Examples Sheet #2

1. *Radius of Gyration and Structure Factor for a Gaussian polymer chain.*
The radius of gyration R_g of a polymer is defined as

$$R_g^2 = \frac{1}{2N^2} \sum_{n=1}^{N} \langle (\mathbf{R}_n - \mathbf{R}_{\rm com})^2 \rangle \ ,$$

where N is the number of links and $\mathbf{R}_{\rm com}$ is the center of mass of the chain. For a Gaussian chain with step length b, find R_g and the scattering function

$$g(\mathbf{k}) = \frac{1}{N} \sum_{m,n}^{N} \langle \exp\left[i\mathbf{k} \cdot (\mathbf{R}_n - \mathbf{R}_m) \right] \rangle \ .$$

Express your result in a scaling form.

2. *Differential geometry of space curves.*
Consider a space curve $\mathbf{r}(s)$ in three dimensions, with velocity

$$\mathbf{r}_t = U\hat{\mathbf{n}} + V\hat{\mathbf{b}} + W\hat{\mathbf{t}} \ .$$

(a) Find the time rate of change of the metric g, and length L in terms of U, V, and W.
(b) What is the condition for local arclength conservation?
(c) Derive the time evolution of the curvature $\kappa(s)$ and of the torsion $\tau(s)$.
(d) Find the equation of motion for the Hasimoto/Darboux function

$$\psi = \kappa \exp\left(i \int_s ds' \tau(s') \right)$$

3. *Euler buckling.*
Analyze the buckling of an elastic rod of length L and bending modulus A under thrusting forces f at each end. Assume (a) clamped and (b) hinged boundary conditions. Give an analytical treatment near the primary bifurcation, and a complete numerical treatment beyond. Just beyond the bifurcation the bent rod behaves like a spring. Find the spring constant.

4. *Twisting instability.*
Completing the discussion from lecture, find the onset condition a linear twisting instability of a rod of length L, bend elastic constant A, twist elastic constant C, and clamped ends, if there is an imposed twist density Ω.

5. *Waves on an elastic filament.*
In lecture we considered an elastic filament with bending modulus A in a viscous fluid. For small displacements $y(x,t)$ from a straight configuration the simplest equation of motion is $\zeta y_t = -A y_{xxxx}$, where ζ is a drag coefficient. Imagine that the left end of the filament is constrained to move as $y(x=0,t) = a\cos(\omega t)$, with zero curvature there. With appropriate boundary conditions at the free right end, find the shape $y(x,t)$ for a semi-infinite filament. Find the solution for very short filaments.

6. *Thermal fluctuations of vesicles.*

Consider a two-dimensional vesicle with bending modulus A and enclosed area $\mathcal{A}$. Assuming the area deficit from the equivalent circle is small, calculate the spectrum of thermal fluctuations.