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Electromagnetic billiards

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Moffatt has argued [J. Fluid. Mech. **218**, 509 (1990)] that it should be possible to control the motion of a metallic sphere immersed in a nonconducting fluid using the electromagnetic force and couple generated by means of a traveling and/or rotating magnetic field applied externally. Such a system has been realized experimentally: one or more aluminum spheres are placed in a vessel containing fluid (air, water, or silicone oil), the whole being placed in an inductor which provides an upward-traveling magnetic field. The spheres move in response to the induced electromagnetic forces, the motion being influenced by gravity, viscous drag, vessel boundary reaction, and collisions. A range of possible behaviors, stable, unstable, and chaotic, are identified and discussed. The term "electromagnetic billiards" seems appropriate to describe this phenomenon.

We study experimentally the behavior of electrically conducting spheres in an insulating bounded liquid placed in a vertical traveling magnetic field. The nonmagnetic spheres develop induced currents. Each sphere is submitted to both electromagnetic torque and net force.

The experimental arrangement, sketched in Fig. 1, consists of a three-phase inductor of six turns, of internal diameter 20 cm, and pole pitch 14 cm. The maximum current amplitude in each turn is 1500 A at frequency 50 Hz. A cylindrical glass vessel of internal diameter $2R = 16$ cm is centrally placed in the inductor with its bottom level with the center of the coil, and may be filled with water, silicone oil, or just air. The maximum magnetic field produced by the inductor varied from about 0.21 T at the wall of the vessel to 0.07 T at the center. Aluminum spheres of diameter in the range 1 to 4 cm are placed in the vessel, and their trajectories and velocities are measured, using a video camera and picture analysis.

The expressions for the force **F** and torque **G** experienced by a sphere of radius a and conductivity σ in a magnetic field of the form $\mathbf{B}(\mathbf{x},t) = \mathrm{Re}[\hat{\mathbf{B}}(\mathbf{x})e^{-i\omega t}]$ are given by[1]

$$\mathbf{F} = 2\pi a^3 \mu_0^{-1}\,\mathrm{Re}(\alpha\hat{\mathbf{B}}\cdot\nabla\hat{\mathbf{B}}^*),$$

$$\mathbf{G} = 2\pi a^3 \mu_0^{-1}\,\mathrm{Re}(\alpha\hat{\mathbf{B}}\times\hat{\mathbf{B}}^*),$$

where $\alpha = \frac{1}{2}(3\eta^{-2} - 3\eta^{-1}\cot\eta - 1)$, $\eta = (1+i)\lambda$, $\lambda = (\frac{1}{2}\mu_0\sigma\omega)^{1/2}a$, and $\mu_0 = 4\pi\times 10^{-7}$ (SI units). For example, for aluminum spheres of radius 2 cm and with $\omega/2\pi = 50$ sec^{-1}, we find $\lambda \approx 0.86$ and $\alpha \approx -0.0065 + 0.0472i$. The electromagnetic force F acting on each sphere is then of order $2\pi a^3 \mu_0^{-1}|\alpha|\;|B|^2/R \sim 2\times 10^{-3}$ N, quite small compared with the weight $\sim 10^{-1}$ N. There is also an unbalanced electromagnetic torque $G \sim 4\pi a^3 \mu_0^{-1} \alpha^{(i)}|B|^2 \sim 6\times 10^{-5}$ N m on each sphere, causing a rolling motion on the bottom of the vessel. In the absence of viscous drag, this generates velocities of the order of 10 cm sec^{-1} as actually observed in the experiments in water described below. However, an experiment with a single sphere of diameter 1 cm *in air* gave a mean velocity of 57 cm sec^{-1}. We have as yet no explanation for this remarkably large effect.

Figures 2–4 show observed trajectories of spheres of radius 1.5 cm placed in water. The picture analysis software actually follows a light spot on each sphere, so that there may be an error of the order of the sphere radius in locating its center. Nevertheless, the trajectories provide a good indication of the possible types of motion. A single sphere [Fig. 2(a)] simply oscillates back and forward along a diameter, with frequency 0.57 Hz (so that the mean speed is 19 cm sec^{-1}). The velocity of the sphere between collisions with the wall [inferred from Fig. 2(b)] appears remarkably uniform, despite the nonuniform force field (stronger near the vessel boundary than near the center). If silicone oil (with viscosity 30 times that of water) is used, or if much weaker electromagnetic forcing is used, then the sphere comes to rest in its metastable position at the center; under some circumstances, a larger sphere (diameter 4 cm) may rotate around the center axis.

If two spheres are used, the motion is more complex

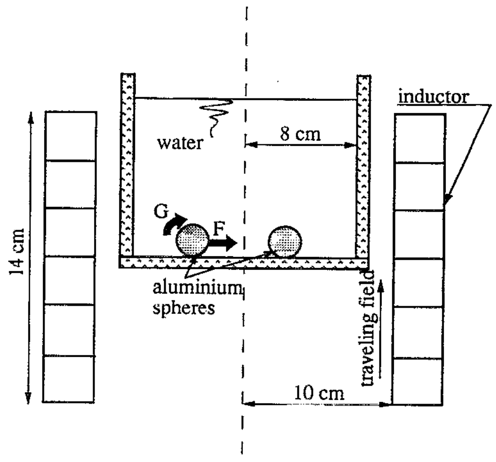

FIG. 1. Experimental configuration.

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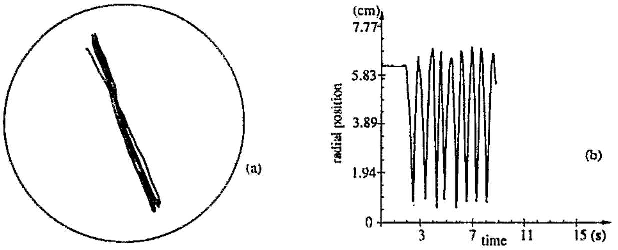

FIG. 2. (a) Trajectory of a single sphere (radius 1.5 cm) in water. (b) Corresponding radial position as a function of time.

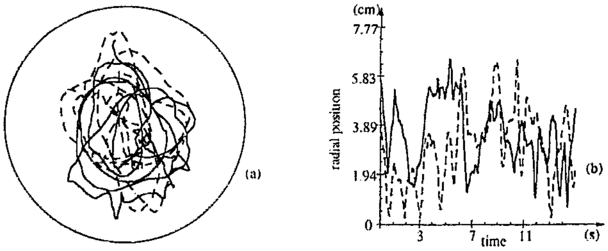

FIG. 4. As for Fig. 2, but showing trajectories of two spheres in a seven-sphere experiment. Note the preferred sense of rotation that is generated.

due to collisions. Depending on initial conditions, the spheres may oscillate independently along different diameters, or one may stay at the center while the other moves irregularly around it. Either regime may be disturbed by collisions. Figure 3(a) shows the rather complex trajectories of two spheres in such circumstances; note that the fundamental periodicity is still detectable in the traces of Fig. 3(b), and that occasional collisions with the vessel wall still occur.

When three or more spheres are used, the motion becomes more dominated by sphere–sphere collisions, and the trajectories become more complex. An interesting new phenomenon now appears: a preferred sense of rotation of the spheres about the axis of the vessel is established, and this rotation is imparted also to the fluid. This behavior may be detected in Fig. 4(a), which shows the trajectories of two spheres in a seven-sphere experiment. A preferred sense of rotation is evident, and this shows up also in Fig. 4(b) through a pronounced reduction in mean radial displacement [as compared with Figs. 2(b) and 3(b)]. If the experiment is repeated many times, this preferred sense of rotation is always present, but may be clockwise or anticlockwise with equal probability. If silicone oil is used instead of water, the viscous drag on each sphere is increased, and the trajectories are more controlled; the range of behavior is then more limited within the range of electromagnetic forcing available.

These experiments are clearly preliminary in nature, but already they indicate an intriguing range of possible behaviors. The possibility of exerting a force and a couple on an immersed body through remote control is clearly demonstrated. The frequency, intensity, geometry, and phase of the applied field are all variables that may in principle be controlled. We have used spheres, but obviously immersed bodies of any shape could be used, and the motion of such bodies under a variable force and couple could be investigated by this technique. The geometry of the containing vessel may also be varied; for example, we find that a slightly convex vessel floor is conducive to more chaotic trajectories. Finally, the conductivity and density of the immersed bodies may be varied; the case of neutrally buoyant particles would be of particular interest.

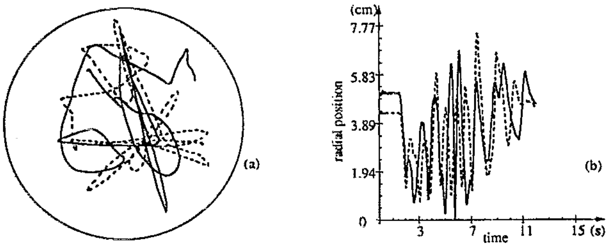

FIG. 3. As for Fig. 2, but with two spheres in water.

The following imperfections in our preliminary experiments should be noted: (i) the aluminum balls used were perhaps not exactly spherical; (ii) the floor of the vessel was not machined to be rigorously flat, and the corner between the vessel wall and the bottom was slightly rounded; (iii) as indicated above, the software for picture analysis followed a light spot on each sphere, but could not follow the exact location of the sphere center; (iv) time sequences were short, so that time series analysis was impracticable without modification of the software.

Despite these shortcomings, the experiments demonstrate a novel technique of potential importance for the fundamental study of the response of immersed particles or bodies to prescribed force and torque distributions. The development of coherent rotation when several spheres are used is of particular interest, and could have potential importance for stirring and mixing in chemical engineering processes.

Finally, we note an interesting complementary with the problem of controlling the motion of *insulating* particles immersed in *liquid metal.*[2] In electromagnetic separation processes, such particles can be removed using differential electromagnetic effects between the particles and the surrounding liquid. Experimental investigation of this process is, however, extremely difficult due to the opacity of the liquid metal. The "inverse" problem described in this Brief Communication may, however, provide some clues concerning the nature of such electromagnetic separation processes.

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[1] H. K. Moffatt, "On the behavior of a suspension of conducting particles subjected to a time-period magnetic field," J. Fluid Mech. **218**, 509 (1990).

[2] G. Gerbeth and D. Hamann, "Dispersion of small particles in MHD flows," in *Liquid Metal Magnetohydrodynamics* edited by J. Lielpeteris and R. Moreau (Kluwer Academic, Dordrecht, The Netherlands, 1989), pp. 97–102.