Chapter 9. Stress relaxation

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Hence

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Important relaxation time τ of stress/microstructure.

Linear visco-elasticity – common to all fluid models

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μ_0 solvent viscosity, G elastic modulus, τ relaxation time.

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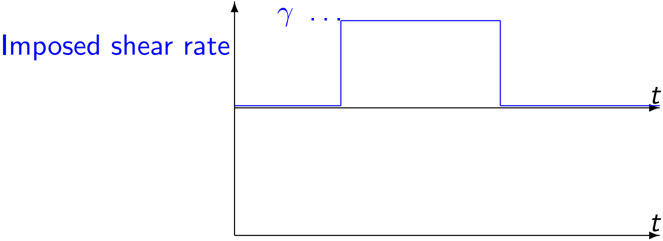

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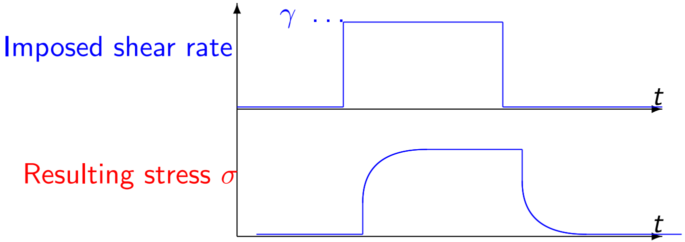

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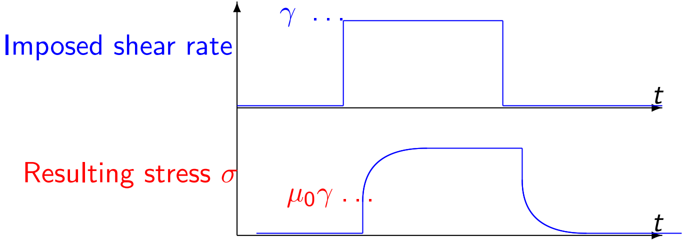

- Early viscosity μ_0

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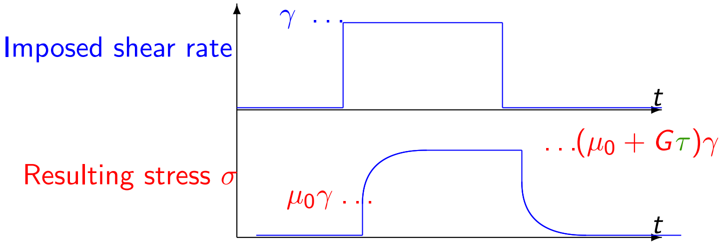

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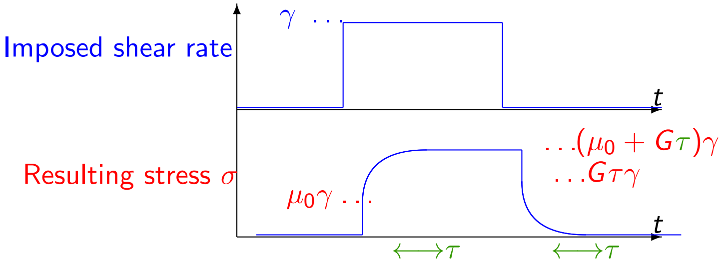

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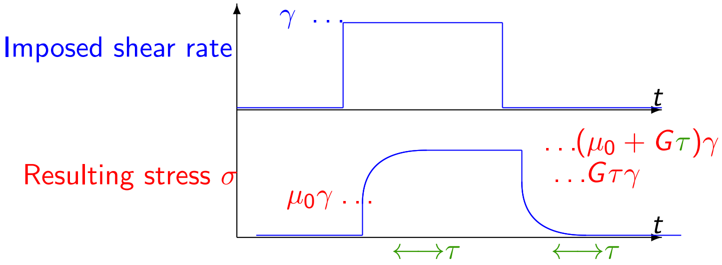

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E.G. linear visco-elasticity for Oldroyd-B

Microstructure A:

$$\frac{DA}{Dt} - \nabla u^T \cdot A - A \cdot \nabla u + \frac{1}{\tau}(A - I) = 0$$

Stress σ:

$$\sigma = -pI + 2\mu_0 E + G(A - I)$$

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Start up:

$$a = \dot{\gamma}\tau\left(1 - e^{-t/\tau}\right) \quad \sigma = \mu_0\dot{\gamma} + G\dot{\gamma}\tau\left(1 - e^{-t/\tau}\right)$$

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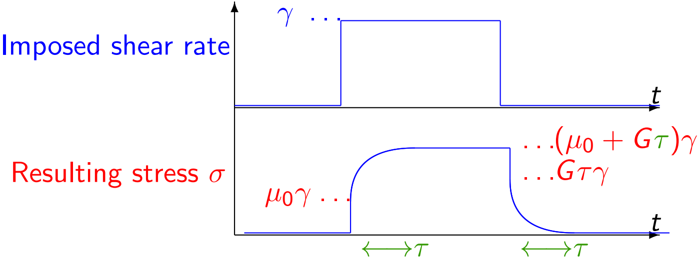

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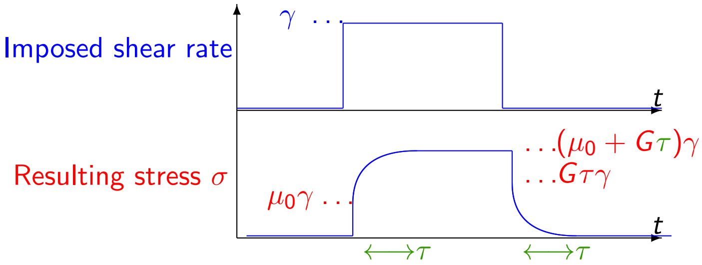

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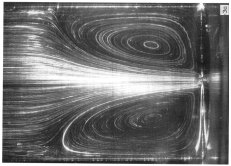
NB steady flows are unsteady Lagrangian.

Contraction flow – Lagrangian unsteady



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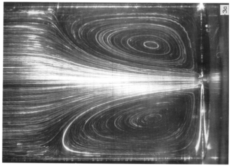
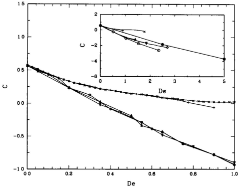


Numerical Oldroyd-B

$$\frac{\Delta P}{\text{Stokes}}$$



$$U\tau/L$$

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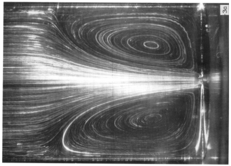

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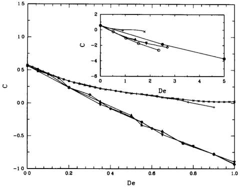

$U\tau/L$

Experiments

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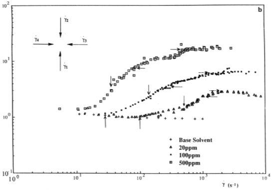

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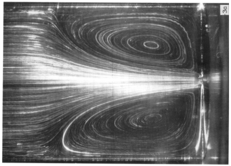

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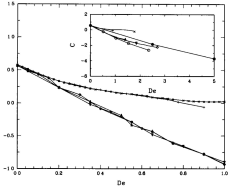

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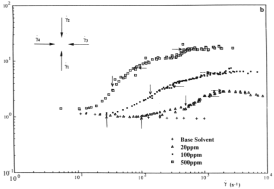

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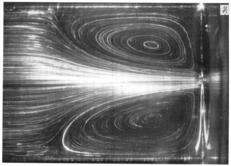

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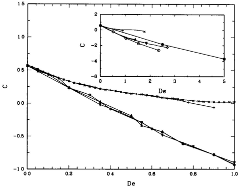

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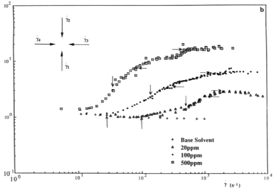

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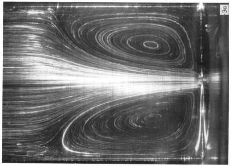

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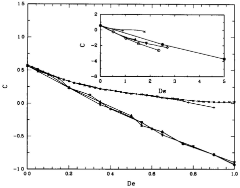

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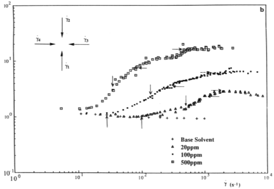

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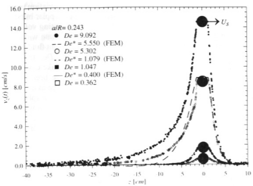
Oldroyd-B has no big increase in Δp, and no big upstream vortex

Flow past a sphere – Lagrangian unsteady



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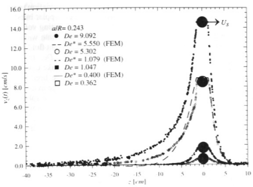

Numerical Oldroyd-B

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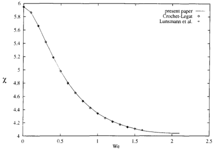

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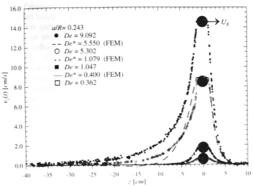

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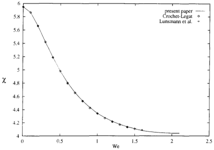

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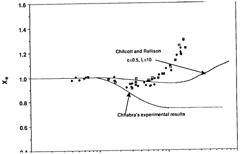

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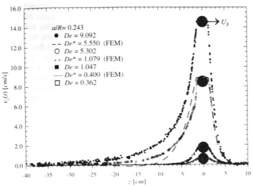

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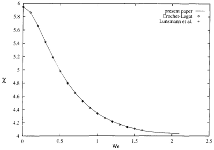

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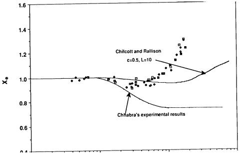

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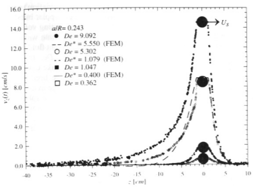

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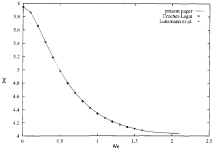

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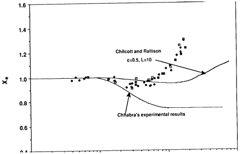

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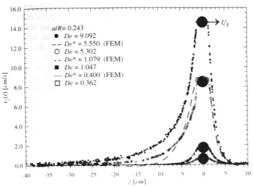

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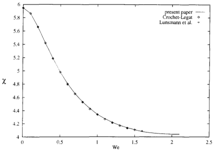

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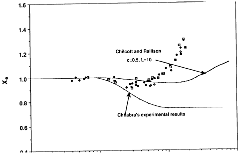

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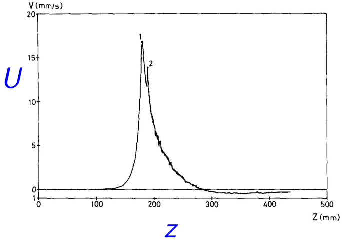
Drag scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

But if flow fast lower, lower drag from early-time viscosity μ_0.

Oldroyd-B has no big increase in drag, and no big wake

. . . and negative wakes

Experiment



Bisgaard 1983 JNNFM

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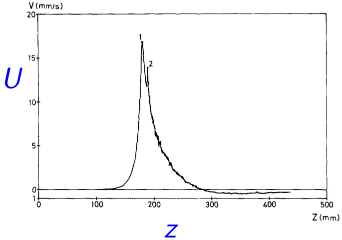


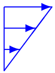
Bisgaard 1983 JNNFM

Unrelaxed elastic stress in wake, cancelled by negative viscous flow.

Tension in streamlines

– relaxation + slightly nonlinear effect

shear γ



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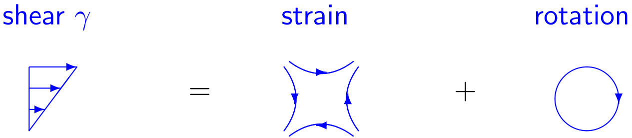

Tension in streamlines

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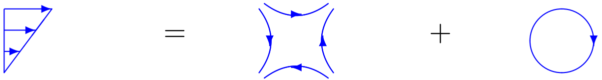
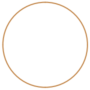
shear γ strain rotation





microstructure

Tension in streamlines

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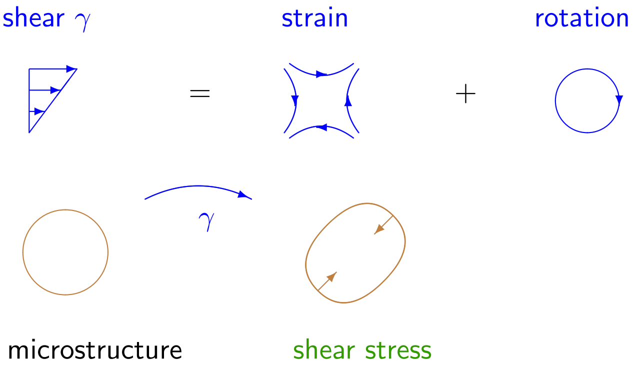

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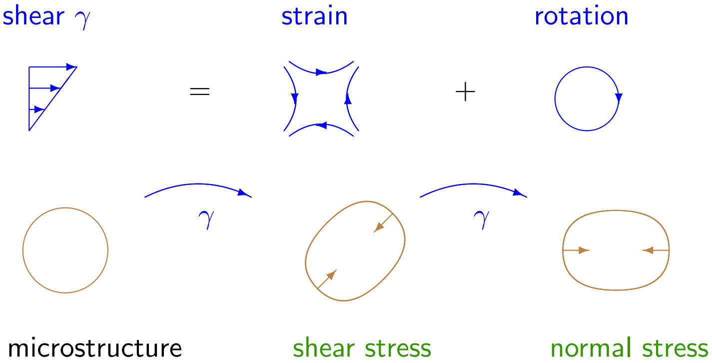

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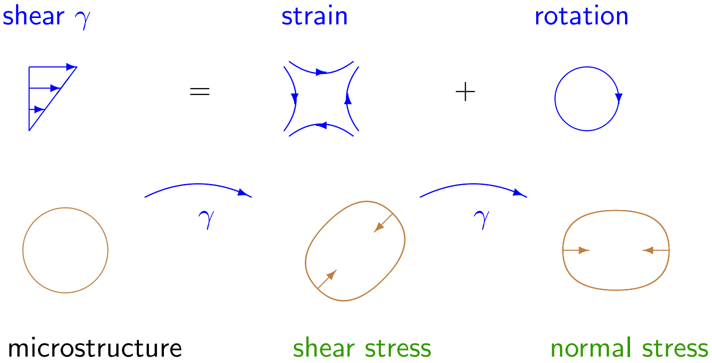
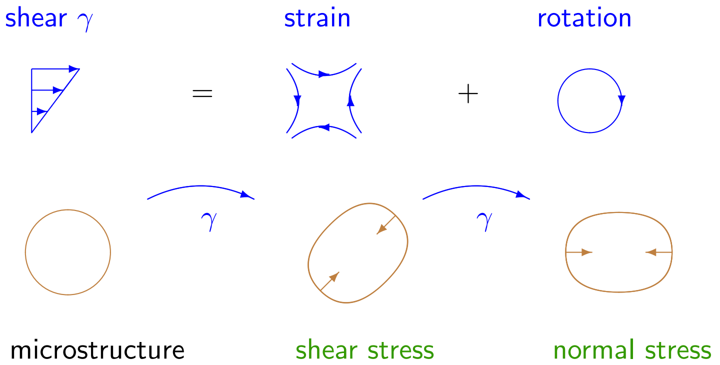


Shear stress $= G \times (\text{rate} = \gamma) \times (\text{memory time} = \tau)$

Tension in streamlines

– relaxation + slightly nonlinear effect



Shear stress $= G \times (\text{rate} = \gamma) \times (\text{memory time} = \tau)$

Normal stress (tension in streamlines) $=$ shear stress $\times \gamma\tau$.

Tension in streamlines

- Rod climbing
- Secondary circulation
- Migration into chains
- Migration to centre of pipe
- Falling rods align with gravity
- Stabilisation of jets
- Co-extrusion instability
- Taylor-Couette instability