

# Unwinding the gordian knot

John D. S. Jones

TAKE a piece of string, tie a complicated knot in it, and then sew the two ends together so you cannot simply slip the knot off. Now, can you unknot the string? Anyone who has been fishing, with the bait on one end of the line and the reel on the other preventing you from slipping off the knot in between, will know that this can be an intricate task. And can two knots made by different processes in fact be the same even though they appear to be different? Such questions are tackled mathematically in knot theory. This theory also extends to links, which arise when you start with several pieces of string, tangle them together introducing knots and at the same time intertwining different threads, and then sew the ends together — like the horror of two fishing lines getting tangled. In his article in this issue (*Nature* 347, 367–369; 1990), H. K. Moffatt proposes to study knots and links by using techniques from fluid mechanics. This leads to a notion of the ‘energy’ of a knot or a link. Moffatt shows that this notion can, in principle, be used to distinguish between different knots.

From the preceding description, it may seem that the theory of knots and links, whatever its internal beauty (a quality highly prized by mathematicians), is a rather frivolous subject. But it is not. Mathematically a knot is a simple closed curve in three-dimensional space and a link is a collection of disjoint simple closed curves; each is simple because the curve does not cross itself, and closed because the starting and end points of the curve are the same.

Simple closed curves arise in many different ways. For example, imagine a particle moving in three-dimensional space under the influence of some external force field, in such a way that after a fixed time  $T$  it returns to its original position. The path followed by the particle is a simple closed curve, provided it does not cross itself at some earlier time, and the knots and links which arise in this way contain important information. In mathematical terms these are closed trajectories of dynamical systems. Linking occurs in biological systems, for example the complicated coiling patterns of strands of DNA. In string theory, which is an attempt to model the structure of the Universe by replacing points in space-time by paths (strings) in space-time, knots and links play a role. Moffatt points out that they also occur in plasma and polymer physics.

One of the most effective ways of studying knots is through their ‘invariants’. In the simplest case this means assigning a number  $n_k$  to each knot  $K$  with the property that if  $n_k$  is different from

$n_l$ , then the knots  $K$  and  $L$  are necessarily different. Here we come across an important point. A knot may have many different representations, and a knot invariant must have the property that it is independent of the particular way we choose to represent the knot. The invariant need not be a number but may be a more sophisticated mathematical object.

The knot invariant Moffatt studies is, in his phrase, the ‘energy spectrum of the knot’. Very roughly, one can think of fluid

flowing in a small tube around the knot and then measure the energy of this flow. There are three main steps in the precise definition. Given a fixed knot  $K$ , one first constructs a knot field, which tells us the direction in which the fluid flows and is constrained to lie within a small tube enclosing the knot. This introduces some dynamics into the picture. The knot field has a well defined energy and the next step is the natural one of letting the knot and its field relax so that it assumes a configuration with minimal energy. It is necessary to specify this process quite carefully to be sure that one does not always end up with a configuration with zero energy. This could happen, for example, if one simply

## B. F. Skinner (1904–1990)

BURRHUS Frederic Skinner, who died on 18 August, was the last of an earlier generation of behaviourists who dominated psychology in the United States for 25 years or more. He established his reputation with *The Behavior of Organisms* (1938), published while he was still in his early thirties. It was a remarkable book, and unlike those of Tolman, Guthrie and Hull can still be read with profit today.

Although he was preceded by the Polish neurophysiologist Konorski, Skinner was the first American psychologist to understand clearly the distinction between Pavlov’s classical conditioning, where an animal learns that one stimulus signals the occurrence of another (usually an event of some significance, like food or water), and Thorndike’s instrumental conditioning, where the delivery of food is contingent on the animal’s own behaviour. Virtually all of his own work was on instrumental or operant conditioning, conducted in a piece of equipment that everyone else came to call a Skinner box. The possibility of precise timing of events and objective recording of responses offered by this automated apparatus had a profound and (largely) beneficial effect on the study of conditioning in the laboratory. One of the perhaps unforeseen consequences was that it now became feasible to generate large amounts of data with relatively little effort. Witness Skinner’s last full-scale piece of empirical work, *Schedules of Reinforcement* (1957) — 700 large pages of experimental results, with many interesting nuggets of information and some acute observations, but mind-boggling in the ratio of undigested data to serious theoretical analysis.

Skinner abandoned the laboratory relatively early in his career: *Walden Two*, a novel about a group of people who set up a community based on sound behavioural principles of positive reinforcement, was published in 1948. It heralded a move to the public platform. If he became the most famous living psychologist, it was

not for his empirical discoveries but for his single-minded advocacy of a rather simple-minded version of behaviourism. Behaviour was to be explained by the ‘contingencies of reinforcement’ (interacting, he acknowledged, with the organism’s physical characteristics) rather than by appeal to beliefs, hopes or feelings, or even to the action of the nervous system — where that was simply inferred from the behaviour itself.

Although frequently misconstrued as denying the existence of consciousness or mental life (it was the appeal to mental states to explain behaviour that he objected to), his views, even when correctly represented, began to seem ever more naive as the ‘cognitive revolution’ swept psychology in the 1960s. Attempts to simulate human behaviour by computer programs, to provide rigorous explanations of the processes by which we perceive the world, talk, plan and act, made the simple story he continued to push seem increasingly irrelevant. And when he turned to the larger problems of society, his advocacy of behavioural engineering, based on principles of reinforcement, seemed not only naive but threatening.

Skinner will remain an important historical figure. He had a lasting effect on the study of learning and conditioning in the laboratory, and his analyses of the ways in which our own behaviour is affected (‘controlled’ in his preferred terminology) by contingencies of reinforcement contain many valuable and valid insights. Perhaps he applied them with too much enthusiasm and insufficient caution to the problems of education or psychopathology; but it is often only those who overstate and simplify their case that have any influence on their fellows.

N. J. Mackintosh

*N. J. Mackintosh is in the Department of Experimental Psychology, University of Cambridge, Downing Street, Cambridge CB2 3EB, UK.*

scaled the field to zero. This is one of the places where Moffatt uses ideas from fluid mechanics to guide him.

The final step is to realize that there may be more than one configuration with minimal energy. The constructions start with a particular representation of the knot and if the knot is sufficiently complicated it is probable that different representations will give different configurations with minimal energy. The energy spectrum of the knot  $K$  is the sequence of numbers  $0 \leq m_0 \leq m_1 \leq m_2 \dots$  given by the energy of the various configurations with minimal energy. The actual minimum energy  $m_0$ , which on its own is insufficient to determine the knot, is the ground-state energy.

M. H. Freedman and Z.-X. He reach similar conclusions from a different, topological starting point (*Topology*, in the press). They show that if the knot has a configuration with sufficiently small energy then the knot is actually unknotted no matter how complicated it might appear to be. This result has the intuitive interpretation that it takes a lot of energy

to make fluid flow around a very complicated knot. It has the same flavour as an earlier result of J. W. Milnor (*Ann. Math.* 52, 248–257; 1950), who proved that if the total  $K$  is sufficiently small then  $K$  is unknotted. This tells us that a very complicated knot is highly curved. Freedman and He also relate the energy of the knot to other topological invariants, in particular the so-called crossing number of the knot.

The whole procedure may be reversed; the topology of knots and links may be used to obtain lower bounds on the energy of fluid flows. This is an example where topological properties, which are normally thought to be rather qualitative, force numerical constraints. Moffatt finishes his article by hinting that in the physical contexts he mentions at the beginning of his article, his energy spectrum plays an important role. But like a writer of serialized fiction, he leaves us hanging on for more. □

John D. S. Jones is in the Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK.

## TECTONICS

# The fate of subducting slabs

Cliff Frohlich and Stephen P. Grand

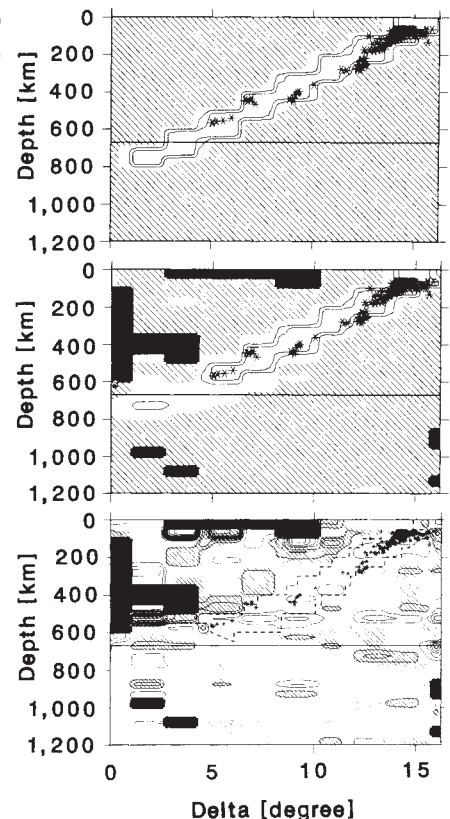
CONVERGING lithospheric plates, it is generally accepted, meet at deep ocean trenches, where one plate subducts beneath another and sinks into the mantle, accompanied by volcanism and deep earthquake activity. But once within the mantle, where does the subducted plate go? Is it absorbed by the mantle within the uppermost 670 km, the region possessing all the earthquake activity? Or does it penetrate far beneath this depth, perhaps even to the core–mantle boundary, almost 3,000 km below the surface? The questions are simple, but the answers are still a matter of controversy. Three new papers by Hua-Wei Zhou and colleagues approach them in two very different ways: residual-sphere analysis, an evaluation of anomalous travel times of individual deep earthquakes<sup>1</sup>; and tomographic inversion of catalogued earthquake travel times to delineate seismic velocity beneath subduction zones<sup>2,3</sup>. Although neither approach is new, the papers are significant because the authors use more data than previous studies, and because they are more careful to show how individual steps in each analysis affect the results.

In residual-sphere analysis, the seismologist first determines whether phases from a single deep earthquake arrive late or early at individual seismograph stations. Mapped onto a hypothetical sphere surrounding the earthquake, the data include a band of points representing phases that

have travelled through the subducting slab. The further the slab penetrates beneath the earthquake's focus, the earlier these phases arrive at the stations.

This is simple in concept but the method allows many opportunities for introducing systematic errors into the results. First, shallow near-station or deep-mantle structures produce travel-time anomalies as large as or larger than the near-source anomalies of interest. Second, many catalogued travel times are inaccurate because phases are often weak or misread. Third, travel times themselves are uncertain, as the location of the earthquake is often poorly known and affected systematically by the very structure of interest. And last, data sampling on the residual sphere is always incomplete, and often very few phases travel in the right directions to test for penetration.

The best-known previous studies, by Creager and Jordan<sup>4,5</sup>, handled these difficulties by such methods as correcting for near-station structure and by statistical 'smoothing' to avoid bad or misleading data. Nevertheless, many geophysicists doubt the outcome. Zhou *et al.*<sup>1</sup> now construct residual spheres for no fewer than 145 deep earthquakes in the northwest Pacific. For each of 11 of these, they present figures of 10 different residual spheres showing smoothed and unsmoothed versions of the observed data as well as predicted residual spheres for four different models of mantle structure. This



Synthetic data, real tomography: how well does tomography resolve structure in the mantle? The top cross-section depicts a synthetic mantle model with higher-than-average velocity (white areas, contours are 1 per cent) in the rectangular regions that lie along a hypothetical subducting slab. If one uses this model to determine travel times at nearby seismic stations from the earthquakes (\*) shown, a tomographic inversion produces the cross-section at the centre (cross-hatched areas are slow, dotted areas within 0.5 per cent of average, and black areas are unresolved by the procedure). Note how the high-velocity regions are resolved best at depths where there are earthquakes, and less well at greater depths. The bottom cross-section shows the model found by applying the same tomographic inversion method to real data beneath Japan. (Modified from ref. 2.)

allows readers to decide for themselves how well the data fit the models. For the models chosen, the authors conclude that the evidence for slab penetration is not compelling, as lower-mantle and near-station structure can account for much (but not all) of the deep-slab-like signal.

But even if the hypothesized slab signal from individual earthquakes is weak, is it somehow possible to combine the signals from all the available travel-time data? This is the approach of travel-time tomography. For earthquakes in the northwest Pacific occurring between 1964 and 1982, Zhou and Clayton<sup>2</sup> analyse more than 130,000 primary (longitudinal) and 56,000 secondary (shear) arrivals selected from the catalogue of the International Seismological Centre. The tomographic procedure divides the upper 1,200 km of the