



$$R_{tot} = \rho \overbrace{\left(\frac{2}{4r}\right)}^{\text{access} \times 2} + \rho \frac{l}{\pi r^2}$$

$$= \frac{\rho}{r^2}\left(\frac{1}{2}r + \frac{l}{\pi}\right)$$



$$r(x) = \frac{x}{l}\left(r_1 - r_2\right) + r_2$$

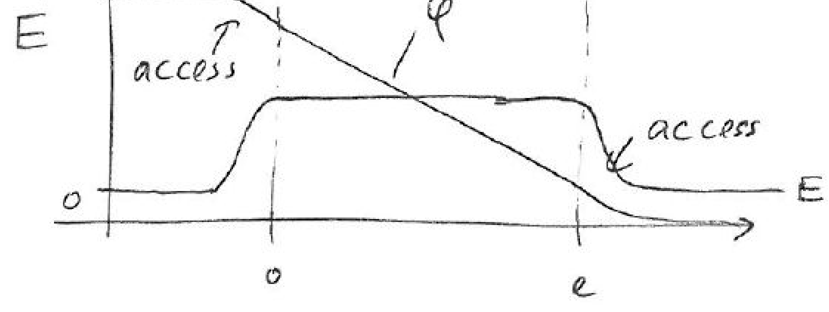
$$R_{cone} = \rho \int_0^l \frac{1}{\pi\left(\frac{x}{l}(r_1-r_2)+r_2\right)} d$$

$$= \frac{\rho}{\pi}\left[-\frac{1}{\frac{r_1-r_2}{l}\left[\frac{x}{l}(r_1-r_2)+r_2\right.}\right]_0^l$$

$$= \frac{\rho l}{\pi}\left[-\frac{1}{(r_1-r_2)^2+(r_1-r_2)r_2} + \frac{1}{(r_1-r_2)r_2}\right]$$

$$= \frac{\rho l}{\pi}\left[\frac{(r_1-r_2)^2}{r_1 r_2 (r_1-r_2)^2}\right] = \frac{\rho l}{\pi}\frac{1}{r_1 r_2}$$

$$R_{tot} = \frac{\rho l}{\pi}\frac{1}{r_1 r_2} + \rho\left(\frac{1}{2r_1} + \frac{1}{2r_2}\right)$$



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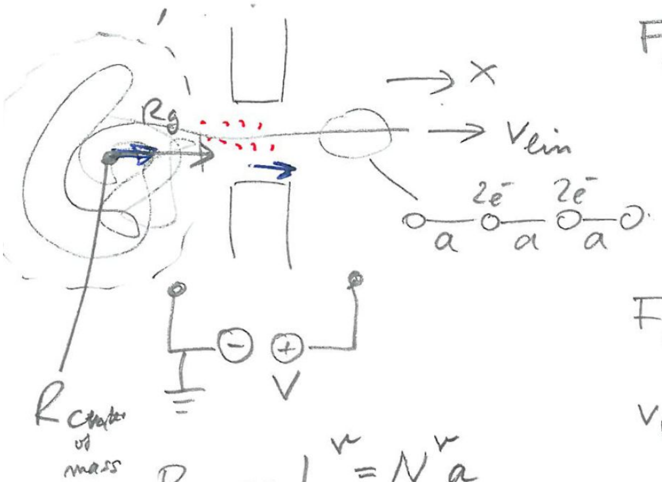
Question 9: Polymer translocation
This solution is described in Storm et al. Nano Letters 2005 http://www.bss.phy.cam.ac.uk/ ufk20/teaching/pdf/storm.pdf. The second part is added just for information.

$\sim$ means $\propto$ prop to!

Fast translocations



translational invariance

$$F_{drive} = \int_{-\infty}^{\infty} \frac{q}{e} E(x)\, dx$$

$$= \frac{q}{a} \Delta V = 110\, pN$$

$$F_{drag} = 6\pi \eta R_g v_{blob}$$

$$v_{blob} = \dot{R}_g = \frac{dR_g}{dt}$$

$$R_g \sim L^{\nu} = N^{\nu} a$$

$$\frac{F_{friction}}{\text{unit length}} \approx \frac{2\pi r v}{R - r} \approx \frac{0.3\, pN}{\text{unit length}}$$

Friction within pore length pore $\ell \ll Na$ much smaller than F_{drag}, F_{drive}

Force balance in steady stat situation $F_{drag} + F_{drive} = 0$ (*)

assum ν = const

$$v_{blob} \propto \dot{R}_g \propto L^{\nu-1} \dot{L}$$

$$F_{drag} \propto R_g \dot{R}_g \propto L^{2\nu-1} \dot{L} \propto -F_{drive} = \text{const}$$

$$V_{DNA} = -\dot{L} \quad \text{with } (*) \Rightarrow -\dot{L} = v_{el} = L^{1-2\nu}$$

Dwell time: $\tau = \int_0^{\tau_D} dt = \int_{L(0)}^{L(\tau)} \frac{dt}{dL}\, dL = -\int_{L_0}^{0} L^{2\nu-1}\, dL$ ($\frac{dt}{dL}$: v^{-1})

$$\Rightarrow \tau \propto L_0^{2\nu} \qquad L_0 = Na \text{ contour length}$$

scaling law, we do not care about prefactors but scaling with L_0

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Question 10: Nanopores II

Solution:

(i) resistive-pulse sensing: label-free detection, detection of molecule by current change through nanopore, detection limit given by diameter, length of nanopore sensor

(ii) calculate resistance for one half of the nanopre and multiply by 2. Parametrization of edge: $d(z) = a + z\frac{b-a}{l}$ and thus calculate resistance R

$$R = 2\rho \int_0^l \frac{1}{\pi d(z)^2} dx$$

after some algebra you should get

$$R = 2\frac{\rho l}{\pi}\frac{1}{ab}$$

since we only look for the resistance from 0 to $2l$ we do not have to mention the access resistance here.

(iii) Calculate the current density $j = I/A = U/(RA)$ you should get two j for 0 to l and l to $2l$ yielding

$$j_1 = \frac{Uab}{2\pi(a + z(b-a)/l)^2\rho l}$$

and

$$j_2 = \frac{Uab}{2\pi(b + (z-l)(a-b)/l)^2\rho l}$$

and the electric field is just $E(z) = j/\rho$ for the two cases. Sketch $E(Z)$ with basically meeting at the center and symmetrically increasing towards both ends. The corresponding ionic current sketch should show to peaks in the current.

The time of flight is simply calculated by

$$\int_{t_1}^{t_2} dt = \frac{1}{\mu}\int_0^{2l} \frac{1}{E(z)} dz$$

with the electric fields from above one gets for the first half 0 to l

$$t = \frac{l}{\mu 3(b-a)}(b^3 - a^3)$$

and for the second part l to $2l$ obviously the same and thus

$$\Delta t = \frac{2\pi l}{\mu(a-b)}(a^3 - b^3)$$

as expected.

Finally for $L << l$ pore-molecule interaction should increase time of flight as well as the entropic barrier for entering the exit constriction. For $L >> l$ correlation between entrance and exit possible, time of flight is dominated by stiffness of molecule, i.e. no relaxation in cavity before 2nd translocation.

Experimental data can be found in paper from Pedone et al. Nano Letters 11 (4) 15611567 (2011).

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Question 11: Thermodynamics of the ATP synthase molecular motor

Q14(i) (Nelson)

change in free energy

$$\Delta G_{proton} = k_BT \ln(c_{in}/c_{out}) + e\Psi_m$$

to make an ATP molecule

$$\Delta G_{ADP\to ATP} = \Delta G^0 + k_BT\left(\ln(c_{ATP}/c^0) - \ln(c_{ADP}/c^0) - \ln(c_P/c^0)\right)$$

(I use $\Delta G^0 > 0$ since this is stored energy. $\Delta G^0 < 0$ also possible)

total change in free energy

$$\Delta G_{tot} = N\Delta G_{proton} + \Delta G_{ADP-ATP}$$

For a fully reversible process $\Delta G_{tot} = 0$

$\Rightarrow$ so combining all from above yields

$$\Delta G^0 = -N(k_BT \ln(c_{in}/c_{out}) + e\Psi_m) - k_BT\left(\ln(c_{ATP}/c^0) - \ln(c_{ADP}/c^0) - \ln(c_P/c^0)\right)$$

$$= k_BT \ln\left(c_{out}^N c_{ADP} c_P / c_{in}^N c_{ATP} c^0\right) - Ne\Psi_m$$

(ii) irreversible process $\Delta G_{tot} < 0$

redo calculation

$$\Delta G^0 < k_BT \ln\left(c_{out}^N c_{ADP} c_P / c_{in}^N c_{ATP} c^0\right) - Ne\Psi_m$$

The answer in (i) is an upper bound. This is intuitive since some energy is lost in the process due to dissipation $\Leftrightarrow$ less energy stored in ATP

Question 12

[bookwork]

Lipid membrane provides scaffold for motor • and provide barrier for ion/protons •
Membrane potential and proton gradient are used to create rotary motion, $Energy = \Delta\Psi_{total} \cdot e$
• •

[4]

[bookwork]

Use formula

$$\Delta\mu_p = e\Delta\Psi - k_BT \ln \frac{[H_o^+]}{[H_i^+]} \quad \bullet$$

$$\Delta\mu_p = 0 \Rightarrow k_BT \ln \frac{[H_o^+]}{[H_i^+]} = e\Delta\Psi$$

$$[H_o^+] = e^{\frac{e\Delta\Psi}{k_BT}} \cdot [H_i^+]$$

$$\overset{!}{=} e^{-4.65} [H_i^+]$$

$$= 1.14 \cdot 10^{14} \quad \bullet$$

$$\Rightarrow [H_o^+] = 9.7 \quad \bullet$$

[3]

[partly bookwork]

Rotation direction

G_+, G_- find ΔG •

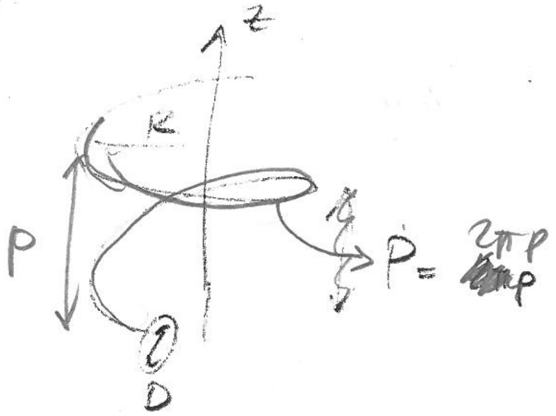
$T = 37°C$ $p_+ = 0.01$
$p_- = 0.99$

$$\Rightarrow \frac{p_-}{p_+} = e^{\Delta G / k_BT} \quad \bullet$$

$$\Rightarrow \Delta G \approx 4.6\, k_BT$$

[3]

$$\approx 1.96 \cdot 10^{-10}\, J \quad \bullet$$



[partly new]

parameterization

$x = R\sin(\varphi t)$

$y = R\cos(\varphi t)$

$z = \rho\varphi$

$s = t\sqrt{R^2+\rho^2}$

curvature is constant

$$\varphi = \frac{t}{\sqrt{R^2+\rho^2}} \qquad ds = dt\sqrt{R^2+\rho^2}$$

outside filament $S_o^2 = 4\pi^2\left(\left(R+\frac{D}{2}\right)^2+\rho^2\right)$

$$S_i^2 = 4\pi^2\left(\left(R-\frac{D}{2}\right)^2+\rho^2\right)$$

$$S_o^2 - S_i^2 = 4\pi^2\left(\cancel{R^2}+2R\frac{D}{2}+\cancel{\frac{D^2}{4}}+\rho^2-\cancel{R^2}+2R\frac{D}{2}-\frac{D^2}{4}-\rho^2\right)$$

$$\overset{!}{=} 4\pi^2\left(4R\frac{D}{2}\right)$$

$$S_o^2 + S_i^2 = 4\pi^2\left(2R^2+2\frac{D^2}{4}+2\rho^2\right)$$

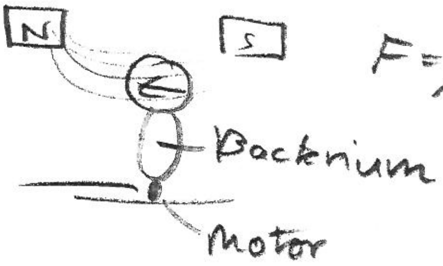
$$\frac{S_o^2-S_i^2}{S_o^2-S_i^2} = \frac{\cancel{4\pi^2}\,\overset{2}{\cancel{4}}R\frac{D}{\cancel{2}}}{\cancel{4\pi^2}\left(\cancel{2}R^2+\cancel{2}\frac{D^2}{4}+\cancel{2}\rho^2\right)} = \frac{2RD}{R^2+\frac{D^2}{4}+\rho^2} =$$

$$D \ll R,\rho \quad \overset{!}{=} \frac{2RD}{R^2+\rho^2}$$

[9]

[partly bookwork]

magnetic tweezers, attach super paramagnetic bead to bacteria



$F = \mu \operatorname{grad} B$

$\underbrace{\text{rotation}}_{\text{of asymmetric particle}}$ can be measured, torque $\underline{\underline{\text{not}}}$

[4]

$M_B(t) = -\gamma_r \omega(t)$ [partly bookwork]

$\xi(t)$ not important as driving by potential difference provides $\simeq 8\,k_B T$

measure rotation at $M_{ex} = 0$, $\omega_0 = -\dfrac{M_B(t)}{\gamma_r}$ •

measure rotation at $M_{ex} = 0$, ω

measure rotation after motor is broken •
and MT dominates

$$\omega_2 = -\frac{M_{ex}(t)}{\gamma_r} \;\bullet$$

$$\omega = \frac{M_B + M_{ex}}{\gamma_r} \;\bullet \qquad \omega_0 = \frac{M_B}{\gamma_r} \Rightarrow \frac{\omega - \omega_0}{\omega_0} = \frac{M_B(\omega)}{M_B(\omega_0)}] \;\bullet$$

even data from motors with different torque etc. can be compared •

[6]

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Soft Matter and Biological Physics

SOLUTIONS for Sheet 1

Version: November 27, 2012
PLEASE DO NOT DISTRIBUTE -for supervisors only - send typos to ufk20

Michaelmas 2012

Revision Question: Linearized Poisson-Boltzmann Equation
Solution:

Poisson - Boltzmann in 1D

$$\frac{d^2\varphi}{dx^2} = -\frac{1}{\varepsilon_0 \varepsilon_r} \sum_{\text{species } i} z e c_0^i \, e^{-\frac{z e \varphi(x)}{kT}}$$

monovalent ions $c_0 = c_0^+ = c_0^-$ $z_0^i = \pm 1 \cdot e$

$$\frac{d^2\varphi}{dx^2} = -\frac{1}{\varepsilon_0 \varepsilon_r} e \, c_0 \left(e^{-\frac{e\varphi(x)}{kT}} - e^{\frac{e\varphi(x)}{kT}} \right)$$

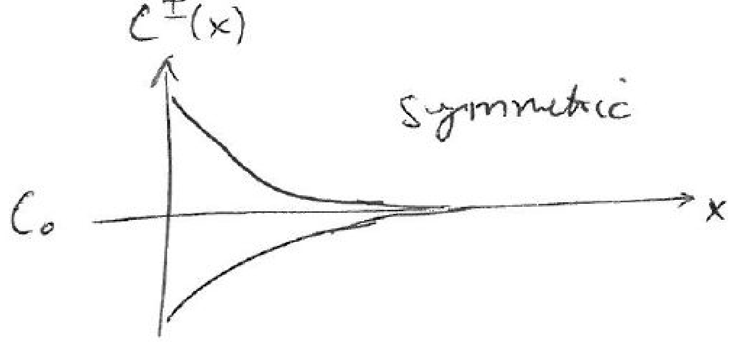
$$= \frac{2 e c_0}{\varepsilon_0 \varepsilon_r} \sinh\left(\frac{e\varphi(x)}{(kT)}\right) \underset{\text{small } e\varphi(x)}{\approx} \frac{2 e c_0}{\varepsilon_0 \varepsilon_r} \frac{e\varphi(x)}{kT}$$

Solution $\varphi(x) = A e^{-x/\lambda} + B e^{x/\lambda}$

$B = 0$ $A = \varphi_0$

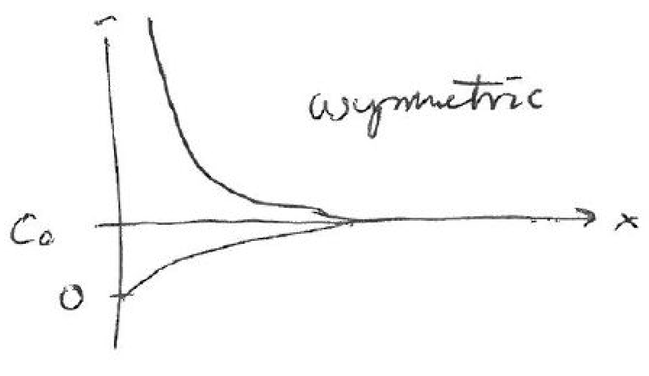
$$1/\lambda^2 = \frac{2 e^2 c_0}{\varepsilon_0 \varepsilon_r kT} \Rightarrow \lambda = \sqrt{\frac{\varepsilon_0 \varepsilon_r kT}{2 e^2 c_0}}$$

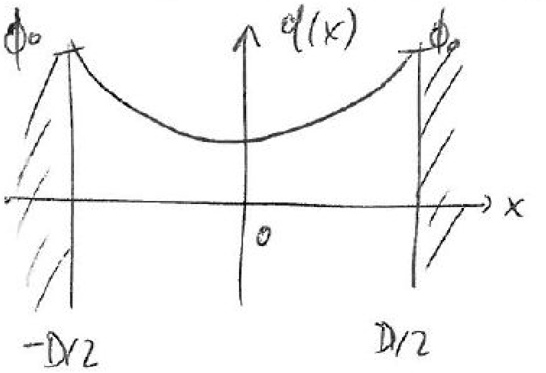
$$\lambda \propto \sqrt{\frac{1}{c_0}}$$



$e\varphi(x) << kT$

$e\varphi(x) > kT$





linearized PB as above different boundary

$$\varphi(x) = A e^{-x/\lambda} + B e^{x/\lambda}$$

$$\Rightarrow A = B = \frac{\varphi_0}{2\cosh(D/2\lambda)}$$

$$\Rightarrow \varphi(x) = \frac{\varphi_0 \cosh(x\lambda)}{\cancel{2}\cosh(D/2\lambda)}$$

$D \to \infty \quad , \quad \varphi(0) = 0$

$D \to 0 \quad , \quad \varphi(0) = \varphi_0$

added pressure:

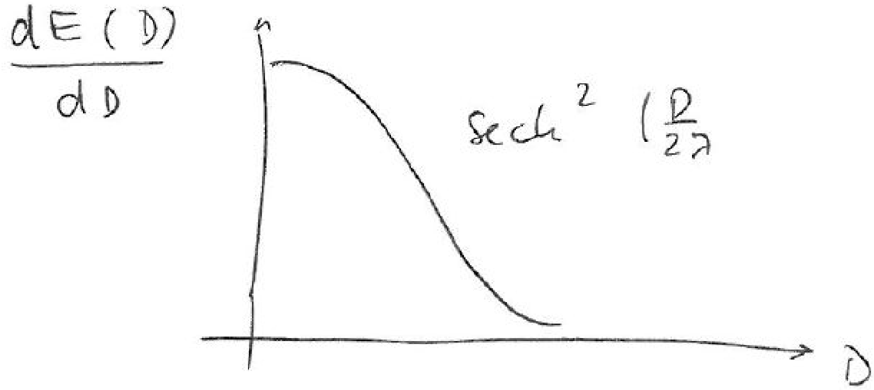
$E(D)$ per unit area

$$E(D) = \frac{1}{2}\int_{-D/2}^{D/2} e c_0 \varphi(x)\, dx$$

$$= \frac{e c_0 \varphi_0 \lambda}{2\cosh(D/2\lambda)} \Big[\sinh\left(\frac{x}{\lambda}\right)\Big]_{-D/2}^{D/2}$$

$$= c_0 e \lambda \varphi_0 \tanh\left(\frac{D}{2\lambda}\right)$$

added pressure is $\dfrac{dE(D)}{dD} = c_0 e \varphi_0 \,\mathrm{sech}^2\left(\dfrac{D}{2\lambda}\right)$



$\frac{dE(D)}{dD}$
$\mathrm{sech}^2 (\frac{D}{2\lambda}$
D