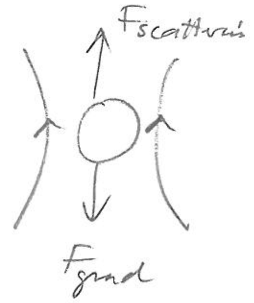
Q1 Michaelmas 2014

$m\ddot{x} + \gamma\dot{x} + \kappa x = \xi(t)$

main forces F_{grad}, $F_{scattering}$

other forces, thermal, viscous



(1) $m \ll \gamma$, long times $\Rightarrow m = 0 \Rightarrow \gamma\dot{x} + \kappa x = \xi(t)$

(1) $\gamma\dot{x} + \kappa x = \xi(t)$

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{+2\pi i f t}\, dt$$

$$\frac{d}{dt}\left[x(t)\right] = \int_{-\infty}^{\infty} x(f) \frac{d}{dt} e^{-2\pi i f t}\, df = -\int_{-\infty} \underbrace{x(f)\, 2\pi i f}_{} e^{-2\pi i f t}\, df$$

Fourier Transform of $\frac{d}{dt} x(t)$

$= -2\pi i f x(f)$

$\Rightarrow$ Fourier transform of (1)

$$\gamma\left(-2\pi i f\, x(f) + \kappa\, x(f)\right) = \xi(f)$$

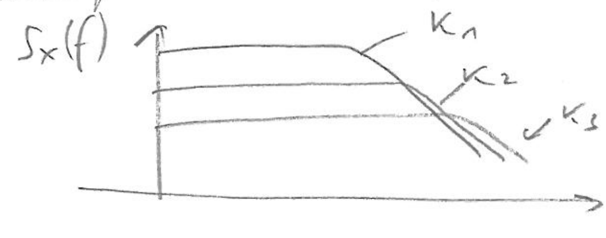
$$\gamma^2\left(4\pi^2 f^2 |X(f)|^2 + \kappa^2 |X(f)|^2\right) = |\xi(f)|^2$$

$$4\pi^2\gamma^2 |X(f)|^2 \left[f^2 + \underbrace{\frac{\kappa^2}{4\pi^2\gamma^2}}_{f_c^2}\right] = 4\gamma k T$$

$$\Rightarrow \quad S_x(f) = |X(f)|^2 = \frac{kT}{\pi^2 \gamma (f_c^2 + f^2)}$$

$\square$

Technique: video, quadrant photo, ...



heating due to absorption

$\gamma = 6\pi\eta(T)R$

$\kappa_3 > \kappa_2 > \kappa_1$

$T_3 > T_2 > T_1$

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Q2

Force on Dipole in static magnetic field:

$$\underline{F} = \nabla \times (\underline{B} \times \underline{\mu})$$

$$\underline{F} = (\underline{\mu} \cdot \nabla)\underline{B} - \underbrace{(\nabla \cdot B)}_{=0}\underline{\mu} = (\underline{\mu} \cdot \nabla)\underline{B}$$

For stationary magnetic fields $\nabla \times \underline{B} = 0$
time constant

$\underline{\mu} = \text{constant}$

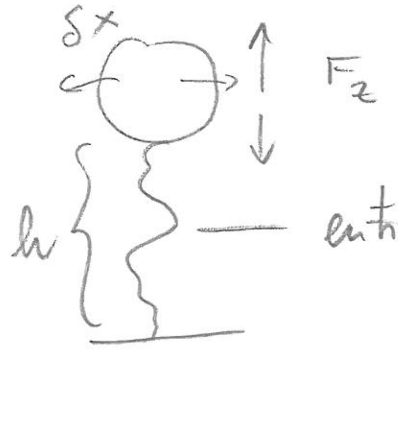
$$(\underline{\mu} \cdot \nabla) B_e = \sum_{i=1}^{3} \mu_i \partial_i B_e = \sum_{i=1}^{3} \mu_i \partial_e B_i = \left[\nabla(\underline{\mu} \cdot \underline{B})\right]_e$$

$$\partial_i B_e = \partial_e B_i$$

$$\underline{F} = \nabla(\underline{\mu} \cdot \underline{B})$$

Force along z $\quad F_z = \frac{\partial}{\partial z}(\underline{\mu} \cdot \underline{B})_z$

as in Lectures



— entropic spring $\quad F = -k_{DNA} h$

Energy $\frac{1}{2} k \langle \delta x^2 \rangle = \frac{1}{2} k_B T$

$$k = \frac{k_B T}{\langle \delta x \rangle^2}$$

$$\Rightarrow F = -\frac{k_B T h}{\langle \delta x^2 \rangle}$$

Force depends on restoring force to center $\Leftrightarrow$ force can be determined by $\langle \delta x^2 \rangle$

Q 13

(i) from lecture (Nelson)

$$E(R) = \left(\frac{k_B T A}{2}\right)\left(\frac{\pi}{2R}\right) \qquad 90° \text{ bend}$$

complete loop

$$E(R) = \frac{k_B T A \pi}{R} \qquad 360° \text{ bend}$$

applied force f

$$\Rightarrow \quad E(R) = \frac{k_B T A \pi}{R} + \underbrace{2\pi R f}_{\text{work against outside force}}$$

(ii)

$$\frac{dE(R)}{dR} \overset{!}{=} 0 = -\frac{k_B T A \pi}{R^2} + 2\pi f$$

$$\Rightarrow \quad R^* = \sqrt{k_B T A / 2f}$$

$f = 1\,pN$, $A = 50\,nm$, $T = 300K$ $\Rightarrow$

$$2\pi R \approx 60\,nm$$

(iii) Torsional energy

$$E_T = \frac{1}{2} k_B T C \theta^2 = \frac{1}{2} k_B T C (2\pi n)^2$$

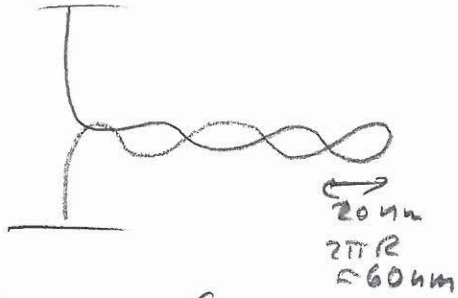
$$\Delta E_T = \frac{1}{2} k_B T C\, 4\pi^2 \left((n+1)^2 - n^2\right)$$

for going from $n \to n+1$ turns

$$\approx 2\pi^2 k_B T C (2n) \text{ for large } n$$

$$\Rightarrow \Delta E_T \simeq 4\pi^2 k_B T C n / L$$

what makes sense



if $\Delta E_T > E(R^*)$ plectoneme forms

$$\frac{4\pi^2 k_B T C n_{buck}}{L} = \frac{k_B T A \pi}{R^*} + 2\pi R^* f$$

(iv) critical torque is

$$2\pi \Gamma_{buck} = \frac{k_B T A \pi}{R^*} + 2\pi R^* f$$

$$\Gamma_{buck} = \frac{k_B T A}{2 R^*} + R^* f$$

$$R^* = 10\,nm, \quad f = 1\,pN, \quad A = 50\,nm$$

$$\Gamma_{buck} \simeq 2 \cdot 10^{-20}\ Nm$$

at $\Gamma > 3.4 \cdot 10^{-20}$ nm p-DNA can be formed!

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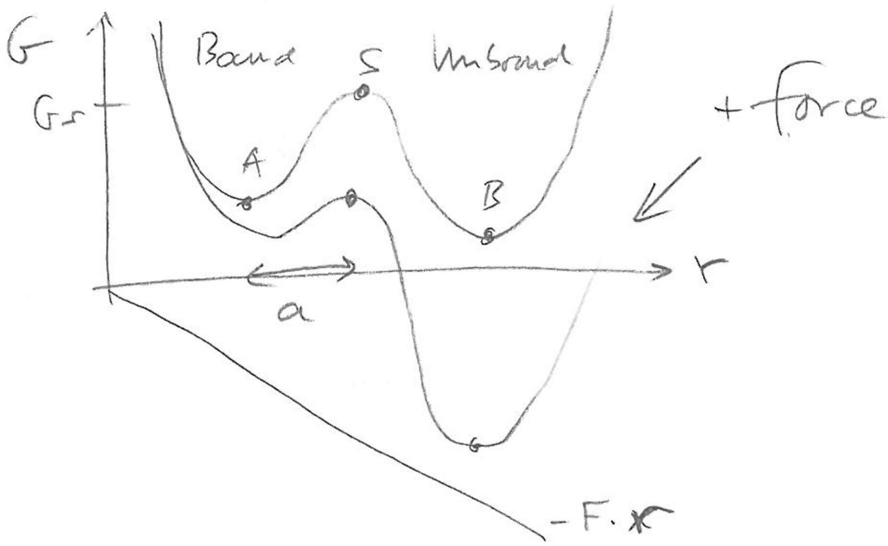
Q4



k_0 increases with F!

$$k \propto \exp\left(-\frac{G_s}{k_BT}\right) \Rightarrow k(F) = e^{-\left(\frac{G_s - Fa}{k_BT}\right)}$$

$$\Rightarrow G_s - Fa = G(F) \qquad = k_0\, e^{\left(\frac{Fa}{k_BT}\right)}$$

$$F_0 = \frac{k_BT}{a} \quad [N] \checkmark$$

bond length

$$k_0 = c\, e^{-\frac{G_s}{k_BT}}$$

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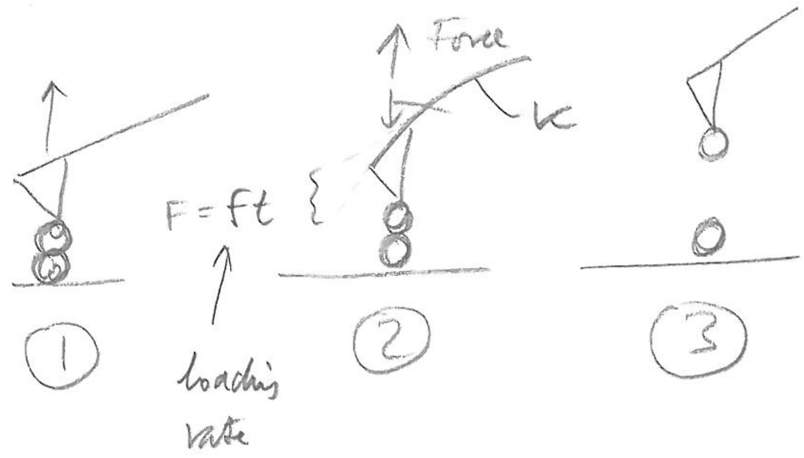
Q 5



$$\frac{dp(t)}{dt} = -k(t)\,p(t) + k_+(t)\,(1-p(t))$$

$k_+ = 0$ due to 3 separation of proteins prevents reformation of bond!

$$F = ft \Rightarrow \frac{dF}{dt} = F \Rightarrow dt = \frac{dF}{F}$$

molecular rate k much faster than experiment

$$\Rightarrow k(t) \rightarrow k(F)\,,\; k_+ = 0$$

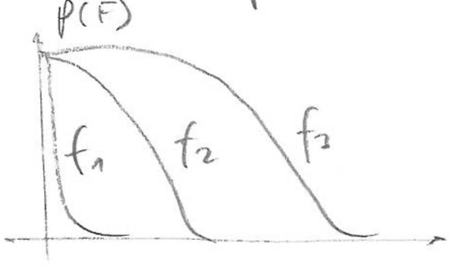
$$f\frac{dp(F)}{dF} = -k(F)\,p(F) \qquad k(F) = k_0 \exp\left(\frac{F}{F_0}\right)$$

$$\Rightarrow \ln p(F) = -\int_0^F \frac{k_0}{f} \exp\left(\frac{F'}{F_0}\right) dF'$$

$$= -\frac{k_0}{f} F_0 \left[\exp\left(\frac{F'}{F_0}\right)\right]_0^F$$

$$= -\frac{k_0 F_0}{f}\left[\exp\left(\frac{F}{F_0}\right) - 1\right] = \frac{k_0 F_0}{f}\left[1 - e^{\frac{F}{F_0}}\right]$$

$$\Rightarrow p(F) = \exp\left(\frac{k_0 F_0}{f}\left[1 - e^{F/F_0}\right]\right)$$



$f_1 < f_2 < f_3$

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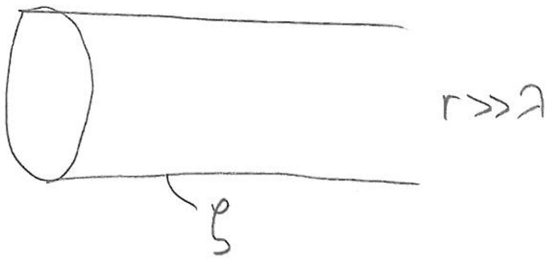
Q 6



Poisson and STOKES

$$\eta \frac{d^2 v}{dx^2} = \varepsilon_0 \varepsilon_r \frac{d^2 \psi}{dx^2} E$$

η, ε_r, E constant

$$\eta \frac{dv}{dx} = \varepsilon_0 \varepsilon_r \frac{d\psi}{dx} E + A$$

$$\eta \left[v \right]_0^{v_0} = \varepsilon_0 \varepsilon_r \left[\psi \right]_{\zeta}^{\psi_e} E + Ax$$

potential in channel

zeta

(no slip boundary)

plug flow profile v_0

$A = 0$ since we need that there is no flow with no field! $v = 0$, $E = 0$! $\underline{\psi_e = 0}$ since large channel!

$$\eta v_0 = \varepsilon_0 \varepsilon_r \left[\psi_e - \zeta \right] E$$

$$v_0 = - \frac{\varepsilon_0 \varepsilon_r \zeta E}{\eta} \quad \square$$

thin Debye layer limit, flow is only determined by charge on surface and <u>not</u> by radius (see question).

if $r \simeq \lambda_{DH}$ pore size should matter!

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Soft Matter and Biological Physics

Michaelmas 2014
Question 7

(i) Stokes - Einstein

$$\frac{k_B T}{\beta} = D \quad \text{with} \quad \beta = \eta N b$$

we get

$$D = \frac{k_B T}{\eta N b}$$

and since we can ignore the interaction with the gel we get for the time τ to diffuse Nb

$$N^2 b^2 = D\tau$$

$$\Rightarrow \tau = \frac{N^2 b^2}{D} = \frac{\eta N b}{k_B T} N^2 b^2 = \eta \frac{N^3 b^3}{k_B T}$$

put in numbers $L = 30{,}000 \cdot 0.34\,\text{nm} \approx 10\,\mu\text{m}$

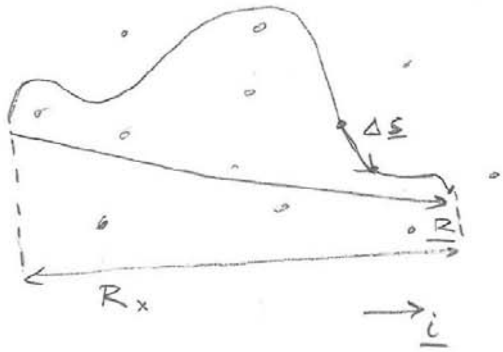
$b = 100\,\text{nm}$ (Kuhn segment)

$$\Rightarrow 100 \cdot 100\,\text{nm} = N \cdot b = L$$

$$\Rightarrow \tau \approx \underline{\underline{240\,\text{s}}}$$

Q11

(ii) ∘← gel fibres $\underrightarrow{\underline{E} = E \cdot \underline{i}}$



$L = Nb$ contour length

Q total charge

$\frac{Q}{L}|\Delta \underline{s}|$ charge on segment $\Delta \underline{s}$ $= \frac{Qb}{L}$

$\underline{R}$ end-to-end distance

R_x in x direction

Force on one segment $\underline{F}_1 = \frac{Q}{L} \Delta s \underline{E}$

Chain can only move along its axis and mainly in x-direction $\Rightarrow$ project

$\underline{\Delta s} \cdot \underline{E} \Rightarrow \quad F_N = \frac{Q}{L} \sum_1^N \underline{E} \cdot \Delta \underline{s}_j = \frac{QE}{L} \sum \underline{i} \cdot \Delta \underline{s}$

$$\Rightarrow F_{N,x} = \frac{QE}{L} R_x = \beta_{DNA} v = \beta_{DNA} \dot{s}$$

$F_{N,x}$ is the total axial force on the chain in direction of $\underline{E}$. So the velocity of the chain along its contour is

$$\dot{s} = \frac{QE R_x}{\beta L} \quad (*)$$

We need now the movement of the centre of mass:

$$M \underline{R}_{cm} = \sum_{i=1}^{N} m_i \underline{r}_i$$

With mass per unit length $m_i = M \frac{\Delta s_i}{L}$ and using that ith segment is $\dot{\underline{r}}_i = \dot{s} \underline{t}_i$

↖ tangent vector $|\underline{t}_i| = 1$

$$M \dot{\underline{R}}_{cm} = \sum_{i=1}^{N} \left(M \frac{\Delta s_i}{L} \dot{s} \underline{t}_i \right)$$

$$\dot{\underline{R}}_{cm} = \frac{\dot{s}}{L} \sum \underline{t}_i \cdot \Delta s_i = \frac{\dot{s}}{L} \underline{R}$$

we just want the movement in x

$$\Rightarrow \quad \dot{x}_{cm} = \frac{\dot{s}}{L} R_x$$

to get V_{drift} we average and use (*) (around)

$$\dot{x}_{cm} = \frac{\dot{s} R_x}{L} \qquad \dot{S} = \frac{QER_x}{\beta L}$$

$$= \frac{QER_x^2}{\beta L^2} \Rightarrow \langle \dot{x}_{cm} \rangle = \frac{QE\langle R_x^2 \rangle}{\beta L^2}$$

$$v_{drift} = \langle \dot{X}_{cm} \rangle = \frac{Q \cdot \cancel{L} \cdot b}{\beta L^{\cancel{2}}} \cdot E = \frac{Q \cdot b}{L} \cdot E \frac{1}{\beta}$$

$\langle R_x^2 \rangle \approx N b^2$ ignoring pre-factors

force on one monomer $f = \frac{Q \cdot b}{L} \cdot E$ $\overset{!}{=} \dfrac{F}{\beta}$ $\square$

$F = Nf$

$\beta = \eta N b$

=

$$V_{drift} = \frac{10\,cm}{1\,hour}$$

$\Rightarrow \quad E \approx 3000\ V/m$ for 30,000 bp DNA

at same voltage:

$$V_{drift} = \frac{F}{Nb\eta} \approx 3.33 \cdot 10^{-5} \frac{m}{s}$$

$L = 25{,}000$ bp $\hat{\approx} 8.5\,\mu m$

$\Rightarrow L_{25kb} \approx 12$ cm $\square$ in 1 hour

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Question 7: Polymers in Confinement

Solution:

(i) The electrostatic potential in the channel is $V(x) = -Ex$ so we can easily write down the energy for the molecule when entering the channel is

$$\Delta U(x) = -\rho E \int_0^x x dx = -\frac{1}{2}\rho E x^2$$

i believe that we do not have to mention that $U(x = 0) = 0$ as a boundary condition. This is of course a major simplification as there will be a finite electric field outside of the channel due to the access resistance.

(ii) First explain the sign: the DNA has to be straight inside the channel so fewer configurations are available so entropy is lower in this state with $\Delta S < 0$.
Dependence on x: entropy is an extensive quantity and therefore proportional to the length L of the strand. Since the DNA in the cavity has a fixed configuration (i.e. no entropy), we have $S = S(L)(1 - x/L$ so obviously $\Delta S \propto -x$. Well in principle this explains also the sign. I believe it is good to split this into two discussions - but do as you see fit.
Remark: In the event that there are questions about entropy as extensive quantity for WLC polymers ... The number of possible configurations will be

$$Z_N = \text{(No. of configs/link1)(No. of configs/link2)...(No. of configs/linkN)} = \text{(const)}^N$$

So we get for the entropy

$$S = k_B \ln(Z_N) = N k_B \ln(\text{const}) \Leftrightarrow S \propto N$$

(iii) Now we can write down ΔG as was asked in the question as

$$\Delta G = -\frac{1}{2}\rho E x^2 + \gamma T x$$

A sketch will look like the plot shown below. Of course the position of the barrier will depend on all parameters as expected from the question. Calculate the stationary points of ΔG:

$$\frac{d\Delta G}{dx} = -\rho E x_c = \gamma T = 0$$

$$x_c = \frac{\gamma T}{\rho E}$$

This is expected since at higher fields the 'barrier'/maximum gets closer and closer to the channel entrance - and smaller of course. No we can get the height:

$$\Delta G^* = \Delta G(x_c) = \frac{\gamma^2 T^2}{2\rho E}$$

(iv) As T is increased the entropic energy cost for reaching the transition state increases while the electrostatic energy remains unchanged. In other words, the entropic chain gets stiffer with increasing temperature resisting the pulling force.

Michaelmas 2014
Question 9

.tive pulse sensing: [Bookwork]

label-free detection •

detection of molecule by ΔI •

detection limit given by diameter / length of nanopore •

[3]

Calculate Resistance [Partly Bookwork]

Total resistance is 2 times upper cone •

variation of radius along z: $d(z) = r + z\frac{(R-r)}{\ell}$ •

Resistance: $\mathcal{R} = 2\rho \int_0^{2\ell} \frac{\ell}{A(z)}\, dz$ •

$$= 2\frac{\rho}{\pi} \int_0^{2\ell} dz \frac{1}{\left(r + z\frac{(R-r)}{\ell}\right)^2}$$

$$= \frac{2\rho}{\pi}\left[- \frac{\frac{\ell}{R-r}}{r + z\left(\frac{R-r}{\ell}\right)}\right]_0^{\ell}$$ •

$$= 2\frac{\rho\ell}{\pi}\left[-\frac{1}{(R-r)r + (R-r)^2} + \frac{1}{(R-r)r}\right]$$

$$= 2\frac{\rho\ell}{\pi}\frac{1}{rR}\left[\frac{-rR + r^2 + R^2 - rR}{(R-r)^2}\right]$$

$$\mathcal{R} \stackrel{!}{=} 2\frac{\rho\ell}{\pi}\frac{1}{rR}$$ ••

[partly bookwork] [6]

Ohm's law $j = \frac{I}{\text{Area}_\perp} = \frac{U/\mathcal{R}}{\text{Area}_\perp}$ ••

has to be split now for $0 \to \ell$ •
$\ell \to 2\ell$

$$j_{0\to\ell} = \frac{\frac{U}{2}\cdot\frac{2}{\mathcal{R}}}{\pi\left(r + z\frac{(R-r)}{\ell}\right)^2} = \frac{URr}{2\pi\left(r + z\left(\frac{R-r}{\ell}\right)\right)^2 \rho\ell}$$ •

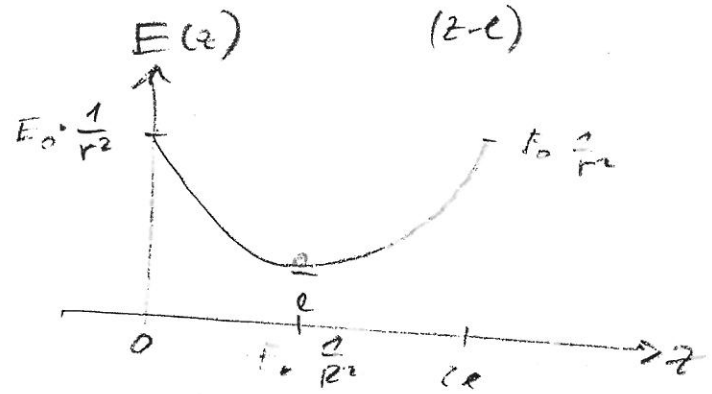
$$j_{e\to 2e} = \frac{\frac{U}{2}\,\frac{2}{R}}{\pi\left(R+(z-e)\left(\frac{r-R}{e}\right)\right)^2} = \frac{URr}{2\pi\left(R+(z-e)\left(\frac{r-R}{e}\right)\right)^2}$$

$$\Rightarrow E_{0\to e} = \frac{URr}{2\pi\varsigma^2 e}\,\frac{1}{\left(r+z\left(\frac{R-r}{e}\right)\right)^2} \qquad E(0) = \frac{URr}{2\pi\varsigma^2 e}\,\frac{1}{r^2}$$

$$E(e) = \frac{URr}{2\pi\varsigma^2 e}\,\frac{1}{R^2}$$

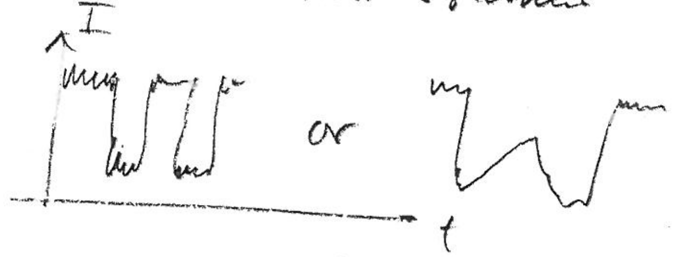
$$E_{e\to 2e} = \frac{URr}{2\pi\varsigma^2 e}\,\frac{1}{\left(R+(z-e)\left(\frac{r-R}{e}\right)\right)^2} \qquad E(e) = \frac{URr}{2\pi\varsigma^2 e}\,\frac{1}{R^2}$$

$$E(2e) = \frac{URr}{2\pi\varsigma^2 e}\,\frac{1}{r^2} \checkmark$$



[9]

With $L_0 \ll e, R \Rightarrow$ Ionic current signature [new]



two maxima [2]

time of flight $0 \to 2e$ [new]

$v = \frac{dz}{dt} = \mu E$

$$\Rightarrow \int_{t_{in}}^{t_{out}} dt = \frac{1}{\mu}\int_0^{2e} \frac{1}{E}\,dz$$

with $E(z) \sim \frac{1}{\pi\left(r+z\left(\frac{R-r}{e}\right)\right)^2}\,,\ \frac{1}{\pi\left(R+(z-e)\frac{(r-R)}{e}\right)^2}$

two separate integrals

$$\Delta t_{0\to e} \sim \frac{1}{\mu}\int_0^e \pi\left(r+z\left(\frac{R-r}{e}\right)\right)^2 dz = \frac{1}{\mu}\pi\,\frac{1}{3}\,\frac{e}{(R-r)}\,\frac{1}{3}\left[R^3-r^3\right]$$

$$\Delta t_{\ell-2\ell} \approx \frac{1}{\mu}\int_{\ell}^{2\ell} \pi \left(R+(z-\ell)\left(\frac{r-R}{\ell}\right)\right)^2 dz$$

$$= \frac{\pi \ell}{\mu(r-R)}\,\frac{1}{3}\left[r^3-R^3\right]$$

$$\Rightarrow \quad \Delta t = \Delta t_{0-\ell} + \Delta t_{\ell-2\ell} \approx \frac{2}{3}\frac{\pi \ell}{\mu(r-R)}\left[r^3-R^3\right]$$

[new]

[6]

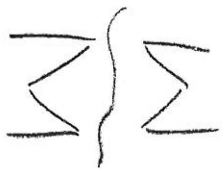
$L \ll \ell$

- pore - molecule interactions increase Δt

- entropic barrier for entering second nanopore not accounted for [2]

$L \gg \ell$

- correlation between two pores possible



no relaxation before 2nd translocation [2]

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Question 10

change in free energy

$$\Delta G_{proton} = k_B T \ln (c_{in} / c_{out}) + e \Psi_m$$

to make an ATP molecule

$$\Delta G_{ADP \to ATP} = \Delta G^0 + k_B T \left(\ln (c_{ATP} / c^0) - \ln (c_{ADP} / c^0) - \ln (c_p / c^0) \right)$$

(I use $\Delta G^0 > 0$ since this is stored energy. $\Delta G^0 < 0$ also possible)

total change in free energy

$$\Delta G_{tot} = N \Delta G_{proton} + \Delta G_{ADP-ATP}$$

For a fully reversible process $\Delta G_{tot} = 0$

$\Rightarrow$ so combining all from above yields

$$\Delta G^0 = - N (k_B T \ln (c_{in} / c_{out}) + e \Psi_m) - k_B T \left(\ln (c_{ATP} / c^0) - \ln (c_{ADP} / c^0) - \ln (c_p / c^0) \right)$$

$$= k_B T \ln (c_{out}^N c_{ADP} c_p / c_{in}^N c_{ATP} c^0) - N e \Psi_m$$

(ii) irreversible process $\Delta G_{tot} < 0$

redo calculation

$$\Delta G^0 < k_B T \ln (c_{out}^N c_{ADP} c_p / c_{in}^N c_{ATP} c^0) - N e \Psi_m$$

The answer in (i) is an upper bound. This is intuitive since some energy is lost in the process due to dissipation $\Leftrightarrow$ less energy stored in ATP

Michaelmas 2014
Question 11

Lipid membrane provides scaffold for motor and provide barrier for ion/protons. Membrane potential and proton gradient are used to create rotary motion, Energy $= \Delta\Psi_{total} \cdot e$

[4]

[bookwork]

Use formula

$$\Delta\mu_p = e\Delta\Psi - k_BT \ln\frac{[H_o^+]}{[H_i^+]}$$

$$\Delta\mu_p = 0 \Rightarrow k_BT \ln\frac{[H_o^+]}{[H_i^+]} = e\Delta\Psi$$

$$[H_o^+] = e^{\frac{e\Delta\Psi}{k_BT}} \cdot [H_i^+]$$

$$\stackrel{!}{=} e^{-4.65}\,[H_i^+]$$

$$= 1.14 \cdot 10^{14}$$

$$\Rightarrow [H_o^+] = 9.7$$

[3]

[partly bookwork]

Rotation direction

G_+, G_- find ΔG

$T = 37°C$ $\quad p_+ = 0.01$
$\quad p_- = 0.99$

$$\Rightarrow \frac{p_-}{p_+} = e^{\Delta G / k_BT}$$

$$\Rightarrow \Delta G \approx 4.6\, k_BT$$ [3]

$$\approx 1.96 \cdot 10^{-10}\ J$$