

Magnetohydrodynamic Flow and Turbulence: a report on the Fifth Beer-Sheva Seminar

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(Received 20 May 1987)

This paper is a summary of the Fifth Beer-Sheva Seminar on Magnetohydrodynamic (MHD) Flows and Turbulence, held in Jerusalem during 2–6 March 1987, with 99 participants from 12 countries. Reviews and research papers were presented on general problems of turbulence, MHD turbulence, fundamental MHD, two-phase flows with and without magnetic fields, and on different applications of liquid-metal MHD, especially in power generation nuclear fission and fusion, and in metallurgy.

1. Introduction

The Beer-Sheva International Seminars on MHD Flows and Turbulence have been held in 1975, 1978, 1981 and 1984. The recent Seminar was the fifth in this series and, unlike the previous ones, it was held in Jerusalem at the Israel Academy of Sciences. The latter is likely to become the permanent home of future Beer-Sheva Seminars, which, however, will still be prepared and organized from the Center for MHD Studies at Ben-Gurion University in Beer-Sheva.

The number of papers presented at this Seminar was again greater than at any of the previous seminars, and concern has been expressed that if the number of participants and papers presented continues to increase the Beer-Sheva Seminar may lose its special character and emphasis on informal discussions and exchange of ideas. Therefore the choice of papers should be even more selective and the number of papers should be limited.

At the present Seminar only the invited and review papers were presented orally while all other contributed papers were included in three poster sessions, each one followed by a panel discussion. The programme of this seminar had a very strong accent on turbulence, primarily on general problems of hydrodynamic (HD) turbulence, but also on MHD turbulence. A number of review papers, panel sessions and extensive discussions focused on such questions as turbulence visualization and advancement of local measurements, coherent structures, helicity, direct numerical simulation, use of the renormalization-group method, etc. In several papers the interrelation of MHD and HD turbulence were discussed, and the significance of results related to MHD turbulence for the understanding of HD turbulence was demonstrated. Regarding applied studies, substantial progress was reported in the area of building and testing integrated liquid-metal MHD energy-conversion facilities as well as in the area of different metallurgical applications of MHD.

In this review the name of the author who presented the paper is printed in italics. Some additional references cited by the author or in the discussions are also listed.

2. Turbulence

Hussain (University of Houston, Texas) analysed a number of fundamental issues related to the phenomenon of coherent structures in turbulent flows, their nature and role. He stressed the point that although virtually every turbulence researcher pursues coherent structures in one form or another, there is yet no consensus on what these structures are and what their significance is. Most opinions are based on flow visualization, which is at best qualitative and sometimes misleading. The author's opinion is that direct flow measurements (by a rake of cross hot wires or by more recently evolving techniques) and direct numerical simulation can throw much light on the whole coherent-structures problem. The present paper as well as other works of the author are based on the definition of a coherent structure as an advected flow module in a turbulent flow characterized by instantaneously phase-correlated vorticity over its spatial extent. A number of new results obtained in the author's laboratory and related to turbulent shear flows, mixing and aerodynamic noise were presented. The relevance of helicity to coherent structures was also discussed.

Klebanoff (US National Bureau of Standards) presented results of experimental investigations of the interaction of a hemispherical roughness element with a laminar boundary layer on a flat plate. This study was aimed towards better understanding of the manner by which single three-dimensional roughness elements induce earlier transition from laminar to turbulent flow in boundary layers. It was shown that the critical behaviour of three-dimensional roughness elements is a consequence of the critical onset of an eddy generating process. The functional dependence of the eddy shedding frequency was evaluated. The behaviour of a three-dimensional roughness element in an oscillatory free stream was also studied which brought into question conclusions drawn from flow-visualization studies pertaining to roughness behaviour. In addition, this part of the study provided a method for generating turbulent 'spots'. The paper, marked by both elegance and profoundness, gave an example of a 'classic' approach to a complicated problem in which the insight and intuition of the researcher permits the maximal use of a thrifty experimental effort.

Wyganski (Tel-Aviv University) addressed the problem of amalgamation (or 'pairing') of vortices (Winant & Browand 1974) and its role in lateral transport of momentum across the mixing layer. Stability theory accounting for the mean-flow divergence and encompassing some weakly nonlinear effects is able to predict many important features associated with large coherent structures in free turbulent shear flows. The linear model, for example, predicts the transverse distribution of amplitudes and phases of externally imposed wavy disturbances. The amplification rate in the streamwise direction can also be correctly predicted by the addition of some nonlinear terms. The special dynamical significance of vortex amalgamation seemingly suggested by experimental evidence, however, is not explainable by wave theory. The author concludes that this stems from the fact that a photograph of tagged particles represents streaklines, which depend on the time and location of the tagging relative to the time and location of the observation. Measurements of Weisbrodt (1981) were used to calculate streaklines, particles paths and vorticity distribution. Results were compared with the Lagrangian vortex approach as well as with the Eulerian stability analysis and resolve the apparent contradictions mentioned above.

Branover & Sukoriansky (Ben-Gurion University, Beer-Sheva) investigated experimentally and theoretically turbulent duct flows with clearly expressed inverse energy cascades. This phenomenon occurs in electroconductive fluids under the

influence of a transverse magnetic field when energy is injected into the disturbed movement at very high wavenumber values. The onset of an inverse energy cascade corresponds to the conditions at which turbulence becomes strongly anisotropic owing to elongation of turbulent structures in the magnetic-field direction. All this occurs against the background of suppression of generation of shear turbulence and ultimately large-scale disturbances are strongly enhanced. Experimental clarification of the phenomena leading to the inversion of the turbulent energy cascade and to the enhancement of turbulence permitted the development of a theory based on the renormalization-group method. This theory enables the calculation of mean-flow characteristics without using empirical correlations. Flows with enhanced turbulence also possess enhanced and anisotropic heat transfer properties, as was proven both theoretically and experimentally.

Branover, *Sukoriansky* (Ben-Gurion University, Beer-Sheva), Greenspan (Israel Atomic Energy Commission) & Hoffman (Energy Technology Engineering Center, Canoga Park, California) analysed the test requirements for establishing scaling laws for anisotropic turbulence in lithium flow in fusion-reactor blankets. Based on results of the paper reported in the preceding paragraph, in which the existence of enhanced MHD turbulence with inverse energy cascades was proven, the analysis identifies the tests required for establishing the nature of the anisotropic turbulence to be expected in fusion-reactor blankets and its dependence on blanket design parameters.

A review of several approaches to analytical turbulence theory was given by *Kraichnan* (Los Alamos). Special attention was paid by Kraichnan to relationships among (1) approximation by finite sets of moments, (2) renormalized perturbation theory, (3) decimation under symmetry constraints, (4) renormalization-group methods, and (5) upper bounding of transport under integral constraints. The procedure of decimation under symmetry constraints (*Kraichnan 1985*), as mentioned by the author, has close connections with other approaches and plays a unifying role.

Montgomery (Dartmouth College) presented results from two recent areas of investigation in MHD turbulence. The first problem that he discussed originated in the subject of fusion plasma configuration. One of the earliest experimental devices was the toroidal Z pinch in which the toroidal magnetic field undergoes sign reversal near the wall. Extending Taylor's theory (1986) to include the dynamics of the decay process, Montgomery presented the results of the numerical calculations performed by *Dahlburg et al.* (1986, 1987). They investigated the concept of 'selective decay' in which it is conjectured that the energy dissipates much faster than the magnetic helicity during the turbulent decay. The predicted magnetic field was observed while the relaxation process stopped before reaching minimum energy. The second subject stems from the area of interstellar and solar MHD turbulence. For this situation, the pressure was assumed to be a function only of the density. With this assumption, the density-fluctuation spectra were calculated by assuming a Kolmogoroff inertial-range spectrum for the velocity and magnetic fields. The results agree with a variety of interstellar and solar-wind measurements.

Moffat (University of Cambridge) reviewed his recently developed topological approach to problems of vortex dynamics and turbulence, based on regarding a turbulent flow as an evolution of a dynamical system in a function space, the fixed points of which acquire special significance. These fixed points are steady Euler flows, characterized by ergodic blobs of maximal helicity, separated by stream-vortex surfaces, generally of toroidal topology, on which vortex sheets may be located. Kelvin-Helmholtz instability of these vortex sheets gives rise to spiral structures

which can in principle account both for an inertial-range spectrum of fractional exponent, and the enstrophy increase characteristic of three-dimensional turbulence.

Mitchell Berger (University of St Andrews) presented a poster on methods of topological description in MHD. He showed that the concept of magnetic helicity can be adapted to the situation when magnetic field lines are 'rooted' on a surface (as for the field of the solar atmosphere). The helicity is related to the linkage of the field lines and to the geometry of the footpoints, and is invariant under field deformations that do not move the footpoints. Berger also considered random motion of the footpoints leading to a slow (logarithmic in time) increase in linkage: it is the classical problem of the drunkard's walk but with a new twist – if two drunkards move independently, how rapidly (in the long-term mean) does the line joining them rotate?

Hameiri (Courant Institute) & *Bhattacharjee* (Columbia University) addressed the problem of turbulent relaxation in MHD plasmas, using essentially the methods of mean-field electrodynamics, and exploiting in an ingenious way the fact that in a plasma with strong magnetic field, characteristic scales for fluctuating fields are much smaller transverse to the field than parallel to it. The procedure leads to an expression for the mean electromotive force ($\mathbf{U} \times \mathbf{B}$) involving the third space derivative of the mean field (\mathbf{B}). The mean-field equation can then be solved and yields solutions of force-free character. This provides a new approach to the problem of explaining the reversed field in the reversed-fields pinch (RFP), complementary to the widely quoted approach of J. B. Taylor involving minimization of magnetic energy subject to the single constraint of invariance of global magnetic helicity – a procedure that is hard to justify in a resistive plasma.

Reshotko (Case Western Reserve University) presented a review of problems of stability and transition to turbulence in boundary layers. He emphasized the continuing importance of linear stability theory in predicting transition, due account being taken of the level and character of various perturbing influences (free-stream turbulence, radiated sound, surface vibrations, etc.). He cited recent work of *Ashpis* (1986) involving asymptotic solution of the non-homogeneous (forced) Orr–Sommerfeld equation which reveals the emergence of Tolmein–Schlichting waves in response to controlled forcing by a vibrating ribbon. The linear regime is now understood to the extent that disturbances may be controlled (e.g. the wave produced by a vibrating ribbon may be cancelled by a second vibrating ribbon), and drag may thereby be reduced. By contrast, the so-called 'by-pass' phenomena, involving large initial disturbances and essentially nonlinear effects are still not well understood.

Fiedler (Technische Universität Berlin) described his current work on the accelerated mixing layer, that is a mixing layer downstream of a splitter plate, constrained in a contracting channel so designed that the width of the layer increases like $X^{\frac{1}{2}}$ and the velocity jump decreases like $X^{-\frac{1}{3}}$, so that the Reynolds number is constant. In this situation, structures can attain a statistically stable form, evolving in a dynamically and geometrically self-similar manner. These structures are approximately elliptical in cross-section, with aspect ratio and orientation influenced primarily by the degree of excitation introduced by means of a vibrating flap at the trailing edge of the splitter plate. Pairing of vortices does not occur for this flow, and the growth of the layer can be explained in terms of simple decay of Oseen vortices. In the discussion that followed this paper, *Hussain* commented that the special characteristics of the flow should make it a good test-bed for the study of three-

dimensional effects (e.g. ribs and braids linking the primary spanwise vortices) and more generally of the turbulent flow topology.

A series of papers was presented on numerical simulations of turbulence by spectral methods. Direct integrations of two-dimensional incompressible Navier–Stokes equations done by Brachet, Meneguzzi & Sulem (1986) at resolution $(800)^2$ and computations using hyper-viscosities by B. Legras, S. Patarnello & P. Santangelo (1977, unpublished) at resolution $(1000)^2$ were discussed in the overview lecture by *Meneguzzi* (CEN-Saclay). Two main points were stressed: (i) the transition from an early k^{-4} energy spectrum reflecting isolated vorticity gradient sheets in the physical space (Saffman 1971) to a k^{-3} energy spectrum resulting from piling up and reconnection of the previous sheets and associated to an enstrophy cascade (Kraichnan 1967; Batchelor 1969); (ii) the persistence of coherent vortices in decaying turbulence. *Brachet* (ENS Paris), *Meneguzzi* (CEN Saclay), *Politano* (Observatoire de Nice) & *Sulem* (Tel Aviv University) presented detailed visualizations in physical space of direct simulations with $(2048)^2$ collocation points. These visualizations, obtained with a raster image processor, validate a simple model (Weiss 1981) which predicts that the location of coherent vortices or of vorticity gradient sheets is determined by the real or imaginary nature of the eigenvalues of the velocity-gradient matrix.

The dynamics of two-dimensional incompressible MHD flows was also addressed by *Meneguzzi* who mentioned the tendency for velocity and magnetic fields to become more and more correlated in freely decaying turbulence (Dobrowolny, Mangeney & Veltri 1980; Matthaeus, Goldstein & Montgomery 1983; Pouquet, Meneguzzi & Frisch 1986). The influence of the rate of correlation, measured as the ratio of the cross-correlation $\langle \mathbf{v} \cdot \mathbf{b} \rangle$ to the total energy $\langle v^2 \rangle + \langle b^2 \rangle$ (which are both invariant in the non-dissipative limit) was discussed by *Meneguzzi* (CEN Saclay), *Pouquet* (Observatoire de Nice) & *Sulem* (Tel Aviv University). Direct simulations at resolution $(512)^2$ indicate that correlations reduce the maximum amplitude of velocity and magnetic-field gradients but do not sizeably influence the characteristic transfer time. They also affect the spectral exponents: at low correlations, the inertial exponents m^+ and m^- of the spectra associated to the Elsasser variables $z^+ = v + b$ and $z^- = v - b$ are close to $\frac{3}{2}$ (Kraichnan 1965). At high correlations, they are significantly different but their sum remains close to 3 in agreement with phenomenology and closure calculations (Grappin, Pouquet & Leorat 1983). The small-scale dynamics of MHD flows is also strongly affected by the presence of a background magnetic field. *C. Sulem* (Ben-Gurion University), *Sulem* (Tel Aviv University) & *Bardos* (ENS Paris) proved that for initial conditions consisting of spatially localized fluctuations around a uniform magnetic field, the solution of the ideal MHD equations remains smooth for all time and that the nonlinear interactions become negligible when the time goes to infinity (Bardos, Sulem & Sulem 1986).

Weil (Hebrew University) demonstrated the existence of homotopic invariants that are related to the linking number of lines of constant magnetic-field direction. Whereas resistivity and viscosity destroy the physical invariants on a reconnection timescale, the homotopic invariants survive until the appearance of a neutral point.

Farge (LMD Paris) studied by direct numerical simulations the interactions between rotational eddies and inertia–gravity waves in rotating shallow water. This system is often termed ‘compressible two-dimensional turbulence’ because the surface height acts as a variable two-dimensional density producing divergence and

inertia-gravity waves. Inhibition of the inverse cascade, transfer blocking due to rotation, a k^0 -spectrum for divergent energy at scales smaller than the deformation radius were reported (Farge & Sadourny 1986). A special effort was devoted to performant representations of different fields in the physical space. Depending on the rotation rate and the excitation of inertia-gravity waves, the formation of coherent monon-like structures was not observed.

Nguyen Duc, *Capelan* & Someria (Madydam, Grenoble) presented an experimental study of two-dimensional flows consisting of a thin horizontal layer of mercury subject to a strong vertical magnetic field which both generates the flow by interactions with injected electric currents at the bottom of the box, and stabilizes it with respect to three-dimensional disturbances. The instantaneous velocity field is obtained by visualizing the trajectories of small particles. It is also measured at 63 points simultaneously and analysed. A k^{-3} energy spectrum is obtained and interpreted as the inverse cascade observed on the visualizations.

Three-dimensional turbulence was also discussed by *Meneguzzi* who reported on dynamo simulations in magnetohydrodynamic turbulent convection and on the (128)³ simulations of incompressible Navier-Stokes equation due to Kerr (1985). The latter simulations show that vorticity is concentrated in tubes with large concentrations of rate of strain nearby. They also display a $-\frac{5}{3}$ spectral regime on about one decade, analogous to that observed with the Taylor-Green vortex (Brachet *et al.* 1983).

Simulations at resolutions (91)² and (33)³ of compressible transitional flows in a cavity with differentially heated vertical walls were presented by Dang & *Loisel* (ONERA, Paris). The influence of compressibility on the transition mechanism was discussed. The latter appears to depend not only on the Rayleigh number but also on the Reynolds and the Froude numbers.

Naot & *Peled* (Center for Technological Education, Holon) presented a statistical model of MHD turbulence and derived a Reynolds-stress transport equation. They showed that a systematic order of magnitude analysis can be performed to discriminate between different effects in the case of small magnetic Reynolds number and large kinetic Reynolds number.

Frisch, She (Observatoire de Nice) & Sulem (Tel Aviv University) reported on a new large-scale instability, the so-called Anisotropic Kinetic Alpha effect (Frisch, She & Sulem 1987) which is analogous to the alpha-effect of MHD (Moffatt 1978), and occurs in three-dimensional, anisotropic, incompressible flows lacking parity invariance. A specific example was given which leads to the growth of a very strong large-scale Beltrami flow where nonlinear saturation of the instability is obtained by feedback on the small-scale flow. Extension of the analysis to MHD flows was considered by Gilbert (Cambridge University) & *Pouquet* (Observatoire de Nice). In the same context, Gilbert & *Frisch* showed that a kinetic dynamo effect can be produced by a non-helical velocity field provided it is non-parity invariant. Such a velocity field can give an alpha effect and a corresponding dynamo effect. Non-helical dynamos are however weaker than helical dynamos and require a larger scale separation between the velocity field and the mean magnetic field (Gilbert, Frisch & Pouquet 1987).

While alpha-effect-like instabilities are characterized by an instability rate proportional to k (the wavenumber of the perturbation), negative viscosity corresponds to an instability rate proportional to k^2 . The two-dimensional Kolmogorov flow which results from a one-dimensional periodic forcing is a simple

example of this phenomenon (Sivashinsky 1985). *She & Frisch* (Observatoire de Nice) presented a detailed analysis of the bifurcations displayed by this flow when the Reynolds number is increased. Special attention was devoted to the phenomenon of vortex pairing which is interpreted in terms of cellular solution competitions in 'zigzag' instabilities.

Fauve (ENS Paris) gave a review of competing instabilities observed in a small-aspect-ratio horizontal layer of mercury heated from below and rotating about a vertical axis (Fauve, Laroche & Perrin 1985). The experimental results are interpreted in the framework of normal-form formalism and the universal character of the dynamic behaviour is traced back to symmetrical properties which constrain the form of the evolution equation governing the amplitude of the unstable modes. It was also stressed that simple scenarios are never observed when several unstable modes interact, as is the case for not highly constrained flows.

Thual & Bellevaux (CNRM, Toulouse) investigated the Ginsburg–Landau equation which models spatial coupling of oscillators. It can be viewed as the generalization of the Hopf bifurcation normal form to a continuum of unstable modes in a spatially extended system. In a particular asymptotic regime, phase-equation theory reduces the dynamics of these oscillators to that of their phase, which is described by the Kuramoto–Sivashinsky equation. By solving numerically the primitive equation and the phase equation, the range of validity of phase dynamics is tested in the one-dimensional periodic case. The original dynamics is qualitatively recovered in the phase dynamics, at least for the first bifurcations obtained by increasing the length of the box.

Pade, Meinhardt & Wolfshtein (Ministry of Defense Scientific Department, Technion) studied the flow of weakly ionized gas inside an annulus of large aspect ratio. The gas is subject to a radial electric field which ionizes it and to an axial magnetic field which rotates it. Numerical results indicated that when a characteristic parameter depending on the electric current, on the magnetic field intensity and on the geometry of the annulus exceeds a critical value, hydrodynamic instabilities develop and annular vortices, analogous to the Taylor cells are established. The vortices alter the heat-transfer characteristics of the flow as they distort the otherwise straight isotherms.

V. Yakhot & Orszag (Princeton University) presented an overview of the recently developed dynamic renormalization-group (RNG) method for hydrodynamic turbulence (V. Yakhot & Orszag 1986). The values of the important constants (Kolmogorov constant, turbulent Prandtl number, Batchelor constant, the von Kármán constant, skewness factor) derived by RNG analysis are found to be in good agreement with experimentally obtained values. The results of calculations using an RNG-based $k-\epsilon$ transport model (heat transfer in a pipe and a pipe pulsating flow) were analysed and the comparison with experimental data exhibits a very good agreement. It seems that the recently reported results of the RNG analysis of turbulence calls for the detailed investigation of different aspects of the turbulence. Some of them – the role of local and long-distance (in Fourier space) interactions, possible Beltramization, anisotropic structures – were discussed during the conference.

The results of RNG analysis of turbulence have been used by *A. Yakhot* (Ben-Gurion University), *V. Yakhot & Orszag* (Princeton University) for large-eddy simulation of a turbulent channel flow. The modified eddy viscosity derived by the RNG method was incorporated into the three-dimensional computer code. Typical

runs use 32 Fourier modes (in the x -, y -directions) and 65 Chebyshev modes (in the Z -direction). The results for Reynolds numbers $Re = 6100$ and 8900 based on a mean velocity and a half-channel width were reported.

The laser-induced fluorescence (LIF) technique has recently been developed. *Logan* (Stanford University) used the LIF technique for simultaneous measurements of temperature, density and mass flux in supersonic turbulence. The two-dimensional boundary layer was investigated and the fluctuating temperature, density and pressure profiles were obtained. The velocity was measured by the hot-wire technique. The LIF method presented certainly has a good potential to be applied in further, more detailed investigations of supersonic turbulence.

An experimental study of turbulent vortex rings was performed by *Perakis & Papailious* (Greece). The entrainment mechanism of ambient fluid into the vortex rings was investigated. A theoretical model based on visualization and measurements has been developed. The results obtained provide useful information on the structure and evolution of turbulent vortices.

Irmay (Technion) presented an analytical solution of the Navier–Stokes equations for unsteady isothermal barotropic flow in the presence of gravity. The solution was obtained by power series in the distance from a given infinite plane.

Lin & Katz (Purdue University) presented a study of cavitation phenomena in water jets. Their results indicate that the cavitation inception index depends primarily on the nozzle geometry and is sensitive to external excitations. A series of photos demonstrating the various effects were shown.

3. MHD power generation

Since the last Seminar, significant progress has been made on both the analysis and the experimental demonstration of liquid-metal MHD (LMMHD) energy-conversion concepts. The optimized MHD or OMACON Cycle was first proposed by *Petrick* (Argonne National Laboratory) & *Branover* (Ben-Gurion University) at the 1984 Seminar. In this cycle, two-phase flow occurs in the vertical riser leading to a gravity separator for the gas (vapour) and liquid metal, and the pressure difference between the pure-liquid downcomer and the two-phase riser drives the liquid metal through the pure-liquid MHD generator. The advantage of this natural-circulation concept is the ability to control the losses. The first experimental results for an OMACON system were presented by *Branover*, *El-Boher*, *Lesin* (Ben-Gurion University), *Petrick* (Argonne National Laboratory) & *Zilberman* (Ben-Gurion University) using two facilities – ER-4 with mercury and steam at 435 K, and ETGAR-3 with a lead-based alloy and steam at 423 K. The authors studied the characteristics of the two-phase riser, the carry-over of liquid metal (from the separator) in the gas (vapour) stream, the carry-under of gas from the gravity separator with the liquid metal, and the performance of the single-phase LMMHD generator. The slip (ratio of gas velocity to liquid-metal velocity) in the two-phase riser, a controlling parameter on the attainable cycle efficiency, was higher than predicted by the *Smissaert* correlation (used for most power-system performance studies), and this could be an important factor controlling the viability of this concept. The liquid carry-over was not a problem. Some gas carry-under was observed, typically a void fraction (ratio of gas volume to total volume) of 0.02 leaving the separator, indicating that more care is needed in the separator design. The pure-liquid generator operated as predicted, and the end and shunt (low-velocity wall-layer) current losses were quite small.

Petrick, *Berry* (Argonne National Laboratory) & *Pierson* (Purdue University)

Calumet) described a hybrid OMACON concept which uses a two-phase nozzle after the two-phase riser to obtain more output power for a given riser height, thus potentially reducing the size and cost of the power system. (The OMACON cycle has high efficiency because the losses are controlled, but it is large and may be uneconomical.) The OMACON, hybrid OMACON, two-phase generator, and homogeneous cycles were compared assuming zero slip in all cases. The homogeneous cycle is penalized by the large fraction of the generator's gross output power required to drive the liquid-metal pump. [This could also be a problem for the two-phase-generator cycle if a liquid-metal (electromagnetic) pump is used.] The authors demonstrated that a two-phase MHD generator will require a magnet that costs substantially more than that for a pure-liquid MHD generator for a generator of the same efficiency and gross power output, because of the reduced electrical conductivity and the higher volume flow rate for the two-phase fluid. (This cost will be still higher if the two-phase-generator cycle uses a liquid-metal pump because of the increased generator power and size.) The magnet cost is very significant – a recent cost study on an OMACON system (Petrick, Berry & Pierson 1985) has shown that the magnet cost for the MHD generator is a significant fraction ($\sim 28\%$) of the total plant cost. The hybrid OMACON system can have a reduced cost compared to a pure OMACON system because the conversion cycle size is reduced and the pure-liquid generator is retained. A hybrid system could also use a two-phase generator after the riser, where the riser-downcomer combination provides the liquid pumping for the two-phase generator, or even all three two-phase expansion devices – riser, generator and nozzle. In the discussion following the paper, several participants questioned the assumption of no slip because the slip magnitude may be different for different LMMHD concepts.

Greenspan, Barak (Israel Atomic Energy Commission), Blumenau, Branover, El-Boher, Spero & Sukoriansky (Ben-Gurion University) described results from coupling LMMHD conversion-cycle concepts to three nuclear heat sources:

A pressurized water reactor producing 330°C water: with a tin-stream LMMHD Rankine system there is a 5% gain in efficiency.

The conventional steam plant has an efficiency of 42%, versus 45% for an LMMHD Rankine cycle.

A sodium liquid-metal-cooled fast-breeder reactor with a top temperature of 550°C . The conventional steam cycle has an efficiency of 39% but suffers from high capital costs and the danger of hazardous sodium-water interactions. An LMMHD Rankine cycle using tin to isolate the sodium from the water would have a conversion efficiency of 42% and improved safety, but not much simplification or cost savings.

Special applications which utilize unique features of LMMHD systems include: (i) arctic applications with a high-temperature reactor, where NaK and xenon form an attractive pair; (ii) space nuclear systems, where the wet vapour cycle using a caesium looks best.

The authors' future plans include searching for novel energy-conversion systems and applications, and economic studies. The critical issues are demonstrating the LMMHD technology for a complete system, maximizing the acceptable void fraction at the generator's exit, demonstrating liquid-metal-containing structures at high temperatures, and determining the acceptability of mercury for power plants. In response to questions, the authors indicated that special applications are the most promising for the near term; large power is attractive but there is currently no demand. Concerning a liquid-metal pump versus a nozzle-diffuser system to

recirculate the liquid metal in the generator loop, the latter was used for the studies but the former is easier to demonstrate.

Berry, *Petrick* (Argonne National Laboratory), Pierson (Purdue University Calumet) & Sukoriansky (Ben-Gurion University, Beer-Sheva) calculated the performance of the OMACON LMMHD concept for several solar applications. Features of the solar LMMHD concepts include inherent thermal storage (a temperature decrease with cloud cover of ~ 0.8 K/min), higher conversion efficiencies, the improved performance of a liquid-metal-cooled solar receiver, no need for liquid-metal pumps, ease of start-up and shut-down (natural circulation, gravity drain), reduced system complexity, and a modular system for ease of construction and expansion. For the operating conditions of SOLAR-I (a once-through boiler-superheater), the OMACON Rankine cycle has an efficiency of 38.5% versus 35% for SOLAR-I. In addition, reheat is inherent in the LMMHD cycle, the extraction points for feedwater heating are built in, a simple superheat low-pressure turbine is used as the last expansion stage, and a direct-contact steam boiler is easily incorporated. For the operating conditions proposed for the Carrisa Plains plant (a sodium-cooled receiver coupled to an advanced steam cycle), the OMACON-Rankine efficiency is 39% to 42.5% depending on steam pressure, compared with 38.5% for the conventional system. The OMACON Brayton cycle system eliminates any liquid metal-steam compatibility problems, and offers higher system efficiencies at higher solar receiver temperatures.

Blumenau & Spero (Ben-Gurion University) proposed a liquid-jet gas pump as the gas-liquid mixer element for a two-phase LMMHD generator or compressor. (An LMMHD compressor is an electromagnetic pump acting on a two-phase flow to increase the gas pressure.) The LMMHD compressor has a calculated efficiency higher than a conventional rotating compressor with zero or one intercoolers, but less than the rotating version with two or more intercoolers. It is necessary to demonstrate experimentally the performance of the jet pump, particularly its ability to produce the uniform two-phase bubbly flow required for efficient two-phase generators and compressors.

Branover, *El-Boher*, *Lesin* & Unger (Ben-Gurion University, Beer-Sheva) investigated experimentally two-phase (liquid metal with steam, vapour or gas) flows in vertical pipes. A variety of experimental techniques were used. Attention was mainly paid to the slip-ratio values because of their importance in MHD energy-conversion systems.

Branover, *El-Boher*, *Lesin*, *Zilberman* (Ben-Gurion University, Beer-Sheva) & *Petrick* (Argonne National Laboratory) presented results of testing of gravitational (so called OMACON type) liquid-metal MHD power facilities. Two integrated facilities, named ETGAR-3 and ER-4, have been built over recent years at Ben-Gurion University and their testing is at an advanced stage. Each facility has operated for more than 1000 hours (accumulatively) with no major failures, and verified theoretical predictions. Special attention was given to the verification of the theory of single-phase-flow MHD generators and of empirical correlations for two-phase flow (liquid metal and steam or gas) in vertical pipes.

Marty (Institut de Mécanique de Grenoble) & *Werkoff* (Commissariat à l'Énergie Atomique, Grenoble) presented experimental results on an induction generator using mercury, and a supporting theoretical analysis. The authors first compared the experimental induction-generator results from previous investigations, covering a frequency range of 30–910 Hz and with a maximum efficiency of 23.5%, and then discussed a.c. versus d.c. generation. They noted that the previous induction-

generator geometries have been flat, while the annular geometry used for most induction pumps is better. The authors developed a criterion for the minimum fluid velocity for self-excitation including the influences of the coil internal resistance and the load. The experimental results for their 0.5 cm by 5 cm flat mercury generator were described. For a synchronous velocity of 3.6 m/s, a minimum velocity for self-excitation of 10.4 m/s was calculated; self-excitation would not start from unexcited conditions up to the maximum velocity of the facility, 10.6 m/s. However, the generator would operate in the self-excited mode if a initial pulse was applied. The magnetic-field distribution in the machine was calculated using finite-element techniques, and the agreement with the measured data was good. This numerical approach was used to design the compensating windings to reduce the finite-length effects, an important loss mechanism.

One practical limitation on LMMHD power systems is the low d.c. voltage produced by the generators. *Pavlik* (Westinghouse Research and Development Center) has completed a feasibility study on using the voltage d.c. low to drive a homopolar motor with conventional liquid cooling coupled to a synchronous generator. The motor efficiency is 92–93.5% for a 5 MWe system with d.c. voltages of 19–23 V, and the overall d.c.-to-a.c. conversion efficiency is 90–91%. The mature system cost is estimated at \$125–150/kWe. The technological data base (including engineering and manufacturing experience) is in place to design, analyse, and build complete systems for sizes to 10 MWe.

The efficiency of the gas–liquid separator is important for the overall efficiency of many LMMHD energy-conversion concepts. *Laborde & Alemany* (Institut de Mécanique de Grenoble) described their separator calculations and initial experimental results. The former concentrated on the liquid film-thickness evolution and velocity profile, the efficiency, and the impact of curved separator surfaces. The latter used air and water, and tested inclined-plate collectors with curved surfaces which may have improved performance. The detailed experimental data include measurements of drop size by laser-light scattering and by photographs, and drop velocity measurements by laser-Doppler anemometry.

Messerle (University of Sydney) summarized the status of plasma MHD power generation for the participants. He covered the alternative concepts, the principles, MHD plant integration, power-system requirements, power conditioning (d.c.-to-a.c. conversion and control), the MHD generator status, plant modelling studies, and recent developments. He stated that technical feasibility is established, and applications can be expected in the next ten years.

4. Metallurgical applications

Moreau (MADYLAM, Grenoble) presented the results of a five-year study of aluminium reduction cells, where a cryolite layer floats on the molten aluminium. Typically the voltage applied to a cell is 4.70 V, of which 1.75 V is needed for the process and the rest is loss. The goal of the study was to reduce the losses, and to reduce turbulence which entrains aluminium in the cryolite. The mean motions of the two fluids were calculated; the vertical motion is more important. The equations of motion and boundary conditions were determined, including the conditions at the interface. The equations were applied to real cells using numerical techniques and assuming small magnetic Reynolds numbers. The measured and predicted velocities agreed.

The interface instabilities were also studied by means of a linear analysis. The

result for finite fluid thicknesses was Helmholtz small-scale (approximately 20 cm wavelength) instability of sheared interfaces slightly modified by MHD effects, and a new, large-scale electromagnetic interaction instability with a large wavelength (approximately 2 m) propagating in the direction of the horizontal current. Increasing the current density excites instabilities and decreasing the cryolite thickness decreases the stability. The results show that the aluminium motion is controlled by the curl of the electromagnetic force, while in the cryolite it is controlled by the anode shape.

M. Garnier (MADYLAM, Grenoble) gave a beautiful review of potential uses of high-frequency (h.f.) magnetic fields in industrial metallurgy and materials science, fields in which problems of MHD are of critical importance. He discussed first the control of cross-sectional profiles and of flapping and capillary instabilities, using suitably engineered magnetic-field distributions (in which spatial non-uniformity is a crucial feature), in the continuous casting of steel, aluminium, and similar metals. He then discussed the exciting developments (largely at MADYLAM) of the so-called 'cold crucible' – a copper crucible constructed in sectors separated by insulting strips to allow free passage of a h.f. field – and of the uses to which this crucible may be put. Up to 1 kg of conducting liquid may be electromagnetically levitated and processed within such a crucible, giving much higher purity than conventional techniques. The cold-crucible technique is particularly relevant for the production of materials such as silicon for the electronics industry, reactive metal titanium, and materials such as zirconia with very high melting points. Again, MHD problems abound in the exploitation of the technique – notably in determining the form and stability of a mass of levitated liquid, and (not unrelated) the structure of the turbulent flow within the liquid which is an inevitable consequence of the rotational Lorentz force in the surface magnetic boundary layer.

Gagnoud, Etay & M. Garnier (MADYLAM) presented two related posters, the first showing computed shapes for a mass of liquid metal levitated within a crucible of realistic geometry, and the second giving a theoretical treatment of the instabilities to which a horizontal slab of cylinder of liquid, levitated by crossed steady \mathbf{j} - and \mathbf{B} -fields, may be subject. Rayleigh–Taylor modes, with wave-vector perpendicular to \mathbf{B} , are generally found to be unstable, the precise conditions depending on the symmetry of the mode, and on surface tension.

Caillault, Perrier, Aubert (LETI/CRM) & *Fautrelle* (Institut de Mécanique de Grenoble) described experimental results with a high-frequency induction furnace for melting oxides. In place of the conventional cold (water-cooled copper) crucible, a container consisting of a helical water-cooled winding is used, resulting in much higher efficiencies. Of the input energy to the aperiodic generator, 72% reaches the coil and 65% is transferred to the material to be heated. The coil is protected from the hot liquid by a solid insulating crust of the material to be melted. For materials that are good electrical insulators at ambient temperature, metallic sheets are inserted for initial heating. The magnetic field and temperature were measured in electrolytic solutions and low-melting-temperature materials having appropriate properties, and the results compared to analytical calculations using a mutual self-inductance method for the magnetic field and difference techniques for the temperatures. The measured and numerical results agreed well, so that the influence of key parameters can be evaluated.

Gliere, Fautrelle & Masse (Madylam, Grenoble) developed a numerical model for electromagnetic stirring in the continuous casting of steel. The electromagnetic field is determined by a finite-element solution; the $\mathbf{v} \times \mathbf{B}$ term is included by a per-

turbation method. The time-average electromagnetic forces are calculated with and without the transport term, and the difference is significant. The hydrodynamic problem is solved numerically, including turbulence by means of the $k-\epsilon$ effective viscosity model. The electromagnetic and hydrodynamic problems are solved iteratively until convergence is obtained. The $\mathbf{v} \times \mathbf{B}$ term causes a 10–30% difference at very low frequencies, but at the actual 50 Hz frequency the velocity effect is not observed.

Vives & Perry (Universite d'Avignon) showed very dramatic micrographs of the effect of rotating pure metals during the solidification process. An annular stainless-steel crucible filled with molten tin was used, where the inner cylinder was cooled, the outer cylinder heated, and the tin rotated by rotating the outer cylinder or by the electromagnetic force from a radial current and a stationary vertical magnetic flux. Local velocity and temperature measurements were made to map the flow and to follow the evolution of the solidification front. In both cases large, inclined, blade-shaped crystals were observed in the solidified tin. The commercial value is unknown.

Lahjomri, Caperan & Alemany (Institut de Mécanique de Grenoble) used a new type of magnetic probe (magnetodiod) in local measurements in wakes of a cylinder. The cylinder was moved in mercury and a homogeneous magnetic field was applied. A good resolution of the velocity as well as the magnetic-field perturbation measurements makes the reported results very interesting.

A complementary group of papers on metallurgical MHD was presented by the Cambridge team, the focus being on the flow structures generated in various geometries by the rotational mean Lorentz force \mathbf{F} associated with an oscillating or rotating magnetic field. Hunt, Davidson & Moros (University of Cambridge) reviewed problems of turbulent recirculating flow, as realized, for example, in the coreless induction furnace, the channel furnace, or electrically stirred systems. The geometry of such systems is such that curl \mathbf{F} is frequently localized near the fluid boundary, particularly where this is cornered or sharply curved. Hunt argued that such flows are dominated by (i) balance of momentum flux and Lorentz force in this localized region, and (ii) turbulent momentum transfer in the bulk of the fluid. The argument requires that nearly all mean flow streamlines should pass through the localized region; although there is some observational evidence that this is in fact the case, it is by no means obvious that it should be generally true. As Hunt made clear, it is at this absolutely basic level of determining which way the fluid moves and how the mean streamlines distribute themselves under a given force distribution that our understanding is still far from complete.

Three poster papers (all University of Cambridge) fell within this field. Davidson described his work on primary and secondary flows in rotary magnetic stirring of molten steel. The primary rotating flow induces a secondary flow in the (r, z) -plane which penetrates a large distance beyond the region of forcing, an effect of importance in the continuous casting process. Moros, Hunt (University of Cambridge) & Lillcrap (ECRC, Capenhurst) presented results on recirculating flows in channel induction furnaces. Here, the problem is to generate a circulation in the channel around an induction coil. Under symmetric conditions, the line integral of $\mathbf{j} \times \mathbf{B}$ round this channel is zero. A circulation may nevertheless be generated by introducing an insulating lip at a corner of the channel, which concentrates the induced current and at the same time stimulates flow separation, the resulting misalignment of \mathbf{j} and \mathbf{u} being conducive to the development of circulation.

5. Laminar MHD flows, single-phase and two-phase flow phenomena

Talmage & Walker (University of Illinois) presented continuing MHD duct-flow studies motivated by the 'lithium blanket' concept in fusion-reactor technology. Flow along a duct of circular cross-section in a region of strong but non-uniform transverse magnetic field was considered. The authors used a simple analogue-circuit approach to educe the main features of the current flow in the fluid and conducting boundary, and described the asymptotic analysis appropriate to large Hartmann number M and small well conductance ratio c . They showed that even when $c \sim 0.02$, the asymptotic regime (which involves a stagnant flow region) is not attained, but that the flow may be resolved by solution of two coupled linear elliptic equations. The predictions of the improved theory are in good agreement with experiments carried out at the Argonne National Laboratory.

Also in the lithium-blanket domain was a poster paper by *Johnston & Cowley* (University of Cambridge) on thermoelectric (TE) MHD, the field initiated by Arthur Shercliff, some ten years ago. Interaction of thermoelectric currents and stray containment magnetic fields will induce significant flows which may affect the operation of the cooling system. Johnston described a theoretical and experimental study of a configuration with applied temperature gradients at the boundary suitably arranged in conjunction with a strong magnetic field (~ 1 T) to generate maximal effect. Measurements of electric potential, temperature, pressure and velocity were presented, and compared with simple theoretical deductions concerning the flow structure.

TEMHD effects occur also in the cooling circuits of liquid-metal fast-breeder reactors, such as the French PHENIX. *Werkoff & J. Garnier* (Centre d'Etudes Nucléaires, Grenobles) reported probe measurements carried out near the secondary cooling circuit of PHENIX. A magnetic field was introduced by means of two coils placed at the outlet of a secondary pump, and amplification of the field associated with flow at a magnetic Reynolds number of order ten was detected. Thermoelectric currents, which occur in response to unbalanced temperature gradients during transient phases of operation of the reactor, were also measured.

Mestel considered the general theory of magnetically forced closed-streamline flows at high Re . An iterative scheme was proposed for the determination of the functional relationship between vorticity and stream function $w(\chi)$. A stability criterion for such flows, related to Arnolds' criterion, was obtained and illustrated with reference to the particular case of magnetically driven flow with circular streamlines.

Laredo, Levy & Timnat (Technion) described experimental two-phase-flow diagnostic results and computer modelling for turbulent reacting flows (combustors) with an axisymmetric sudden expansion. The sudden expansion provides recirculation, a large pressure loss, a much hotter region in the recirculating zone, and acts as a flame holder. The gas and droplet velocities, and the droplet size distribution are measured by laser-Doppler anemometry, and temperatures by a traversing thermocouple. The spray is simulated by a finite number of droplet size groups, and the two-phase flow is calculated using the basic single-phase TEACH code modified to include two-phase droplets and evaporation. The calculated and measured recirculation zone lengths are the same.

Manzini, Lopez de Haro, Castrejon & Ramos (Mexico) have reported the results of their numerical calculations of the developing distance in MHD flow. These calculations were carried out under the assumption of laminar flow. The results obtained include velocity and temperature profiles for various Reynolds numbers.

Nathenson, Alexion, Slepian, Groupe (Westinghouse Research and Development Center) & Gray (Electric Power Research Institute) extended the flow-coupler results presented at the 1984 Seminar. A flow coupler consists of an MHD generator in one flow loop driving an electromagnetic pump in another flow loop, with the generator and pump connected electrically in series and located in a single magnet. The new results include flow-coupler designs for nuclear reactors.

Block (Technische Hochschule Aachen) performed experiments to determine the effect of the oscillating part of the time-varying electromagnetic force density. This oscillating or periodic part is normally ignored and only the time-average force considered, but at low frequencies it may be significant, causing momentum exchange in an inhomogeneous flow which results in an augmented apparent viscosity. He measured the flow of mercury in a 2 mm ID, 1.5 m long glass tube with a pressure difference of 4–6 kPa. The flow was first measured with no magnetic field, and found to be turbulent. With 21 pairs of magnets producing a field of 0.1 T placed at different spacings along the tube the mercury velocity increased; obviously the laminarization effect is larger than the braking effect (changes in the velocity profile caused by the electromagnetic forces). If an axial current of 3 A is imposed, the flow is decreased for a d.c. current, and for an a.c. current with a frequency less than approximately 10 Hz. For frequencies above 20 Hz there is no effect on the flow. A resonance effect occurs at approximately 3 Hz, and this frequency yields the minimum flow. The flow rate could be decreased by up to 25%. The results of these experiments give insight into the electromagnetic mixing processes inside a mould. Estimated flow patterns including this momentum exchange are given.

Slepian & Walk (Westinghouse Research and Development Center) have presented a simplified model, for calculation of magnetofluid (MFD) devices such as flat and annular linear induction pumps, linear and rotary electromagnetic stirrers. The authors point out that practical MFD machines are characterized by the presence of different layers of material (electrical coils, iron, air gap, duct walls, liquid metal) with different thermal and electrical properties. Hence, the model represents MFD devices as a series of layers each having constant thermal and electrical properties and a constant velocity. The limitations of such a 'multilayered' model are discussed in the paper. The resulting equations were solved numerically for various MHD machines.

Oshima & Yamane (Tokyo Institute of Technology) presented a study of nonlinear waves in liquid metals under a transverse magnetic field. In their experimental study, they investigated the influence of a vertical magnetic field on a solitary wave and on a hydraulic bore. In the latter case, the strong (0.75 T) vertical magnetic field creates a reflected wave and the velocity of the bore's front changes in the areas of non-uniform magnetic field. A reflected wave was also observed in the case of solitary wave.

Mond (Ben-Gurion University) presented a poster paper which demonstrated the effects of an oscillating electric field on bubbly liquid-metal flow. He showed that the coupling of the external electric field with the internal normal modes of the system might give rise to parametric instabilities. Two types of such instabilities were observed. In the first one, the bubbles grow without oscillations while the other type of instability is oscillatory. The cases of spatially homogeneous as well as sinusoidal external electric fields were investigated. Criteria for the onset of the various instabilities were derived as well as the maximal growth rates. This mechanism is relevant to bubbles' size control in LMMHD devices and lithium blankets in fusion reactors.

Kalman & Letan (Ben-Gurion University) presented a theoretical study of the apparent saturation temperature in the condensation process of a bubble. An exact expression is obtained which contains also effects of non-condensibles. The expression reproduces various limits treated by previous models and extends them to regimes of higher temperature differences.

Marc Berger (Weizman Institute of Science) described two numerical methods for solving the equations that govern two-phase flow inside a structural network. In the first approach he used the method of characteristics in which special attention was given to the refraction of the characteristics across interfaces. The other method treats the various components of the system as volume elements. This allows the problem to be cast in the shape of a large system of coupled ordinary differential equations. Applications to the study of safety valves with liquid seals in nuclear reactors were presented.

6. Panel discussions

Three panel discussions were held: Applied MHD, Turbulence and Flow Phenomena. The first was chaired by Hunt (University of Cambridge) and the other two by Moffatt (University of Cambridge). The panelists of each session had to review and summarize the papers presented in poster sessions and related to the topic of the panel, and then analyse the current status and the problems which they regard most important and acute in the respective areas of the panel sessions.

Four panelists participated in the Applied MHD panel session: Braun (Studsвик, Sweden), Moreau (Madylam, Grenoble), Petric (Argonne National Laboratory) and Pierson (Purdue University Calumet). In relation to liquid-metal MHD power generation the opinion was expressed that substantial progress had been achieved, mainly because integrated power-generation facilities had been put in operation (Ben-Gurion University) and others are in advanced stages of construction (University of Grenoble). The necessity was stressed of finding the best possible industrial applications for such systems with the most advantageous coupling to specific heat sources. Specific problems of upscaling were also addressed.

In the field of metallurgical applications further substantial progress was made in electromagnetic stirring of molten metals, shaping, smelting in levitation conditions, etc. Flow and heat-transfer phenomena were better understood and some advancement had been made also in engineering development of devices. A number of young researchers in France and England had entered interdisciplinary applied science areas and this fact was well reflected both in participants of the Seminar and in the papers presented.

Five panelists addressed the current status and main problems in turbulence research: Brodkey (Ohio State University), Frisch (Observatoire de Nice, France), Hussain (University of Houston, Texas), Kraichnan (Los Alamos), and Wygnanski (University of Tel-Aviv). The following problems received the greatest attention: coherent structures, helicity, advanced measurement techniques versus visualization, new statistical approaches, particularly the renormalization-group method, and direct numerical simulation.

Regarding coherent structures, a lively debate developed regarding the actual concept and term and one of the arguments was that the introduction of this concept and term was unnecessary since it actually refers just to large-scale disturbances. However, most of the participants did not support this argument. Another quite emotional discussion concentrated on deficiencies of different visualization methods

for investigating instabilities and turbulence. The consensus was in favour of acknowledging the necessity for more detailed and advanced measurements in addition to visualization studies.

Regarding direct numerical simulation of turbulence, the opinion was that this approach should gradually replace certain physical experimentation. However, the view was expressed that spatial resolution will remain poor even with the use of the next generation of supercomputers, not only with the supercomputers available today, and therefore for a long time to come results of direct simulation will still miss phenomena that occur close to the dissipation scales.

The panel session on flow phenomena had four panelists: Moffatt (University of Cambridge), Moreau (Madylam, Grenoble), Mestel (University of Cambridge) and Berger (Weitzman Institute). In the area of two-phase flow a brief review of the presented papers was given by Berger. The participants pointed out directions towards which the investigations could proceed in order to make them more general, and expressed appreciation regarding the progress in laser-Doppler-anemometry techniques which were used by the Technion group for flow visualization.

The desire to hold the next, Sixth, Beer-Sheva Seminar in early spring of 1990 in Jerusalem was unanimously confirmed.

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