

Fluctuations in the velocities of sedimenting particles

John Hinch

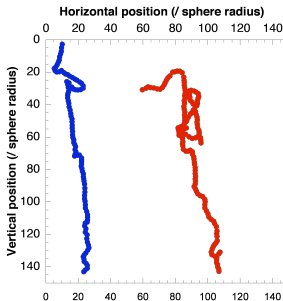
DAMTP, Cambridge

In collaboration with Élisabeth Guazzelli & Laurence Bergougnoux
and their students

Fluctuating velocities

Particles do not fall at a constant speed in a suspension

Trajectories of two spheres at $\phi = 0.3$



Nicolai, Herzhaft, Hinch, Oger & Guazzelli. (1995) *Phys. Fluids* **7**, 12–23.

The divergence paradox

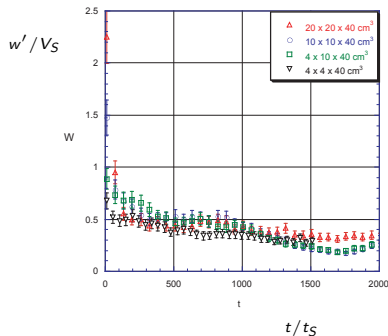
- ▶ Theory: depend on size L of box

$$w' = V_S \sqrt{\phi \frac{L}{a}}$$

The divergence paradox

- ▶ Theory: depend on size L of box
- ▶ Experiments: no such dependence

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Theory of scaling

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$\frac{N}{2} + \sqrt{N}$	$\frac{N}{2} - \sqrt{N}$
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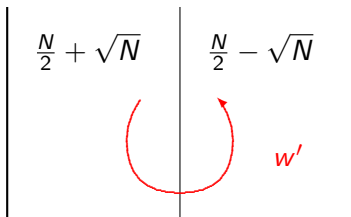
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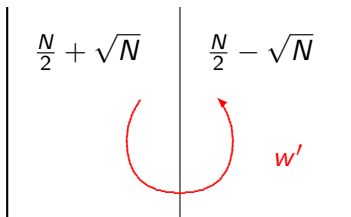
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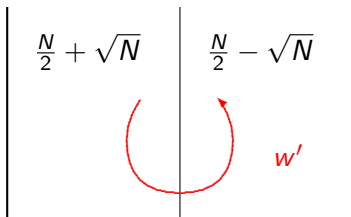
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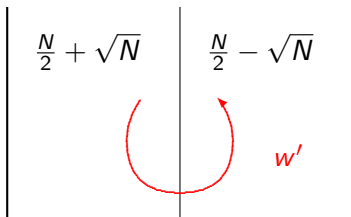
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'Poisson' value

Big effect of a little stratification

Bławdziewicz c1995, private communication - ignored.

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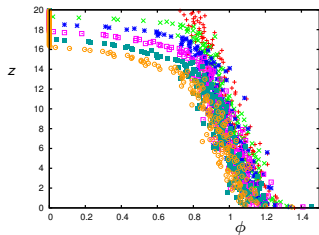
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Computer simulations to test effect of stratification

Initially stratified

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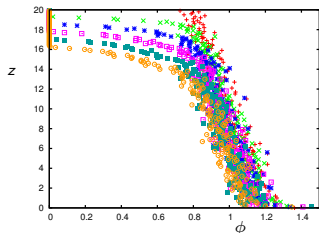


Concentration profile
at different times

$\Delta\phi/\phi = 0.4$, 2500 particles,
average over 40 realisations

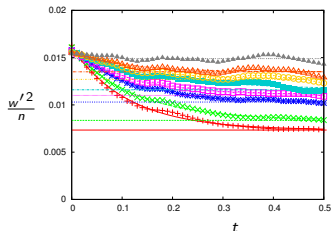
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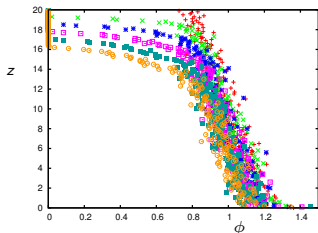


Velocity fluctuations for
 $\Delta\phi/\phi = 0, \dots, 0.4$

10^4 particles, $h = 10$

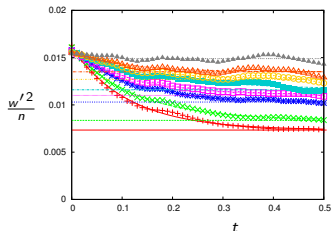
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Decay to a plateau value w'_∞

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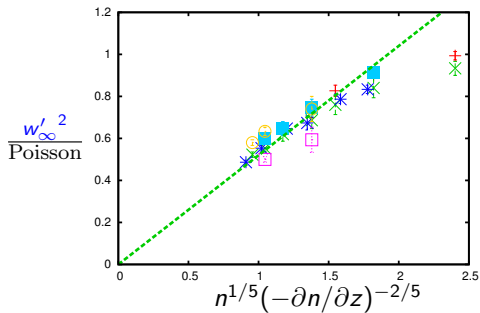
Initially stratified

Plateau value w'_∞ ² plotted against stratification

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Plateau value $w'_\infty{}^2$ plotted against stratification

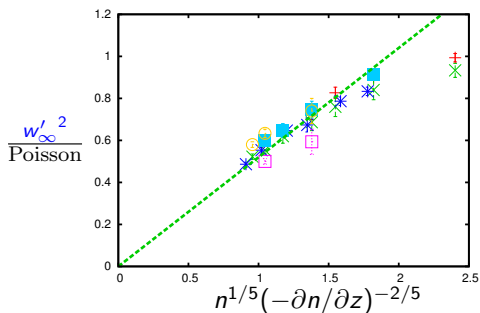


Different n & δx

Computer simulations to test effect of stratification

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Plateau value $w'_\infty{}^2$ plotted against stratification



► Hence

$$w'_\infty = 0.94 V_S \phi^{3/5} \left(-a \frac{\partial \phi}{\partial z} \right)^{-1/5}$$

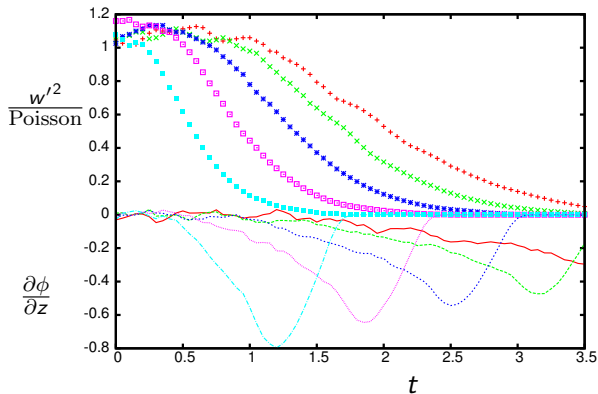
Computer simulations to test effect of stratification

Initially uniform - stratified in descending front

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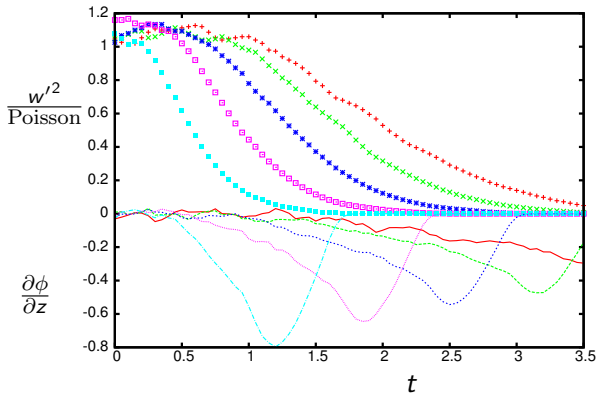
Viewed in windows at different heights: **top**, **bottom**



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Velocity fluctuations reduced when front arrives in window

Computer simulations to test effect of stratification

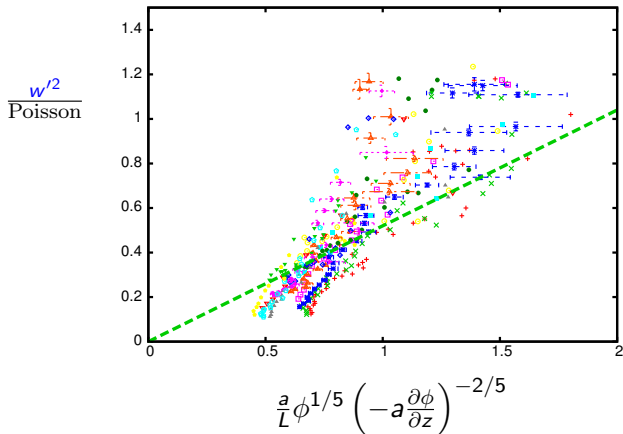
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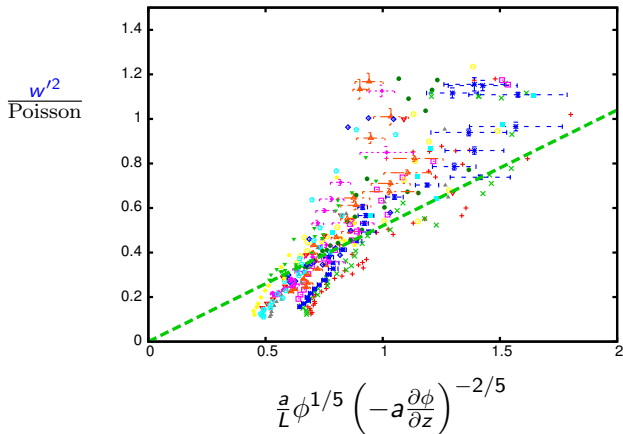
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Fair agreement only, but recall time delay for initial value to decay

Experiments

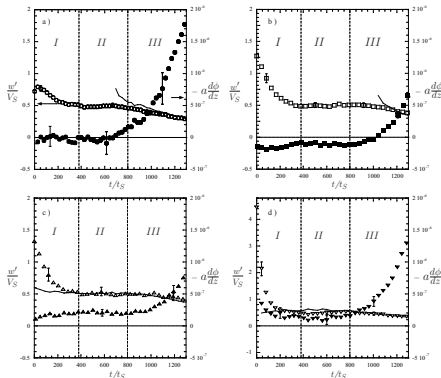
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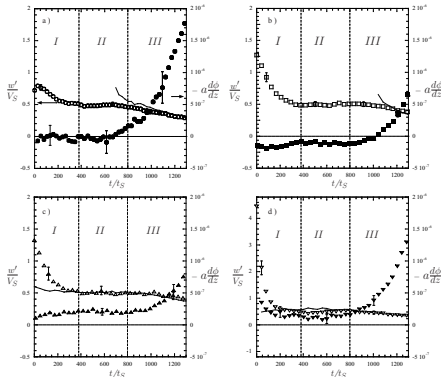
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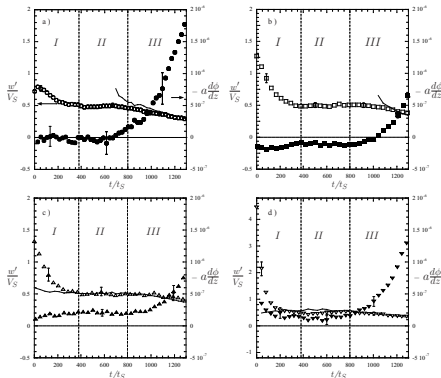


I – Decay of initial state,

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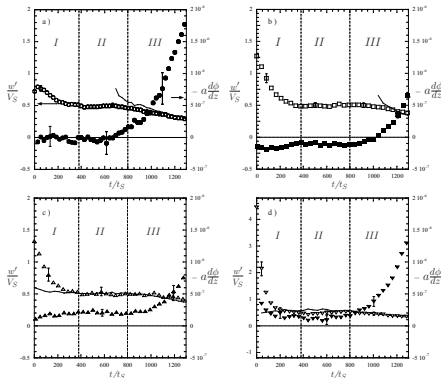


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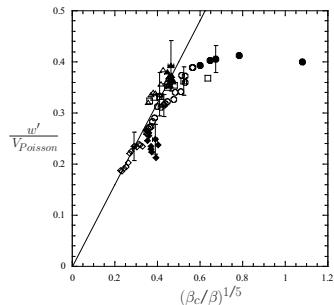


I – Decay of initial state, II – plateau, III – in front

Experiments

Initially uniform - stratified in descending front

- ▶ Velocity fluctuations inhibited by stratification



Filled symbols on plateau (II),
open in front (I).

Experiments

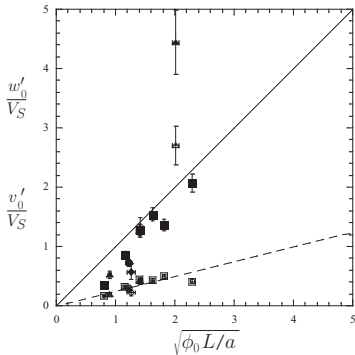
And initial values are the old divergent scaling

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Mucha & Brenner (2003) Phys. Fluids 15: 1305-13

- ▶ Numerical value **2.75** of diffusivity from similarity solution ...

Diffusing front

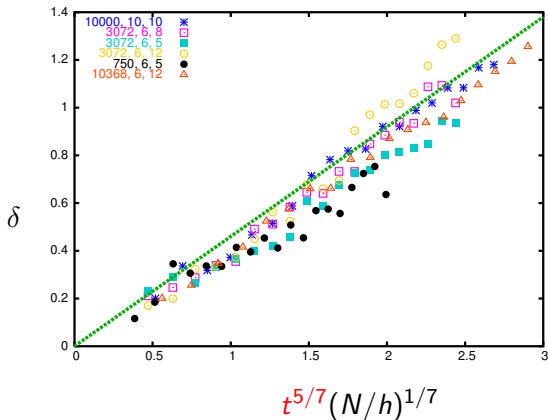
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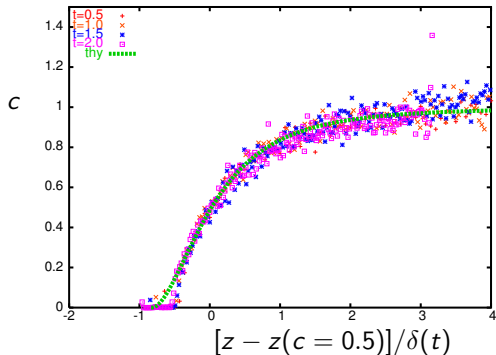
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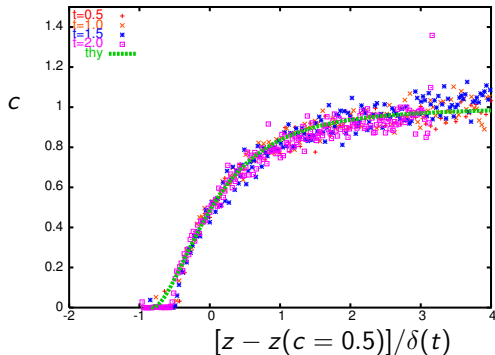
Diffusing front

Similarity plot of concentration profile



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Similarity plot of concentration profile



- ▶ Nonlinear diffusion equation predicts concentration profile in diffusing front at top of suspension

Open question: effect of small inertia

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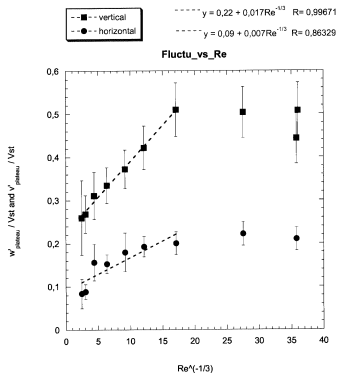
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Recent preliminary experiments
(Bergognoux 2011)