LETTER TO THE EDITOR

[Brief letters to the Editor that make specific scientific reference to papers published previously in THE JOURNAL OF GENERAL PHYSIOLOGY are invited. Receipt of such letters will be acknowledged, and those containing pertinent scientific comments and scientific criticisms will be published.]

Access Resistance of a Small Circular Pore

Dear Sir:

The access resistance of a small circular pore is of some importance in estimating the conductance of pores in biological membranes (Hille, 1968,1970). This resistance is usually approximated as the convergence resistance to a hemisphere of the same radius as the pore. The approximation assumes that the contribution of the hemisphere itself is negligible. It turns out that this is not true and that the resistance of the hemisphere is in fact of the same order of magnitude as the resistance of the material in the half-space outside the hemisphere.

We can estimate the resistance of the hemisphere by assuming field lines inside the hemisphere are straight and perpendicular to the mouth of the pore. With this approximation the resistance of the hemisphere is

$$R_h = \frac{\rho}{2\pi a},\qquad (1)$$

where a is the radius of the hemisphere and ρ resistivity of the medium. This is equal to the value obtained for the convergence resistance to a hemisphere (Hille, 1970, p. 21). Hence making the approximation that the convergence resistance to a circular pore is given by the convergence resistance to a hemisphere neglects a contribution possibly as large as the one calculated from the original approximation.

Fortunately a more satisfying physical approximation can be solved exactly using a well-known result of electrostatics. If the mouth of the pore is an equipotential surface, the convergence resistance can be calculated exactly. This problem is equivalent to that of calculating the resistance between a conducting disk on an insulator and a half-spherical electrode very far from the disk.

It can be shown that problems of resistance between electrodes in conducting media and problems of capacitance between electrodes in insulating media are related by a simple transformation. (See, for example, Smythe pp. 234 and 237.) For a given electrode geometry, resistance and capacitance are related by the equation

$$R = \frac{\epsilon\rho}{C},\qquad (2)$$

where R is the resistance, C the capacitance, ϵ the permitivity of the medium where the

capacitance is measured, and ρ the resistivity of the medium where the resistance is measured. The result is in mks units.

The problem thus reduces to one of calculating the capacitance of a conducting disk to a half-spherical electrode at infinity. Such a problem has already been solved. The capacitance of a charged conducting disk is well known to be:

$$C = 8\epsilon a, \tag{3}$$

where a is the radius of the disk. The units are mks (see Jackson, p. 89).

The field and equipotential lines of this problem have exactly the same shape in one half-plane as those of the problem we are interested in. It is only necessary to realize that our problem involves only one half-space. Consequently the appropriate value of the capacitance is half the above value. Eq. 2 then gives a value

$$R_{\text{conv}} = \frac{\rho}{4a}, \tag{4}$$

for the access resistance of the pore. This is a factor of $\pi/2$ greater than the simple convergence resistance to a hemisphere, an increase of almost 60%. The value of the access resistance of a circular pore is an important one in estmating the conductance of possible pores in biological membranes, and it is useful to realize it can be obtained almost exactly by this simple procedure.

James E. Hall
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REFERENCES

HILLE, B. 1968. Pharmacological modifications of the sodium channel of frog nerve. *J. Gen. Physiol.* **51**:199–219.

HILLE, B. 1970. Ionic channels in nerve membranes. *In* Progress in Biophysics and Molecular Biology. J. A. V. Butler, editor. Pergamon Press, N. Y. **21**:1.

JACKSON, J. D. 1962. Classical Electrodynamics. John Wiley & Sons, Inc., New York.

SMYTHE, W. R. 1950. Static and Dynamic Electricity. McGraw-Hill Book Company Inc., New York.