10 Instabilities

Spinline draw resonance

Buckling instability

Purely elastic instability of curved streamlines

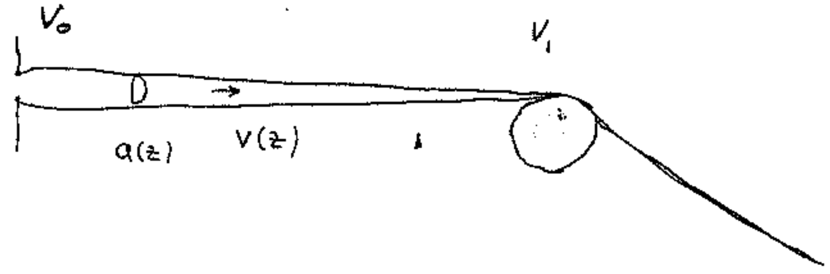
Coextrusion instability

Turbulent Drag Reduction & Couette/Poiseuille

High speed elastic jet

Spin line Draw Resonance

Ch15 "Mech of Poly Processing
JRA Pearson (85) Elsevier



Mass $\quad \frac{\partial a}{\partial t} + \frac{\partial}{\partial z}(aV) = 0$

Momentum : simplest - constant tension T & Newtonian viscous

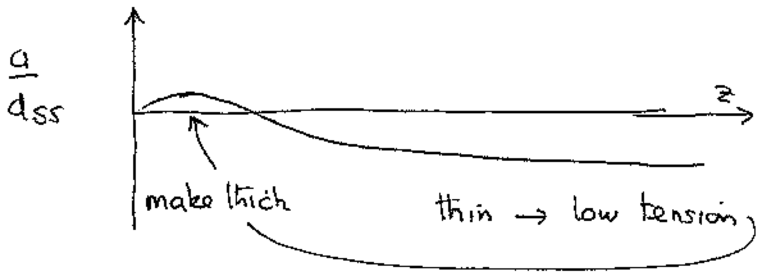
$$\frac{T}{a} = \sigma_{zz} = 3\mu \frac{\partial v}{\partial z}$$

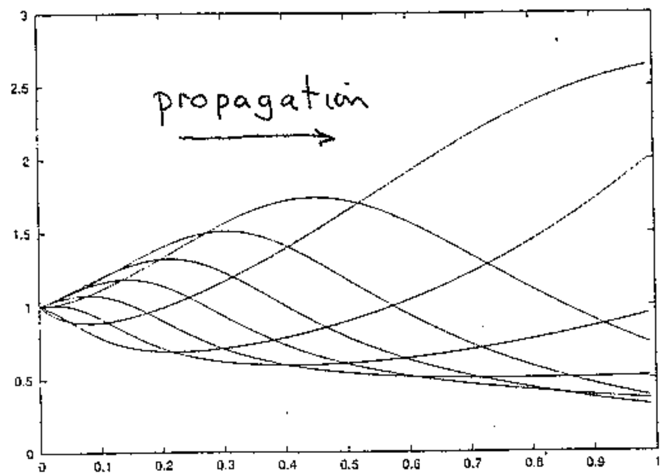
Unstable if Draw Ratio

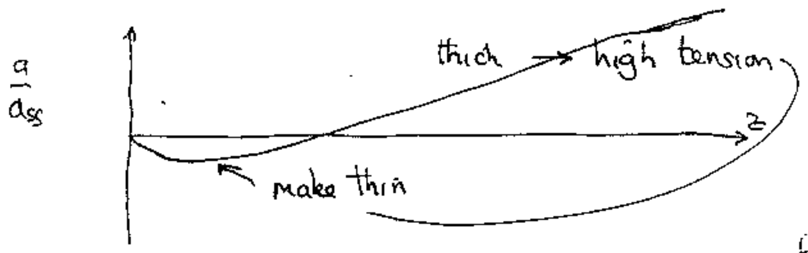
$$D_r = \frac{V_1}{V_0} > 20.3$$

Gelder (1971)
Ind Eng Chem Fund

Mechanism







Time delay (propagation) + amplified feedback

→ Unstable

Above critical draw ratio → Limit cycles

Destabilised

by surface tension   to $D_r = 3$!

by shear thinning

Stabilised

by inertia

by cooling (increasing viscosity)   to $D_r = 10^3$!

by elastic effects

Also

2D sheets drawn, film blowing

Buckling instability

while stretching a filament

S.H. Spiegelberg, G.H. McKinley / J. Non-Newtonian Fluid Mech. 67 (1996) 49–76

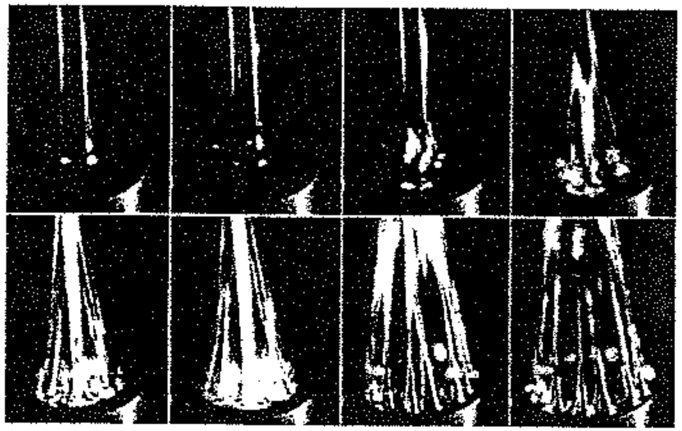


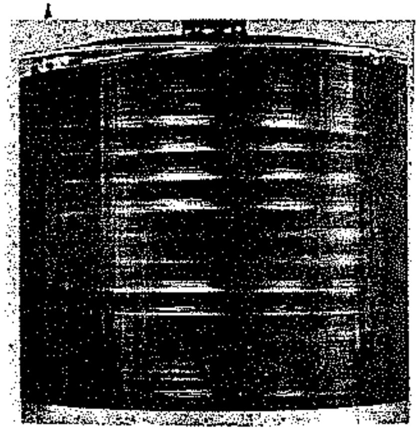
Fig. 9. Evolution of instability in a 0.2 wt.% PS-based Boger solution stretched at a strain rate $\dot{\epsilon}_0 = 0.90\ s^{-1}$. The first image is 3.42 s after the start of stretch, or $\epsilon = 3.1$. Images are spaced 0.13 s apart. The camera is inclined at 22° relative to the horizontal.

Purely elastic instabilities of curved streamlines

Shaqfeh (1996) Ann Rev FM 28

Taylor - Couette

$Ta = 10^{-7}$



MLS (1989)

Large elastic effect with no inertia thy + exp

Muller/Larson/Shaqfeh (1989) Rheol Acta 28
(1990) JFM 218
(1993) JNNFM 46

Earlier : small destabisation of classical inertia

Thomas e Walters (1964) JFM 18

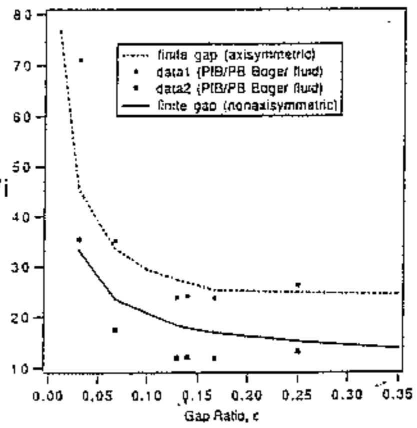
Critical

$\varepsilon^{1/2}\, Wi \simeq 8$

$\uparrow$

De Wi

Wi

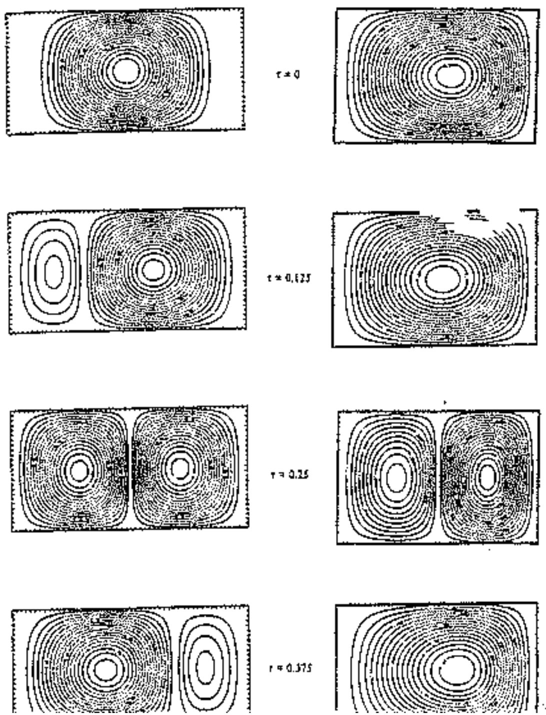


ε = Gap ratio

Oscillating
+ nonaxisymmetric

Joo & Shaqfeh (1996)
JFM 266

Axi sym $\qquad$ n = 1



Supercritical Hopf

"Elastic turbulence"

Grossman & Steinberg
(1998) PoF 10

Mechanism - need clearer explanation

Hoop stress in curved tensioned streamlines

$\longrightarrow$ oscillation

Phase delay by stress relaxation

$\longrightarrow$ growth

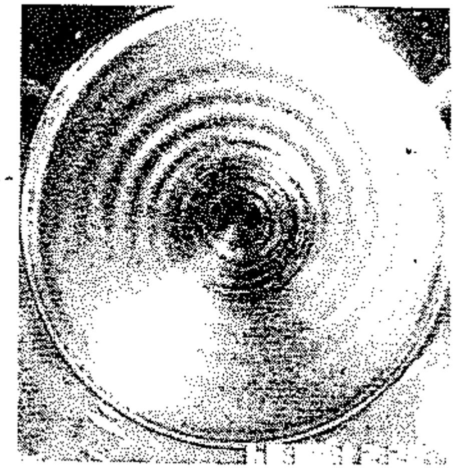
More flows with curved tensioned streamlines

Taylor - Dean

Plate - plate

Cone & Plate

Cone & Plate



McKinley, Byars, Brown, Armstrong (1991) JNNFM 60

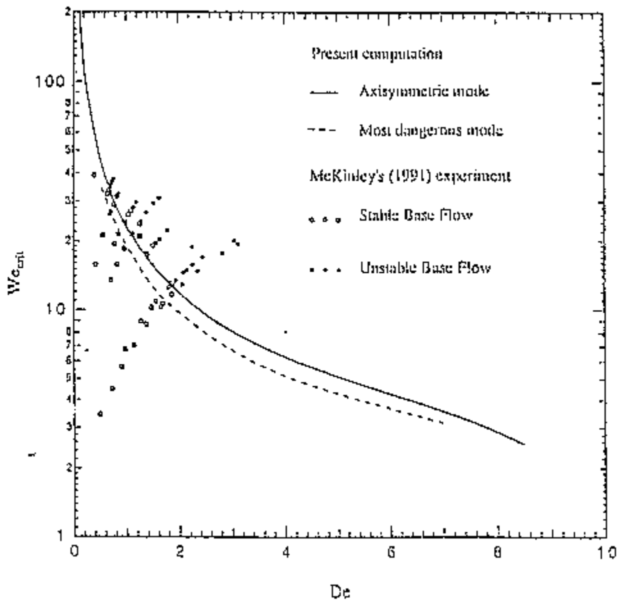
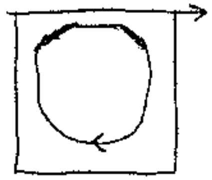


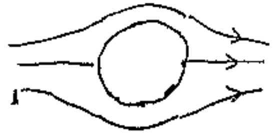
Figure 17 Comparison of the critical conditions predicted by the theory for the elastic plate-and-plate flow (Oztekin & Brown 1993) and flow measurements made by McKinley et al (1991b).

Oztekin Brown (1993) JFM 255

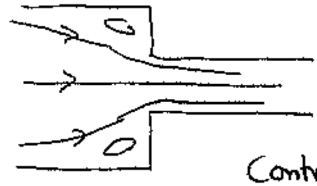
Yet more flows with curved tensioned streamlines



Lid driven cavity



Past cylinder



Contraction

McKinley criterion

McKinley, Pakdel & Öztekin (1996) JNNFM 67

$$\left(\frac{\tau U}{R} \frac{\sigma_{xx}}{\sigma_{xy}} \right)^{1/2} >$$

8.4	Taylor-Couette
4.6	Cone & plate
4.8	Lid-driven cavity
6.1	Flow past cylinder

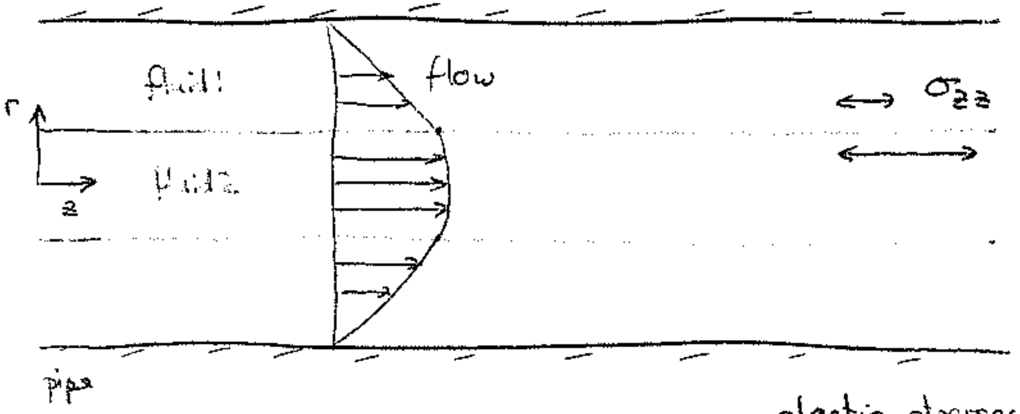
τ = relaxation time

R = radius of curvature

11 25

Instability of coextrusion

DJ Harris, JM Rallison & EJH 92



elastic stresses

(normal stresses in simple shear flow)

$\underline{\underline{\sigma}} \cdot \underline{n}$ continuous at interface

$\therefore$ σ_{zz} can jump across undisturbed interface

12. (26)

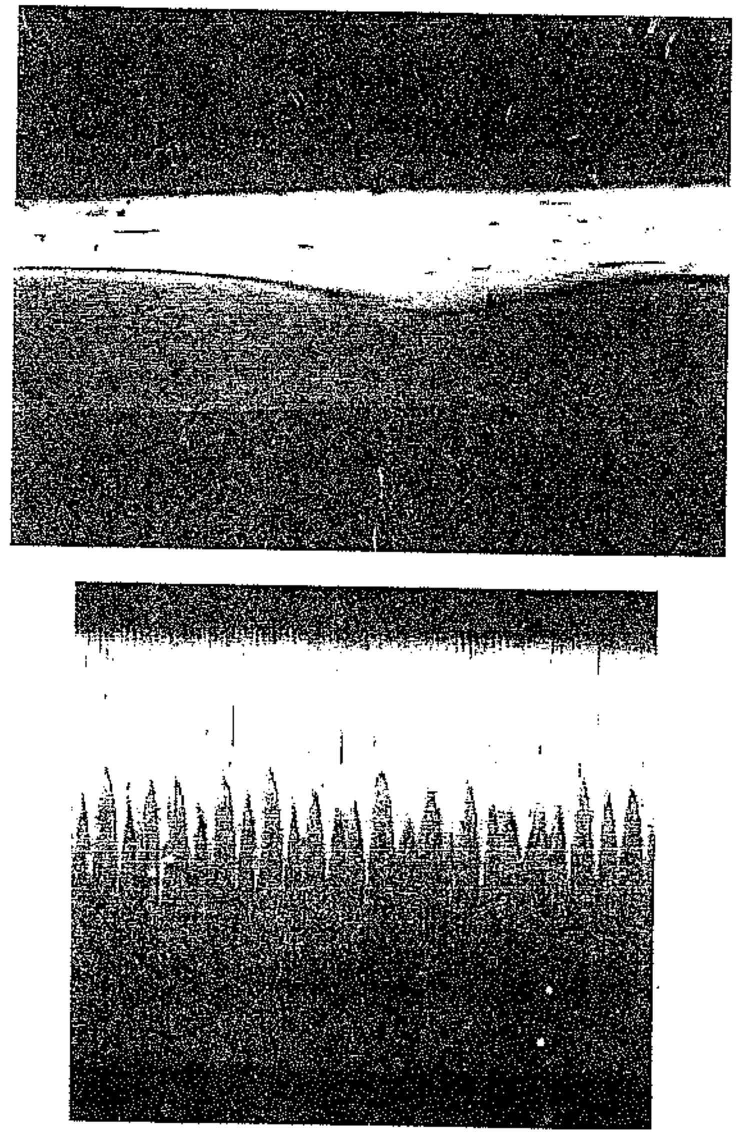
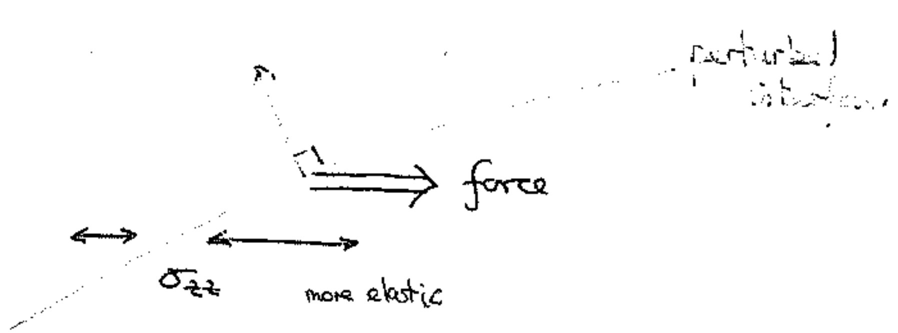
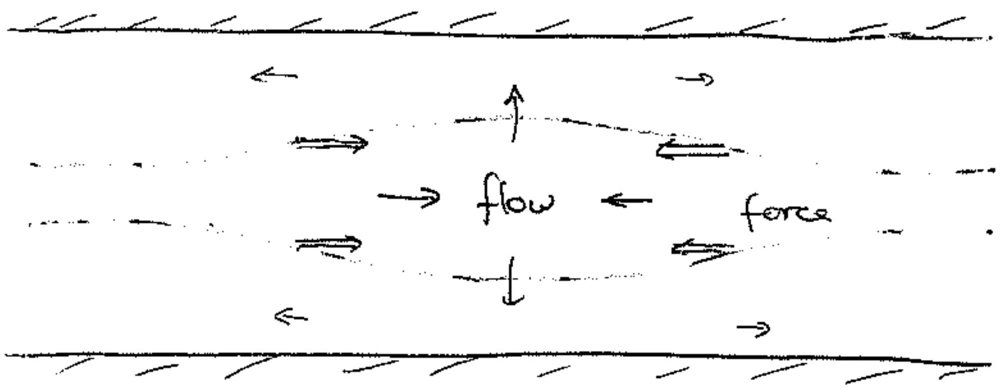


Fig. 7. Interfacial instability of the PP/HDPE system (PP in upper layer); random disturbances. Top: single video frame from window no. 4 with "random" coextrusion disturbances; bottom: composite image from window no. 4 with "random" coextrusion disturbances.

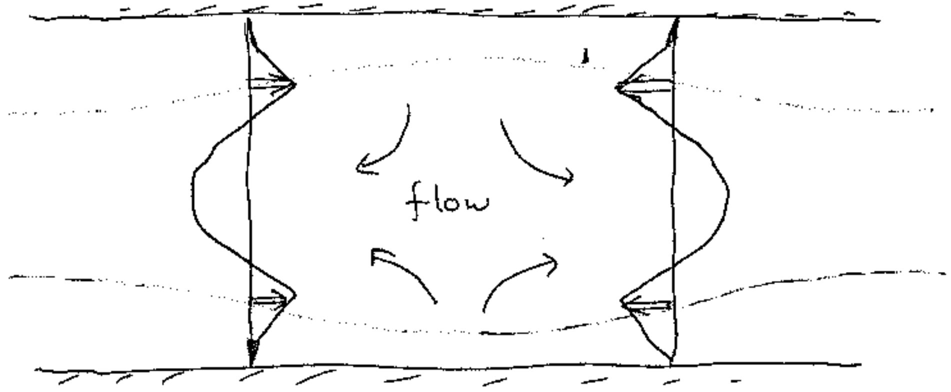


Core more elastic



∴ Unstable

But stable with large core

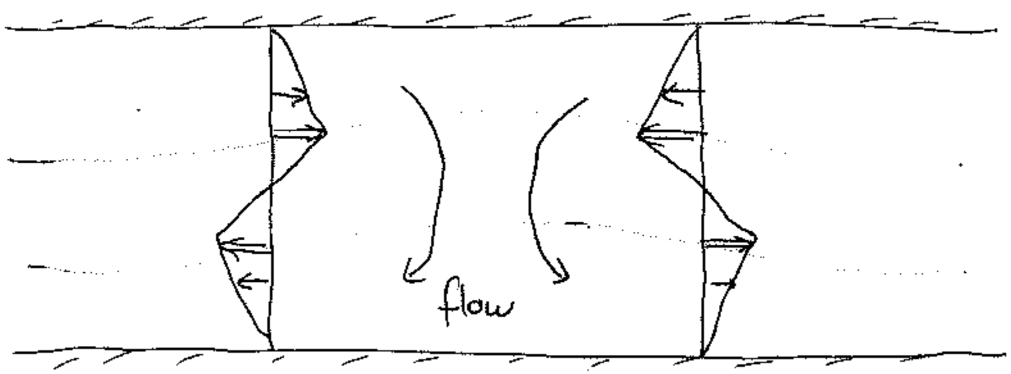


∴ stable

if core more elastic & > 32% section

core less elastic & < 68% section

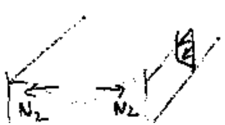
Sinuous mode



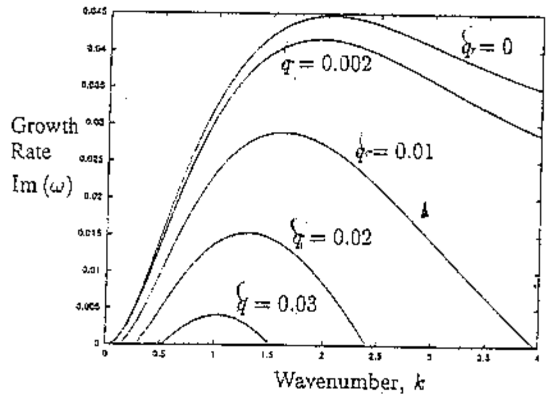
∴ Stable if core more elastic

Also N_2 driven instabilities

recent N_2 instability by Brady & Carpen (2002) JNNFM 102



Coextrusion : stabilised when discontinuity smoothed



Wilson & Rallison (1999) JNNFM 85

Short waves stabilised if $k\delta > 0.5$

Long waves stabilised by destructive phase mixing

eg if $\omega \sim k U(y) + i a k^2$

$$\begin{cases} t_{growth} = 1/ak^2 \\ \text{phase change over layer} \quad k \frac{dU}{dy} \delta . t \end{cases}$$

∴ Destructive (stable) if $k < \frac{dU}{dy} \delta / 2\pi a$

Turbulent Drag Reduction

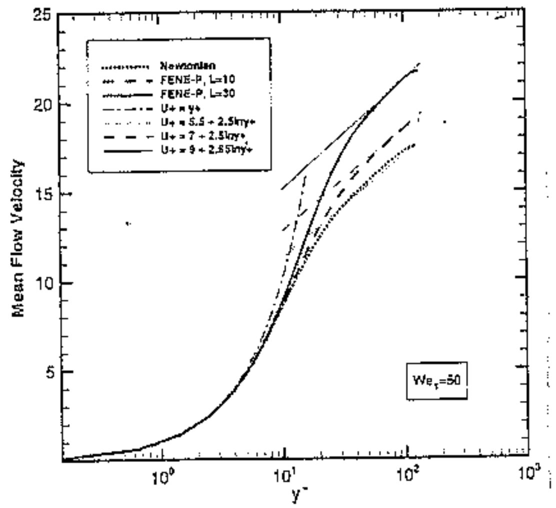
Dimitropoulos, Sureshkumar & Beris
(1998) JNNFM 79

At given drag

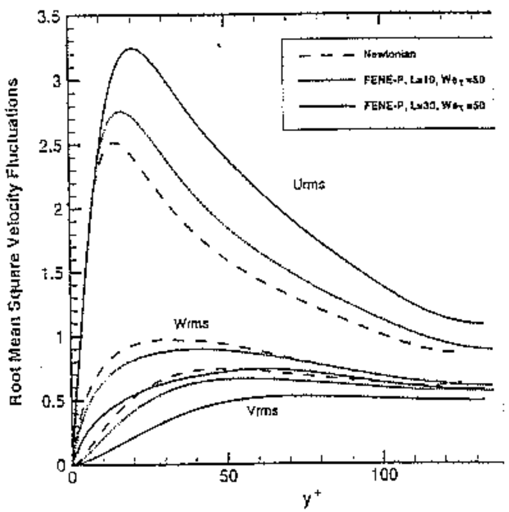
mean flow increases →

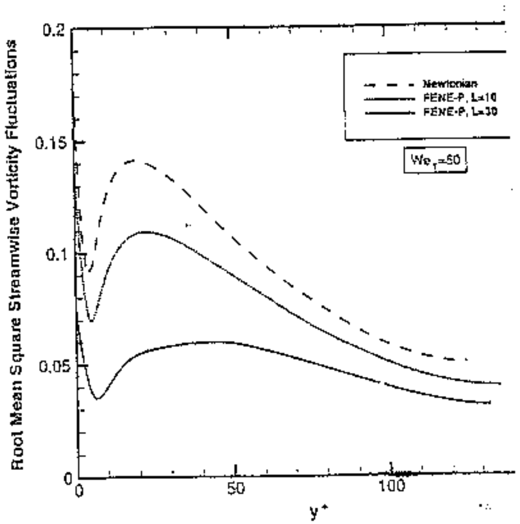
downstream u′ increases

↙ vortex spacing increases ↘



$Re \sim 2000, \ \frac{\eta_p}{\eta_s} \approx 9, \ 64^3$



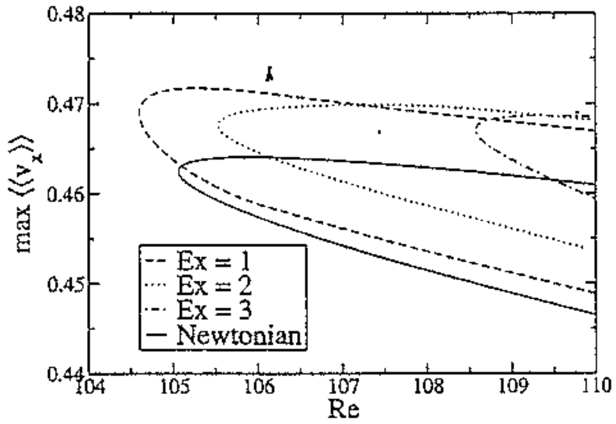


Nonlinear states in plane Couette & Poiseuille

Couette

'Exact Coherent States'

Stone, Waleffe & Graham
(2002) PRL 89

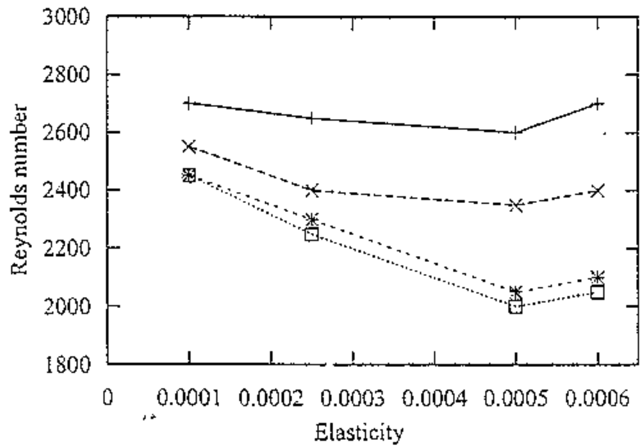


FENE-P $\qquad Ex = \rho_{ext}/\rho_{shear}$

Poiseuille

'Non-normal'

Atalik & Keunings
(2002) JNNFM 102



$C = 0.9$

$C = 0.5$

$C = 0.1$

$C = 0.01$

$C = \frac{\eta_s}{\eta_s + \eta_p}$

$E = \frac{2\lambda}{h^2}$

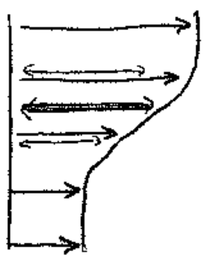
Instability of a high speed submerged elastic jet

by J M Rallison + E J Hinch

DAMTP, Cambridge

(1995) JFM 288

[illegible] :



Elasticity ⇒ tension in streamlines

[illegible] : [illegible] + "surface tension"

But problems setting up basic state.

Governing equations

$$\nabla \cdot u = 0$$

$$\rho \frac{Du}{Dt} = -\nabla p + \mu \nabla^2 u + G \nabla \cdot A$$

elastic stress GA

$$\frac{DA}{Dt} = \underbrace{A \cdot \nabla U + \nabla U^T \cdot A}_{\text{stretch by flow}} - \underbrace{\frac{1}{\tau}(A - 1)}_{\text{relaxation}}$$

Basic state

$$u = (U(y,z), 0, 0)$$

tension in streamlines

$$A = \begin{pmatrix} 1 + 2\tau^2 (U_y'^2 + U_z'^2) & \tau U_y' & \tau U_z' \\ \tau U_y' & 1 & \\ \tau U_z' & & 1 \end{pmatrix}$$

High speed

High Reynolds number $\quad \frac{\rho U L}{\mu + G\tau} \gg 1$

High Weissenberg number $\quad \frac{U\tau}{L} \gg 1$

Linearise

$$u_x + v_y + w_z = 0$$

$$\rho\left[u_t + U u_x + v U_y + w U_z \right] = -p_x + G\left[a_{11,x} + a_{12,y} + a_{13,z}\right]$$

$$\rho\left[v_t + U v_x \right] = -p_y + G a_{12,x}$$

$$\rho\left[w_t + U w_x \right] = -p_z + G a_{13,z}$$

$$a_{11,t} + U a_{11,x} + v A_{11,y} + w A_{11,z} = 2A_{11} u_x + 2a_{12} U_y + 2a_{13} U_z$$

$$a_{12,t} + U a_{12,x} = A_{11} v_x$$

$$a_{13,t} + U a_{13,x} = A_{11} w_x$$

Solution

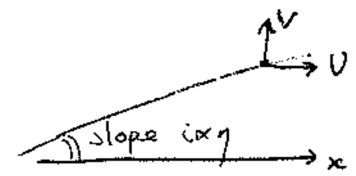
$$e^{i\alpha(x-ct)}$$

real wavenumber α

growth rate $\sigma = \alpha c_i$

Transform to streamline displacements

$$y \longrightarrow y + \eta \qquad z \rightarrow z + \zeta$$



$$v = i\alpha\eta\,(U - c)$$

$$u = -\eta U_y - \zeta U_z \quad - (U-c)(\eta_y + \zeta_z)$$

U constant on displaced streamlines

acceleration from crowding of streamlines

$$a_{12} = i\alpha\eta A_{11}$$

stress tilted

$$a_{11} = -\eta A_{11,y} - \zeta A_{11,z} - 2A_{11}(\eta_y + \zeta_z)$$

a_{11} constant on displaced streamlines

stress concentrated by crowding of streamlines

Elastic Rayleigh equation

$$x: \quad i\alpha \left[\rho(U-c)^2 - GA_{11}\right](\eta_y + \zeta_z) = i\alpha\, p$$

acceleration from crowding streamlines (~ Bernoulli)

extra stress from crowding streamlines

x - pressure gradient

$$\begin{array}{l} y: \\ z: \end{array} \quad \left[\rho(U-c)^2 - GA_{11}\right] \begin{pmatrix} \alpha^2 \eta \\ \alpha^2 \zeta \end{pmatrix} = \begin{pmatrix} p_y \\ p_z \end{pmatrix}$$

centrifugal acceleration

hoop stress

curved streamlines

Self-adjoint $\longrightarrow$ semi-circle theorem

Time-scale of instability $\ll$ vorticity diffusion, stress relaxation, shear wave propagation

Two-dimensional jet

$$U = \begin{cases} U_0\left(1 - \frac{y^2}{b^2}\right) & |y| \le b \\ 0 & \text{outside} \end{cases}$$

$$GA_{11} = \begin{cases} 8G\tau^2 U_0^2\, y^2/b^4, & |y| \le b \\ 0 & \text{outside} \end{cases}$$

Elasticity number

$$E = \frac{GA_{11}}{\rho U^2} = \frac{G\tau^2}{\rho b^2}$$

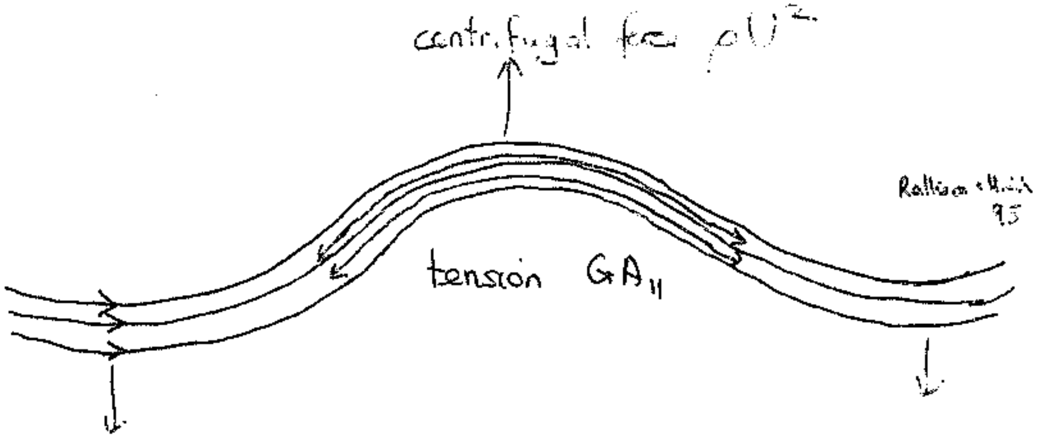
Outside jet

Potential flow : $\quad p = e^{\mp \alpha y}, \quad \eta = \mp \frac{e^{\mp \alpha y}}{\alpha c^2}$

7

Stabilisation of jets

Sinuous mode, Long waves $^{23}E\,4$



x-mtm : $\eta \sim 1$ across jet

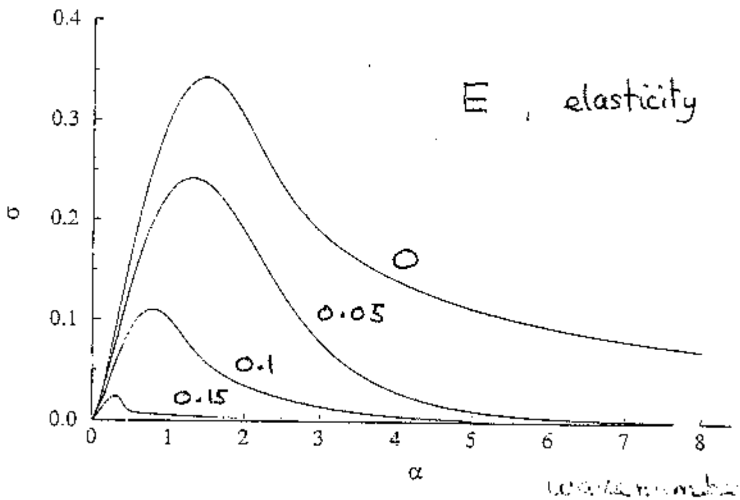
y-mtm $\dfrac{dp}{dy} = \left[\rho (U-c)^2 - GA_{11} \right] \alpha^2 \eta$

Match to potential flow outside $-\alpha c^2 = \left[c^2 - \frac{4}{3} c - \frac{8}{15} - \frac{8}{3} E \right] \alpha^2$

$\therefore \sigma_{max} = \dfrac{9\sqrt{3}}{2} \left(\dfrac{1}{5} - E \right)^2$

Sinuous mode (2D)

– more unstable at E = 0



E, elasticity $\dfrac{G\tau^2}{\rho b^2}$

Stabilized, longwaves less stable

Conclusions

Two-dimensional jet

Sinuous mode stablised at $E = 0.2$

hoop stress vs centrifugal

Varicose mode reduced growth

complex mechanism

Axisymmetric jet

Sinuous mode stablised at $E = 0.3756$

Varicose mode stablised at $E = 0$ & $E = 0.228$

New mode : elastic waves on a shear flow

Effects of $Re \neq \infty$, $We \neq \infty$?