Jet break-up and polymers: a numerical study of beads-on-string

J. Etienne\textsuperscript{1,3}, J. Hinch\textsuperscript{1} & J. Li\textsuperscript{2}

\textsuperscript{1} DAMTP & \textsuperscript{2} BP Institute, Cambridge University
\textsuperscript{3} SPECTRO, UJF & CNRS, Grenoble
EPSRC collaboration

Companies

Universities: Cambridge, Durham, Leeds, Manchester, Wales
Effect of polymer

©2007 Steve Hoath, Ian Hutchings & Graham Martin
Difficult to break non-Newtonian jets

Polymers resist stretching – need rheology equation
Oldroyd-B model fluid

\[ \sigma = -pI + 2\mu_0 E + GA \]

\( \sigma \) stress, \( \mu_0 \) viscosity, \( G \) elastic modulus

with \( A \) microstructure.

\[ \frac{DA}{Dt} = A \cdot \nabla u + \nabla u^T \cdot A - \frac{1}{\tau} (A - I) \]

\( \frac{DA}{Dt} \) deform with fluid, \( \tau \) relaxation time
Numerical: Finite Elements

\[
\begin{align*}
\mathbf{u} & \quad P_2 \cap C^0 \\
\mathbf{p} & \quad P_1 \cap C^0 \\
\mathbf{A} & \quad P_1 \cap C^{-1}
\end{align*}
\]

"C++ code embedded in the free-software FEM environment rheolef (P. Saramito, N. Roquet, J. E.)"
Auto-adaptive mesh

Generated by free-software Bamg (F.Hecht)
Lagrangian approach

\[
\frac{DA}{Dt} = A \cdot \nabla u + \nabla u^T \cdot A - \frac{1}{\tau} (A - I)
\]

deform with fluid \hspace{1cm} \text{relaxes}

\[
u(x) + \delta x \nabla u
\]

\[
u(x) \hspace{1cm} \text{relax}
\]
Oldroyd-B results for low inertia

Slow relaxation: \( De = \tau \sqrt{\frac{\gamma}{\rho R^3}} = 95 \)

Concentration: \( c = \frac{G\tau}{\mu} = 3 \)

Ohnesorge (capillary Reynolds): \( Oh = \frac{\sqrt{\rho \gamma R}}{\mu} = 3.2 \)
Oldroyd-B results for high inertia

Slower relaxation: \[ De = \tau \sqrt{\frac{\gamma}{\rho R^3}} = 300 \]

Concentration: \[ c = \frac{G\tau}{\mu} = 3 \]

Ohnesorge (capillary Reynolds): \[ Oh = \frac{\sqrt{\rho \gamma R}}{\mu} = 10 \]
Oldroyd-B thinning

Mass
\[ \dot{a} = -\frac{1}{2} E a \]

Momentum
\[ \frac{\chi}{a} = 3\mu_0 E + G (A_{zz} - A_{rr}) \]

Microstructure
\[ \dot{A}_{zz} = 2EA_{zz} - \frac{1}{\tau} (A_{zz} - 1) \]

Solution
\[ a(t) = a(0)e^{-t/3\tau} \]

Never breaks!
FENE modification

Finite Extension Nonlinear Elasticity

\[
\frac{DA}{Dt} = A \cdot \nabla u + \nabla u^T \cdot A - \frac{f}{\tau} (A - I)
\]

\[
\sigma = -pI + 2\mu_0 E + GfA
\]

\[
f = \frac{L^2}{L^2 - \text{trace } A} \quad \text{keeps } A < L^2
\]
FENE results

Slower relaxation: $De = \tau \sqrt{\frac{\gamma}{\rho R^3}} = 30$

Concentration: $c = \frac{G\tau}{\mu} = 1.5$

Ohnesorge (capillary Reynolds): $Oh = \frac{\sqrt{\rho \gamma R}}{\mu} = 2.5$
FENE thinning

Now breaks:

Hence design of inks:
Need concentration $\times$ molecularweight$^2$ less than critical.