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DEFLECTION OF A STREAM OF LIQUID METAL BY MEANS OF AN ALTERNATING MAGNETIC FIELD

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ABSTRACT. The deflection of a thin, two-dimensional stream of liquid metal due to external high-frequency currents is investigated. A simple theoretical model assuming uni-directional flow is presented. The relationship between the angle of deflection of the stream and the power supplied to the perturbing currents is determined. Experiments are performed in which a free-falling mercury sheet is deflected by two anti-parallel line currents. The agreement between theory and experiment is reasonable, despite a tendency towards three-dimensionality in the latter.

1. Introduction

For many purposes involving the processing of liquid metals, it may be desirable to be able to deflect a stream through a given angle. In this paper we describe how such deflection can be achieved by the action of the magnetic pressure p_M associated with a high-frequency magnetic field produced by current sources outside the liquid stream. We give a simple one-dimensional analysis of the phenomenon and compare the theoretical predictions with the results of some experiments performed with mercury. A fuller description of both the theory and experiments may be found in Etay, Mestel & Moffatt [1].

We consider a two-dimensional configuration, in which a sheet of metal, of initial thickness d_0 and uniform velocity u_0, moves under the influence of alternating line currents. Such sheets are used in industrial processes such as the manufacture of metallic ribbon. In order to analyse the effect, we make certain simplifying assumptions, as follows:

(a) We assume that the field frequency, $\omega/2\pi$ is sufficiently high for it to be reasonable to treat the effect of the fields entirely in terms of the magnetic pressure on the liquid surface. This requires that the magnetic skin-depth $\delta = (2/\mu_0\sigma\omega)^{1/2}$ (where σ is the electrical conductivity of the liquid and μ_0 its permeability) be small compared with the undisturbed thickness of the stream, d_0.

(b) We assume that d_0 is small compared with the scale L on which the magnetic pressure varies over the surface, as determined by the current source distribution and the surface curvature.

(c) Finally, we assume that gravity may be neglected at least over the scale L on which the deflection takes place. In terms of u_0, the length-scale on which gravity acts is $l_g = u_0^2/g$.

These three assumptions imply a hierarchy of length-scales as follows:

$$\tfrac{1}{2}\delta \ll d_0 \ll L \ll l_g \qquad (1)$$

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The factor of 1/2 is included in (1) because the magnetic pressure is quadratic in the magnetic field and thus decays twice as quickly. Assumptions (a), (b) and (c) are obviously restrictive, but they permit significant progress to be made. The limitations of the analysis are considered in [1].

2. Quasi-one-dimensional analysis

The assumption (b) above allows the use of a quasi-one-dimensional analysis, in which the liquid stream is in effect located by the position of its left-hand boundary. This is a curve C parameterised by arc length s from some fixed point O, as in figure 1. The stream thickness, $d(s)$, is (weakly) non-uniform when deflection occurs. We suppose that, upstream of the region of magnetic influence, the stream has uniform velocity u_0 and thickness d_0 so that the volume flux is $Q = u_0 d_0$. As viscous effects are negligible, the flow is irrotational by virtue of the assumption (a) which ensures that the sole effect of the magnetic field is to provide a magnetic pressure distribution over the liquid surface.

Let (s, n) be taken as coordinates tangent and normal to C, as in figure 1, and let $u(s, n)$ be the velocity within the stream, effectively parallel to C. To leading order in the small parameter d_0/L, the velocity is uniform, $u \approx u_0$, and the n-component of the equation of motion is

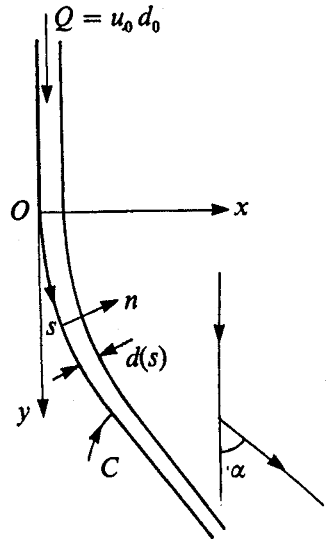


Figure 1: The coordinate system.

$$\frac{\partial p}{\partial n} = -\rho u^2 K(s), \tag{2}$$

where ρ is the liquid density, $p(s, n)$ the pressure, and $K(s)$ is the curvature of C at position s. To the same approximation, the appropriate boundary conditions are

$$\begin{aligned} p(s, d_0) &= p_0 - \gamma K && \text{on the right-hand boundary,} \\ p(s, 0) &= p_0 + \gamma K + p_M && \text{on the left-hand boundary,} \end{aligned} \tag{3}$$

where p_0 is atmospheric pressure, γ is the surface tension and $p_M(s)$ is the magnetic pressure. In writing (3), we are assuming that all the current sources occur on the left-hand side of the metal stream. Integrating (2) using (3) we get

$$p(s, n) = p_0 - \gamma K + \rho u_0^2 K (d_0 - n) \tag{4}$$

and hence the required magnetic pressure $p_M(s)$ is given by

$$p_M = \rho u_0^2 d_0 K(s) \lambda \qquad \text{where} \qquad \lambda = 1 - \frac{2\gamma}{\rho u_0^2 d_0} \,. \tag{5}$$

The perturbation to the velocity $u(s, n)$ and width $d(s)$ may now be found from Bernoulli's theorem,

$$p + \tfrac{1}{2}\rho u^2 = p_0 + \tfrac{1}{2}\rho u_0^2 \,, \tag{6}$$

and mass conservation. From (4) and (6), we have

$$u = u_0 + K \left[\frac{\gamma}{\rho u_0} - u_0 (d_0 - n) \right], \qquad \text{and so} \tag{7}$$

$$Q = \int_0^{d(s)} u\,\mathrm{d}n = u_0 d - \tfrac{1}{2} K u_0 d_0^2 \lambda\ . \tag{8}$$

As also $Q = u_0 d_0$, the stream thickness, d, is given by

$$d(s) = d_0 + \tfrac{1}{2} K d_0^2 \lambda\ . \tag{9}$$

It is clear from the form of (5) that the inertia of the uniform stream acts somewhat like a negative surface tension. As the magnetic pressure is positive, the direction in which the stream is deflected (as determined by the sign of K) may be seen from (5) to depend on the sign of λ. We are mainly concerned here with parameter values such that λ is positive, i.e. when the magnetic pressure generates a momentum flux away from the currents. When $\lambda < 0$, the magnetic pressure is strongly resisted by surface tension, with the momentum flux being less important. In both these cases the jet thickness, $d \geq d_0$ everywhere. It is of interest to note that when both λ and p_M are zero, any shape C gives rise to an admissable steady state, to lowest order in the jet thickness. Somewhat curiously, the path of the stream is maintained by its own surface tension.

Equation (5) defines a non-linear relation between the curve C and the external currents. In theory, any desired stream path C (for which $K > 0$) can be obtained by choosing the distribution of current sources to give the required magnetic pressure in (5). One such distribution (and there will be many others) is provided by placing coils so as (in effect) to provide a current sheet $J(s)\cos\omega t$ near to the deflected stream. This current sheet produces a magnetic field $B(s)\cos\omega t$ in the gap between the coils and the stream, where $B(s) = \mu_0 J(s)$, and an associated time-averaged magnetic pressure $p_M = B^2/4\mu_0 = \frac{1}{4}\mu_0 J^2$. Hence the required magnetic pressure (5) is achieved provided

$$\mu_0 J^2 = B^2/\mu_0 = 4\rho u_0^2 \lambda K d_0\ . \tag{10}$$

The required deflection of the stream can then be maintained.

If we use the expression $K = \mathrm{d}\psi/\mathrm{d}s$, where ψ is the angle that the tangent to C makes with the horizontal, then by integrating (10) we obtain the result

$$4\rho u_0^2 d_0 \lambda\, \alpha = \int_C \frac{B^2}{\mu_0}\,\mathrm{d}s \tag{11}$$

where α is the total angle of deflection of the stream. We may express α in terms of the power supplied to the coils (per unit length in the z-direction). This power, W, is balanced by the Joule heating in the metal stream

$$W = \int \frac{|\nabla \wedge \underline{B}|^2}{\mu_0^2 \sigma}\,\mathrm{d}V = \frac{\sigma \delta^2}{\mu_0^2} \int_0^\infty e^{-2n/\delta}\,\mathrm{d}n \int_C B^2(s)\,\mathrm{d}s = \frac{\delta\omega}{4} \int_C \frac{B^2}{\mu_0}\,\mathrm{d}s \tag{12}$$

in terms of the skin-depth approximation. Together (11) and (12) give a simple relationship between the angle of deflection and the power supplied to the external coils,

$$\alpha = \frac{W/\omega\delta}{\rho u_0^2 d_0 \lambda}\ . \tag{13}$$

Recalling the definition of δ, we see that the power needed to obtain a given deflection α behaves as $\sqrt{\omega}$, provided ω is sufficiently large for the skin-depth approximation to apply. Thus, in practice there will be an optimal frequency at which the deflection effect is still pronounced, but for which the power dissipated in the metal is relatively low. This optimum value will occur when δ/d_0 is $O(1)$.

3. Deflecting action of weak line currents.

It is difficult to find the stream path C analytically for a given current distribution, as the magnetic field is strongly coupled with the shape of C. A problem of this type was solved numerically by Shercliff [2] in his work on the shaping of liquid metal columns, when the balance is between surface tension and magnetic pressure. If the deflection is weak, however, then progress is possible by perturbation analysis. We illustrate this with reference to the action of a concentrated line current $I\cos\omega t$ placed at a distance L from a stream whose undisturbed position is $0 < x < d_0$. When $I \neq 0$, we suppose that the stream is symmetrically perturbed as indicated in figure 2. To leading order, however, the magnetic field distribution may still be calculated as if the stream were in the undisturbed position. There is then an image current $-I\cos\omega t$ at $x = L,\ y = 0$, and the net magnetic field on $x = 0$, is $\underline{B} = (0, B_y, 0)$ where

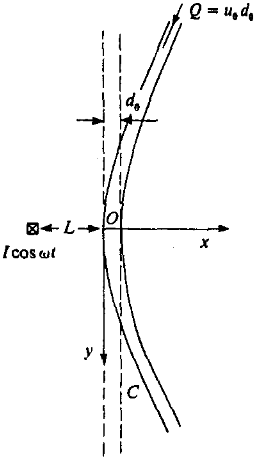


Figure 2: Deflection by a weak current source.

$$B_y = \frac{\mu_0 I}{\pi}\,\frac{L}{L^2 + y^2}\cos\omega t, \qquad \text{so that} \qquad p_M = \frac{\mu_0 L^2 I^2}{4\pi^2}\,\frac{1}{(L^2+y^2)^2}\,. \tag{14}$$

The deflection of the surface, from (5) with $K \approx \mathrm{d}^2x/\mathrm{d}y^2$, is given by

$$\frac{\mathrm{d}^2x}{\mathrm{d}y^2} = \frac{N}{4\pi^2}\,\frac{L^3}{(L^2+y^2)^2} \qquad \text{where} \qquad N = \frac{\mu_0 I^2}{\rho u_0^2 d_0 L\lambda}\,. \tag{15}$$

We integrate this with 'initial' conditions $x(0) = \frac{\mathrm{d}x}{\mathrm{d}y}(0) = 0$ with the result

$$x(y) = \frac{N}{8\pi^2}\,y\tan^{-1}\left(\frac{y}{L}\right)\,. \tag{16}$$

When $|y| \gg L$, we have $x \sim \frac{N}{16\pi}|y|$, so that the net angle of deflection is given by

$$\alpha = \frac{N}{8\pi}\,. \tag{17}$$

Equation (17) is strictly valid only provided the deflection is weak, i.e. $\alpha \ll 1$; however the results of section 2 suggest that an arbitrarily large deflection of the stream may be achieved if the dimensionless parameter N is increased to a sufficiently large value. This parameter may be regarded as the magnetic interaction parameter, giving a measure of the transverse flux of momentum generated by the magnetic forces relative to the flux of momentum $\rho u_0^2 d_0$ in the incoming stream.

The magnetic field lines of a line current and its image consist of a family of co-axial circles. Thus, the analysis of this section and the result (17) in particular may be extended to the more realistic case when the line current is replaced by a circular wire, of radius r, whose axis lies on $x = -b,\ y = 0$. At high frequency, the current in the wire flows near the surface and is equivalent to a line current at $x = -L,\ y = 0$, where $L^2 = b^2 - r^2$. For this value of L, the deflection obtained is given by (17) with (15). It should be noted that no such exact result holds for low frequencies (or D.C.) for which the surface of the wire is not a magnetic field line.

In the experiments that follow we use for electrical convenience a device which may be modelled by two opposite line currents a fixed distance, a, apart. It is a simple matter to extend the above theory to include this case. The magnetic field may be represented as a sum of two terms similar to

(14) which can then be squared and integrated to calculate the deflection angle α. When the two line currents lie in a horizontal plane a distance a apart, the result corresponding to (17) is

$$\alpha = \frac{N_h}{8\pi} \qquad \text{where} \qquad N_h = N\frac{a^2}{(L+a)(2L+a)} \tag{18}$$

whereas when they lie in a vertical plane we find

$$\alpha = \frac{N_v}{8\pi}\left(1 + \frac{1}{1+\frac{a}{L}\frac{N_v}{8\pi}}\right) \qquad \text{where} \qquad N_v = N\frac{a^2}{a^2+4L^2}\ . \tag{19}$$

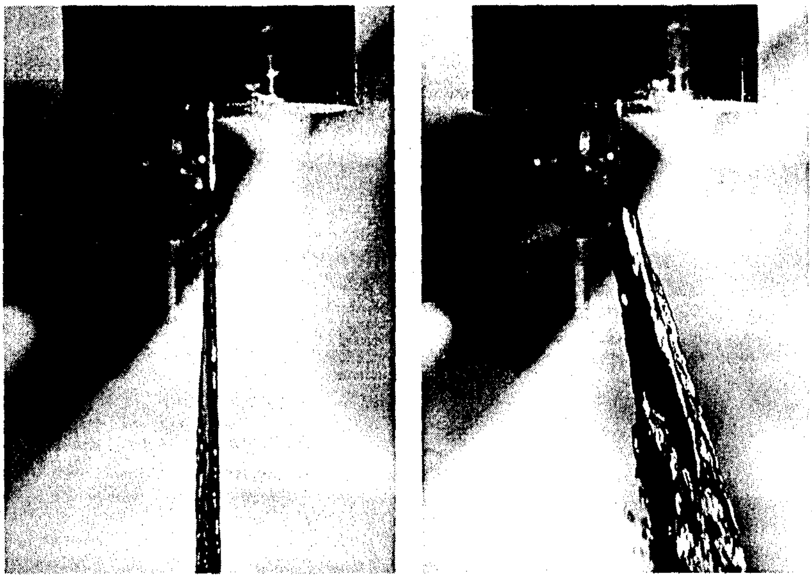
In (19), α varies between a value due to deflection by both branches of the inductor in turn, and one where only the uppermost branch is important, according to the relative size of a/L and N.

So far, we have supposed that the current sources are all placed to one side of the stream. More complex stream deformation may clearly be achieved if current sources on both sides are used. Because of the two-dimensionality of the problem and the high frequency approximation, the magnetic fields on either side of C do not interact. The stream path, C, will still be determined by the differential equation (5) with the appropriate value of p_M being the difference between the magnetic pressures on either side of C. In [1], it is described how two line currents, one on each side of the metal stream, can act as a sensitive control over the ultimate direction of the stream.

4. Experiments

Experiments were performed in order to measure the deflection of a nearly two-dimensional stream by line currents for various values of the interaction parameter N. The experimental set-up used is described in full by Etay & Garnier [3]. Essentially, the facility consists of a mercury-filled hydraulic circuit containing a freely falling column. The initial cross-section of the column is determined by a gently converging nozzle ending in a slit 1mm. by 39mm. The alternating magnetic field is supplied by insulated copper inductors connected to adjustable capacitors, and powered by a 100kWatt generator. The inductors are made from hollow copper tubing of 3-4mm. diameter through which cooling water is passed. A single line current would give rise to a large self-inductance with resultant inconvenience for the tuning capacitor. Instead, hairpin-shaped inductors were built, with the current flowing along one branch of the hairpin and returning along the other. In the middle of the inductor the field is approximately that due to two line currents. The distance between the branch axes is 15mm. The current flowing in the inductor varies from 0 to 1520 Amps. at a frequency of 350kHz. The skin depth, δ, is therefore 0.8mm. and the magnetic field near the sheet varies from 0 to 2000 Gauss. A higher current could not be obtained with this kind of copper tube inductor. In Riga we were taught the phrase "The experiment was a success: the apparatus burned out!" Indeed, one of the inductors used in our experiments proved very "successful!"

A truly two-dimensional geometry cannot be achieved experimentally, even in the absence of electromagnetic effects. Surface tension causes rolls to form at both ends of the initally flat sheet which grow as the metal falls, as in figure 3a. Of course, in any industrial application, the sheet would be longer and thus end effects less important than in these experiments. A further departure from two-dimensionality occurs due to the slight twisting of the sheet when it leaves the nozzle. When an alternating, high frequency current flows in the inductor the departures from two-dimensionality increase. The ends of the sheet are repelled further from the inductor than is the middle, due to the three-dimensionality of the magnetic field there. The resulting curved cross-section aggravates the tendency of the sheet to contract and its thickness increases. The measurement of the deflection angle can thus become difficult. This effect can be seen in the photograph of figure 3b which shows the "worst" kind of deflection that can occur.

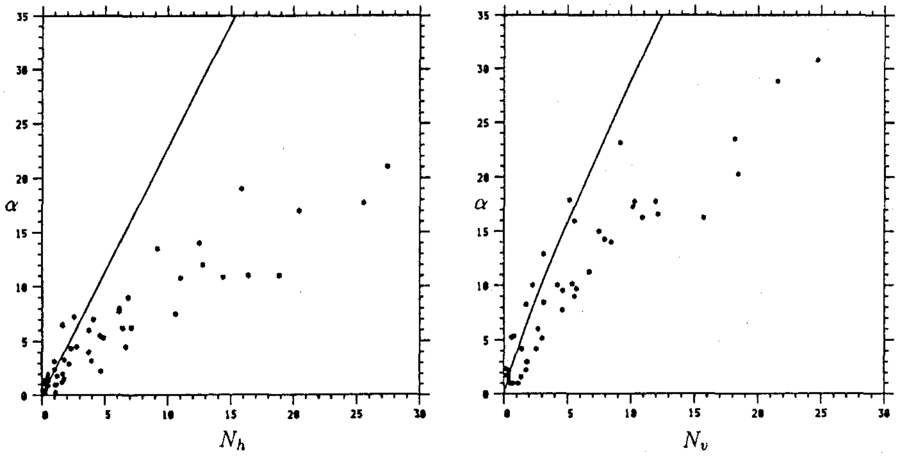


(a) No current, $I = 0$. (b) With current, $I = 1520$Amps.

Figure 3: Evolution of a two-dimensional mercury sheet.

The greatest error in calculating the experimental value of N derives from the estimate of L, defined in terms of the distance between the centre of the coil and the liquid stream. The value of L was measured on a photograph taken with a 45° mirror placed below the inductor. For maximal effect, L should be small, when the accuracy of this measurement is low. The deflection angle, α, was measured on photographs taken by a camera mounted adjustably in the plane perpendicular to the metal sheet. The three-dimensionality of the deflected jet ensures that the measurement of α is not easy. It was decided to define α as the average of the maximum and minimum deflections attained over the sheet. A curious feature of the observed deflection was that the stream did not appear to bend until below the position of maximum pressure, whereas theoretically this should occur very slightly above that position. This was probably an observational effect.

Two sets of experiments were performed, one with the inductor horizontal (so that only one branch of the inductor has a large effect on the stream) and the other with it vertical. The deflection was noticably greater when the inductor was vertical. The measured values of α (in degrees) are plotted in figure 4 as functions of N_v and N_h. The theoretical line (18) and curve (19) are drawn on the figure for comparison. The agreement is satisfactory for α small, but not surprisingly, the theory over-estimates the obtained deflection angle for N (and α) large. The manner in which this occurs appears to be fairly systematic. There are a number of reasons why this might be expected. First, equations (18) and (19) apply only when α is small, whereas the experiments cover a wide range of α. The "weak deflection" approximation, whereby the magnetic field may be easily calculated, thus breaks down. Secondly, assumptions (a) and (b) are only weakly satisfied in the experiments. As a result, the unidirectional irrotational flow assumed in the theory may not be wholly accurate. Thirdly, it is clear that the three-dimensionality of the experiments will lead to a reduced deflection. It also hinders observation of α as we have already discussed. Finally, we should recall the problems inherent in the measurement of L. It may well be that there is a systematic under-estimation of L when α is appreciably large. Bearing in mind the practical difficulties, it was felt that the agreement between theory and experiment was satisfactory.



Inductor horizontal   Inductor vertical

Figure 4: Deflection angle α in degrees plotted against N_h and N_v.

6. Concluding remarks

The use of electromagnetic fields as controlling devices in the metallurgical industry is growing. Liquid metal may be stirred, heated shaped and transported without resort to mechanical means. Usually the fluid flow that results is complex in nature, and is often turbulent. In this paper, by contrast, we have investigated a process for which the flow is particularly simple, in which the ultimate position of a stream of liquid metal is controlled. The analysis is fairly general and may be extended to cover particular geometries of industrial interest. Although the fluid dynamical techniques we have used are elementary, they do seem to give a reasonable description of the real behaviour, as witnessed by experiment.

The main limitations of the process derive from its reliance on two-dimensionality and a very high-frequency. In [1] a second configuration is considered which avoids these difficulties, relying instead upon the repulsive force that exists between electric currents. If a thin, cylindrical metal jet is made to carry a current, then it can be deflected by other suitably positioned currents. The equilibrium position for such a jet is calculated, and the question of its stability is briefly discussed.

In conclusion, the authors would like to offer their congratulations to the organisers of this very successful and enjoyable symposium, and to express their gratitude for the lavish hospitality shown to them in Riga.

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