

# Jet break-up and polymers: a numerical study of beads-on-string

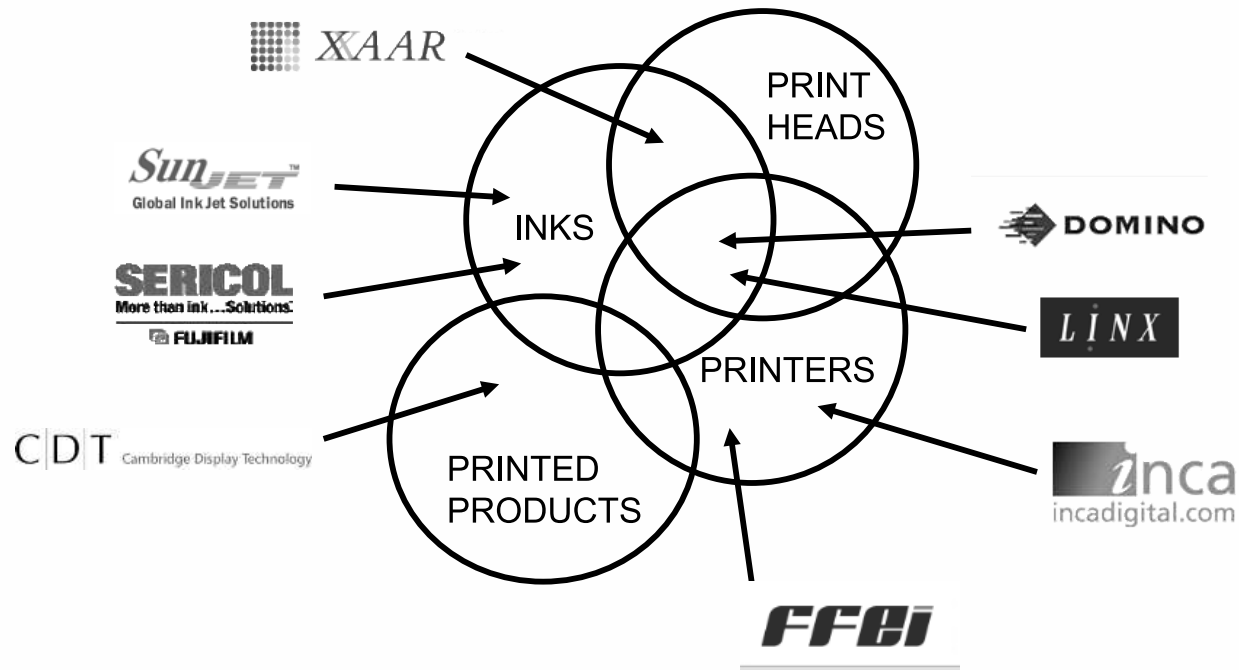
J. Etienne<sup>1,3</sup>, J. Hinch<sup>1</sup> & J. Li<sup>2</sup>

<sup>1</sup> DAMTP & <sup>2</sup> BP Institute, Cambridge University

<sup>3</sup>SPECTRO, UJF & CNRS, Grenoble

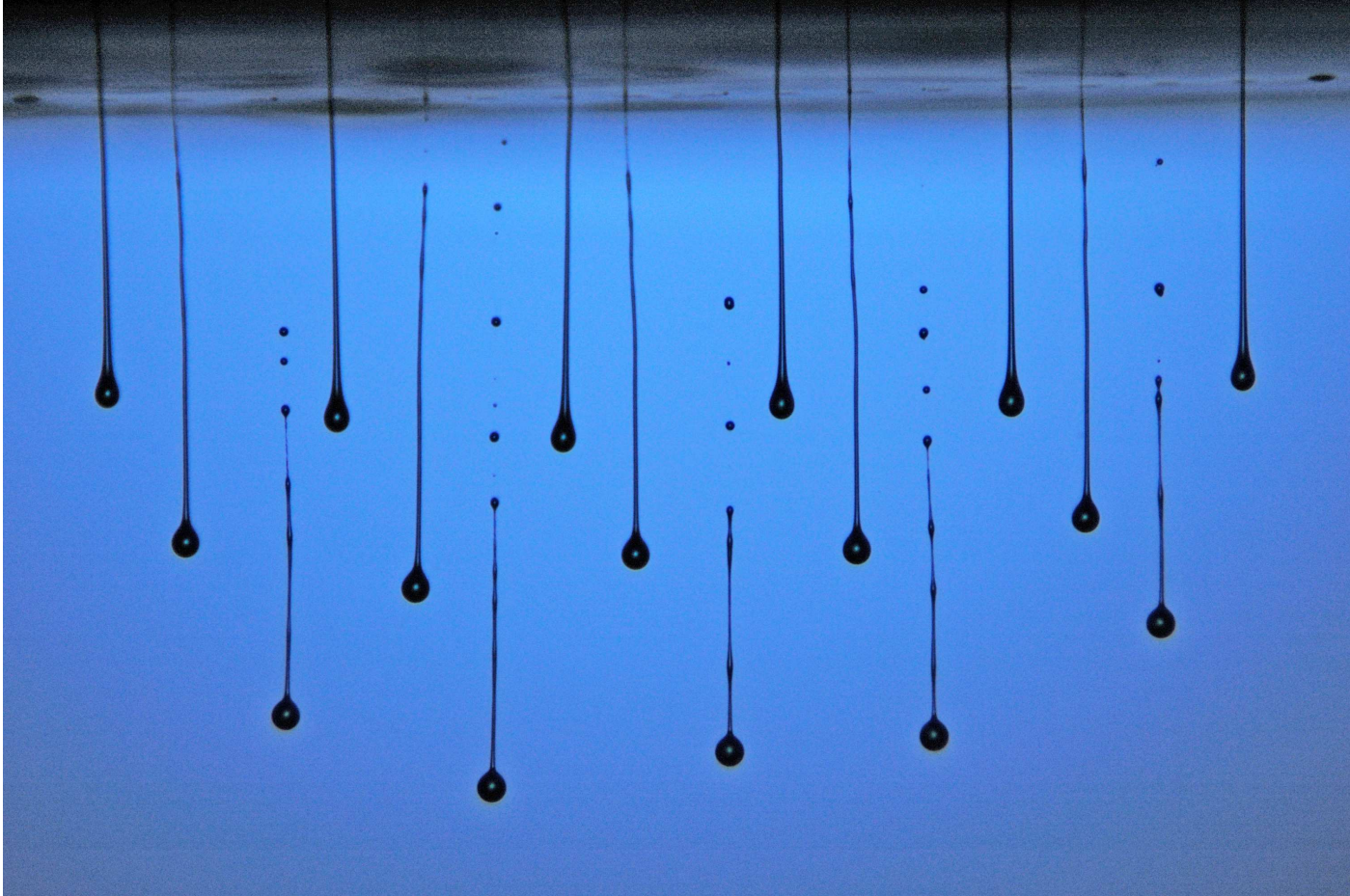
# EPSRC collaboration

## Companies



Universities: Cambridge, Durham, Leeds, Manchester, Wales

# Drop-on-Demand printer



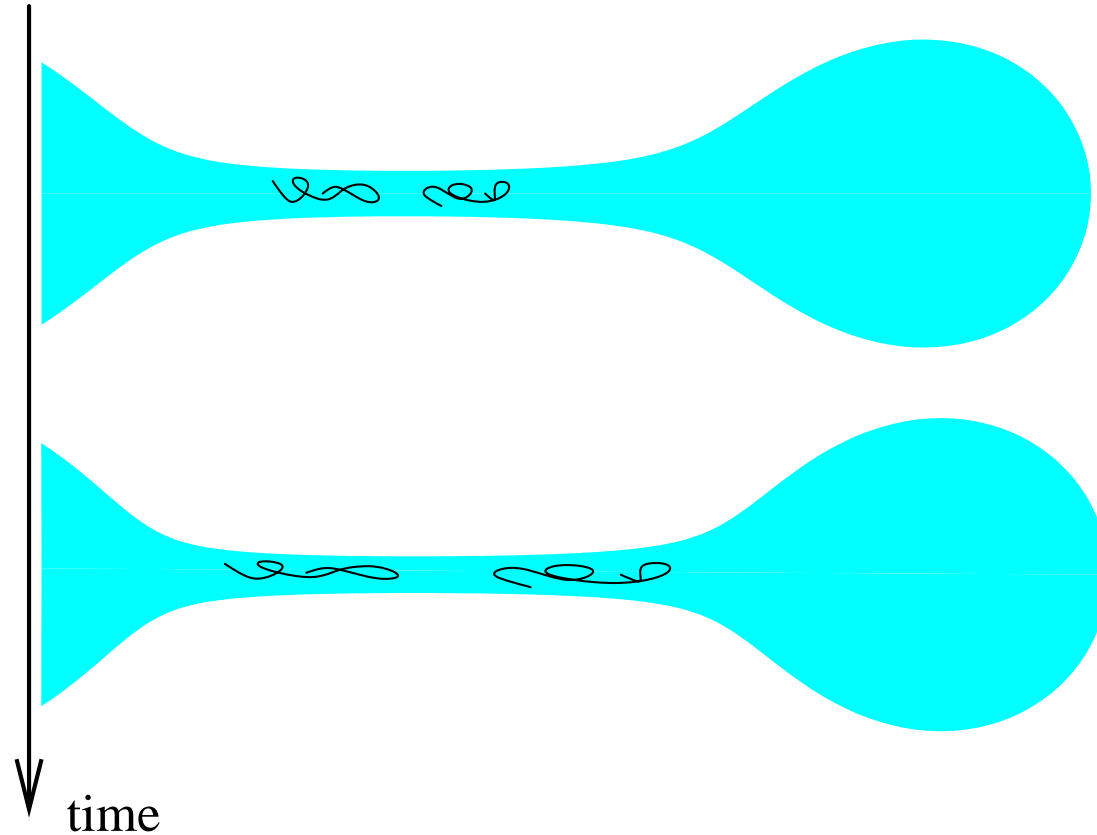
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# Effect of polymer



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# Difficult to break non-Newtonian jets

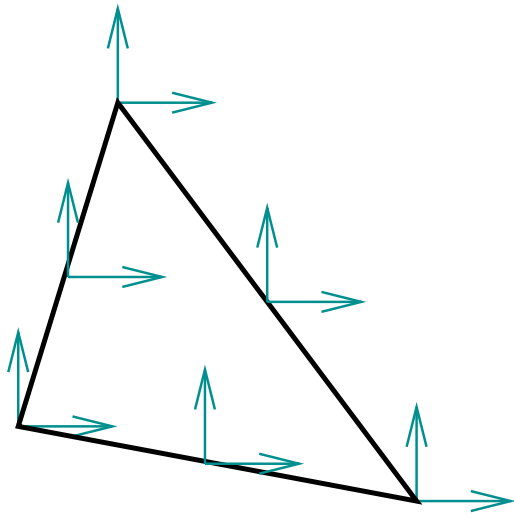


Polymers resist stretching – need rheology equation



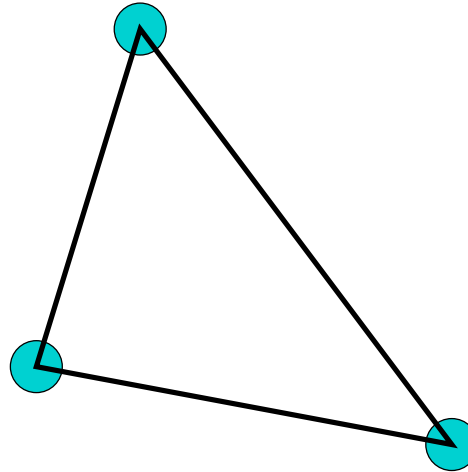
# Numerical: Finite Elements

$u$



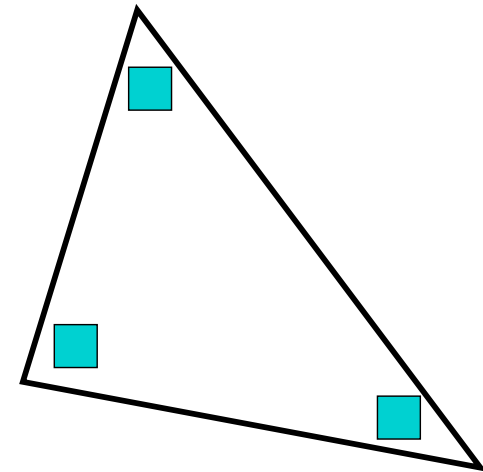
$P_2 \cap C^0$

$p$



$P_1 \cap C^0$

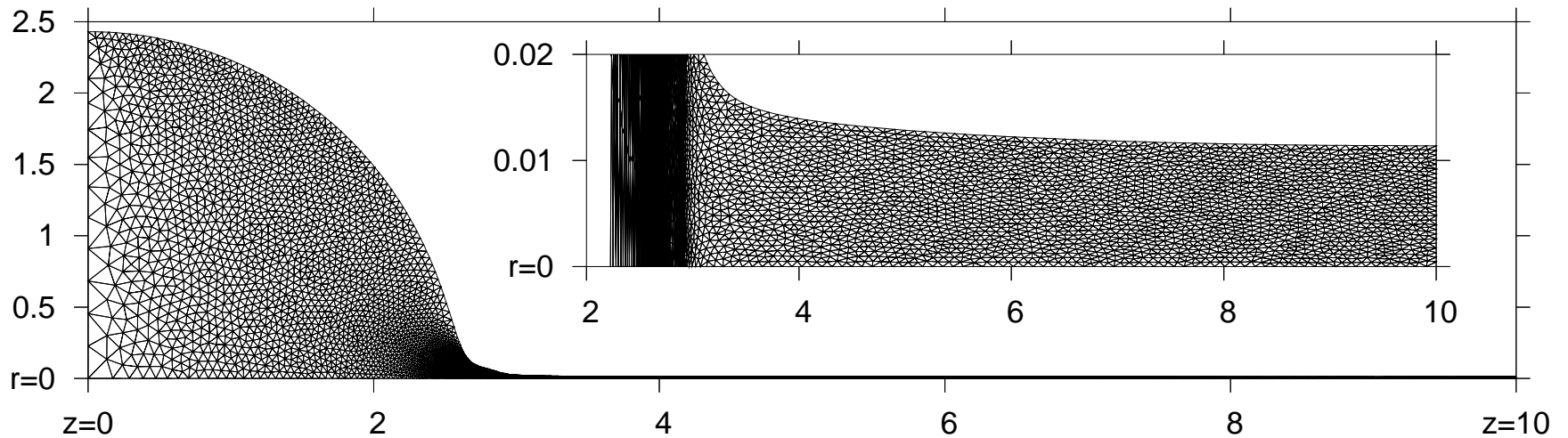
$A$



$P_1 \cap C^{-1}$

$C^{++}$  code embedded in the free-software FEM environment  
rheolef (P. Saramito, N. Roquet, J. E.)

# Auto-adaptive mesh

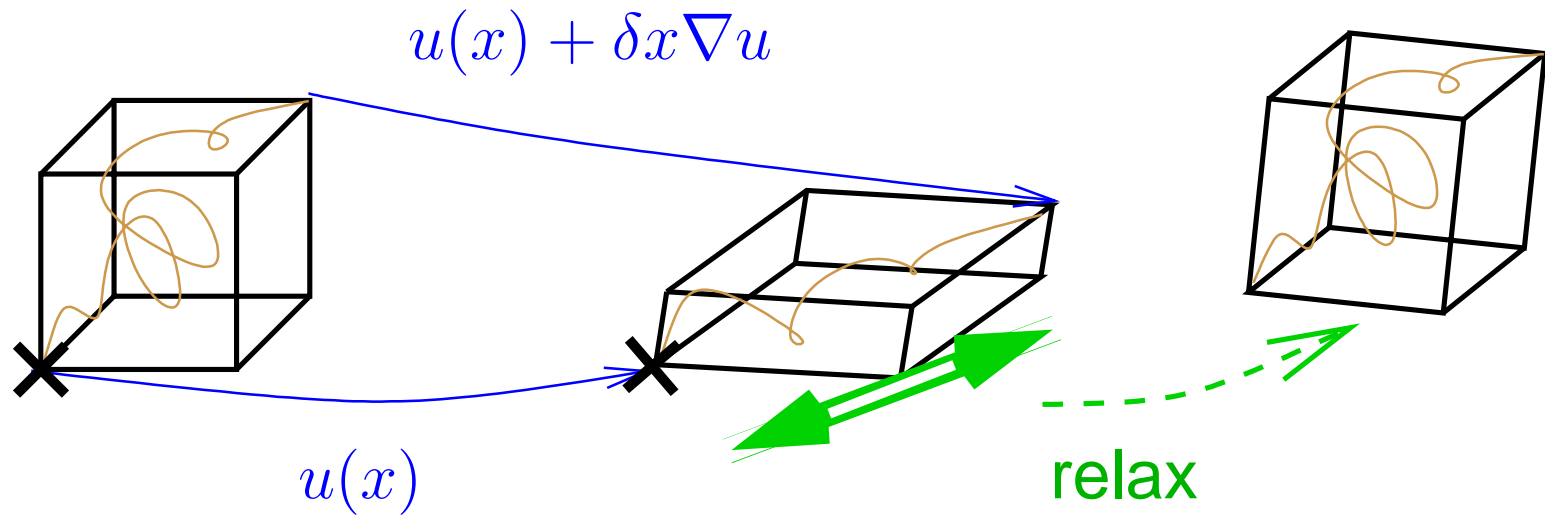


Generated by free-software Bamg (F.Hecht)

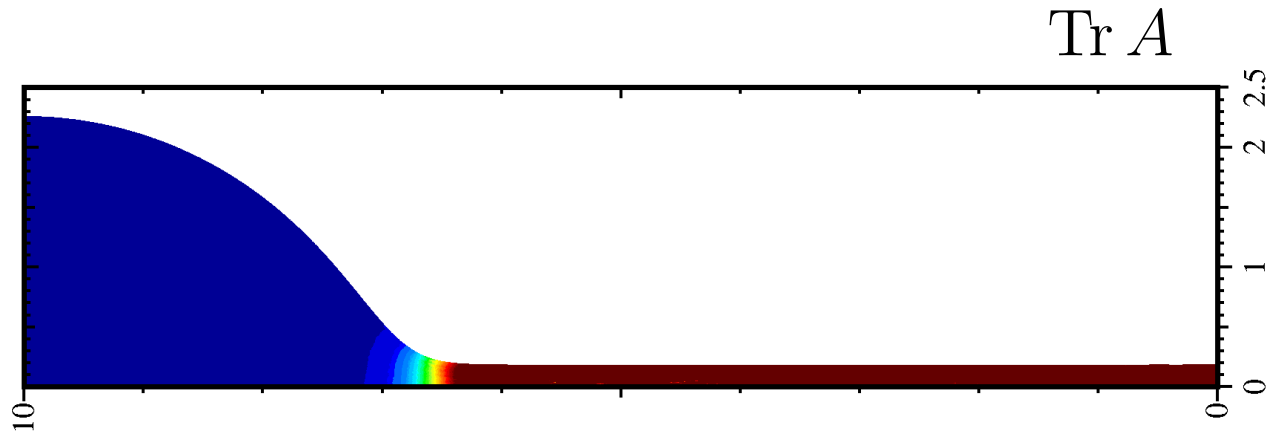
# Lagrangian approach

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot \mathbf{A} - \frac{1}{\tau} (\mathbf{A} - \mathbf{I})$$

deform with fluid                      relaxes



# Oldroyd-B results for low inertia

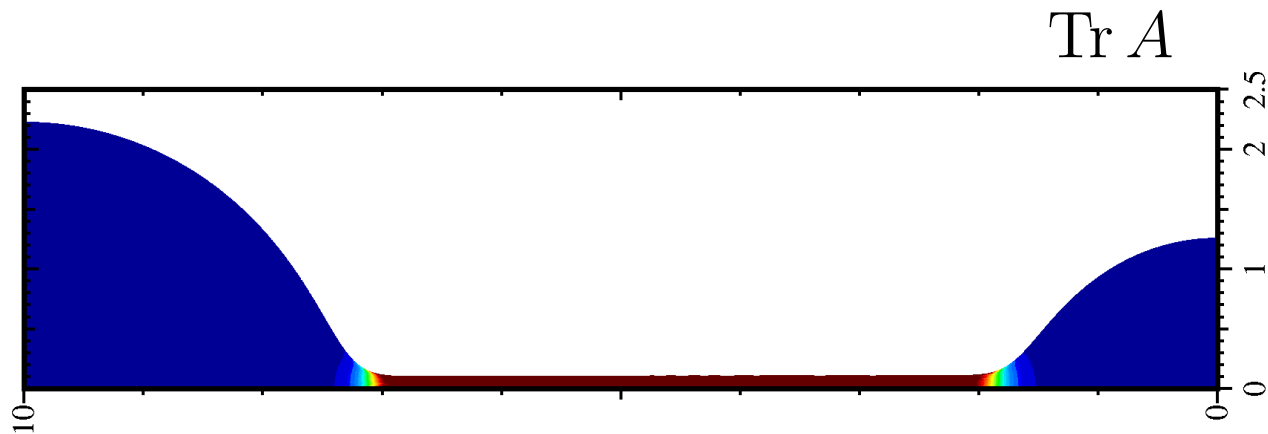


Slow relaxation:  $De = \tau \sqrt{\frac{\gamma}{\rho R^3}} = 95$

Concentration:  $c = \frac{G\tau}{\mu} = 3$

Ohnesorge (capillary Reynolds):  $Oh = \frac{\sqrt{\rho\gamma R}}{\mu} = 3.2$

# Oldroyd-B results for high inertia



Slower relaxation:  $De = \tau \sqrt{\frac{\gamma}{\rho R^3}} = 300$

Concentration:  $c = \frac{G\tau}{\mu} = 3$

Ohnesorge (capillary Reynolds):  $Oh = \frac{\sqrt{\rho\gamma R}}{\mu} = 10$

# Oldroyd-B thinning

Mass

$$\dot{a} = -\frac{1}{2}Ea$$

Momentum

$$\frac{\chi}{a} = 3\mu_0 E + G(A_{zz} - A_{rr})$$

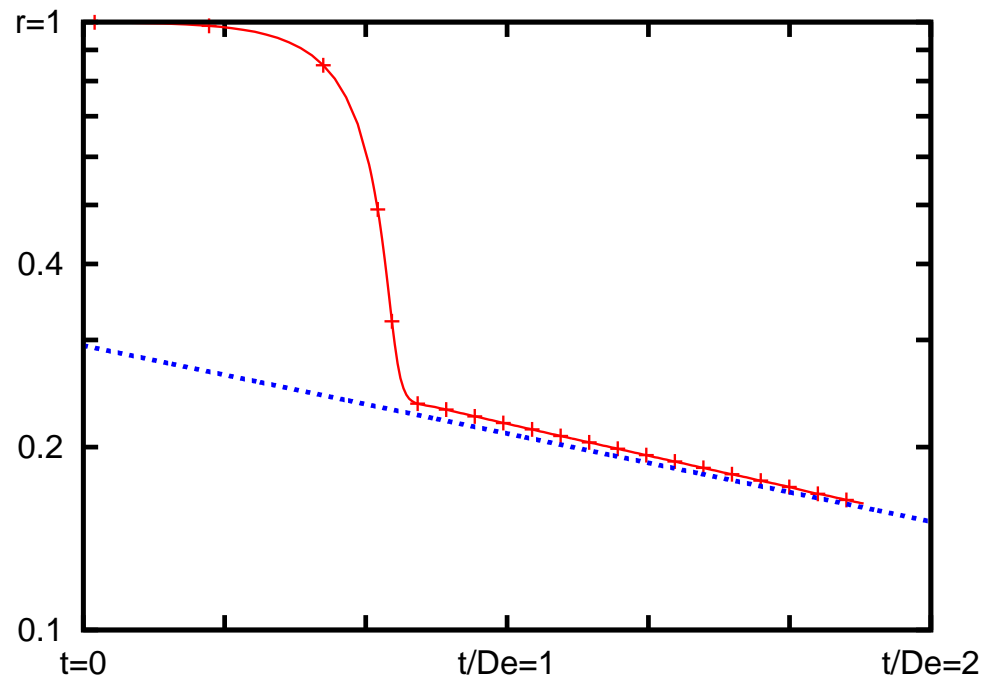
Microstructure

$$\dot{A}_{zz} = 2EA_{zz} - \frac{1}{\tau}(A_{zz} - 1)$$

Solution

$$a(t) = a(0)e^{-t/3\tau}$$

Never breaks!



# FENE modification

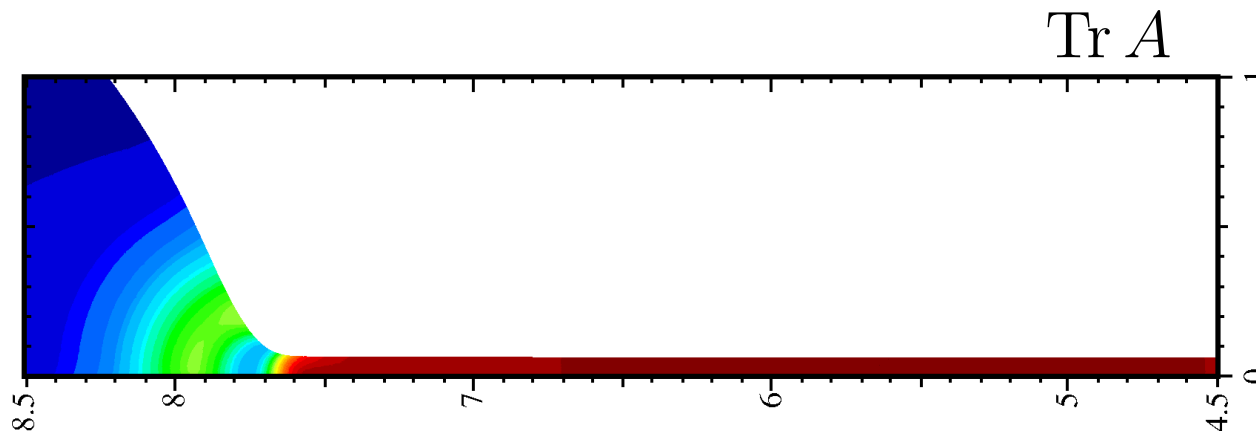
Finite **E**xtension **N**onlinear **E**lasticity

$$\frac{DA}{Dt} = A \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot A - \frac{f}{\tau} (A - \mathbf{I})$$

$$\sigma = -p\mathbf{I} + 2\mu_0 E + G f A$$

$$f = \frac{L^2}{L^2 - \text{trace } A} \quad \text{keeps } A < L^2$$

# FENE results



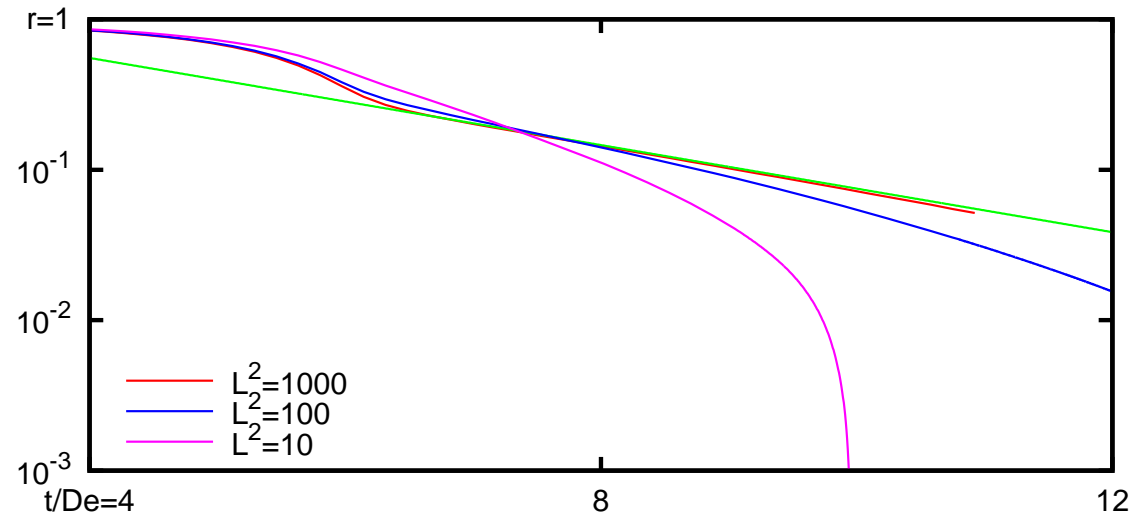
Slower relaxation:  $De = \tau \sqrt{\frac{\gamma}{\rho R^3}} = 30$

Concentration:  $c = \frac{G\tau}{\mu} = 1.5$

Ohnesorge (capillary Reynolds):  $Oh = \frac{\sqrt{\rho\gamma R}}{\mu} = 2.5$

# FENE thinning

Now breaks:



Hence design of inks:

Need concentration  $\times$  molecularweight<sup>2</sup> less than critical.